STATISTICS

Paper II

(CONVENTIONAL)

Time allowed : Three Hours | *Maximum Marks : 200*

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. 1 and 5 are compulsory. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections **A** and **B**.*

Attempts of questions shall be counted in chronological order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Answer Book must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question / part is indicated against it.

*Answer must be written in **ENGLISH** only.*

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

SECTION A

Q.1. Answer the following : *8×5=40*

(a) Define a linear programming problem (LPP). What is meant by basic solution and feasible solution to a LPP ?

Show that the optimal solution to a LPP corresponds to one of the extreme points of the convex set generated by the set of basic feasible solutions to a LPP. *8*

(b) Describe a CUSUM control chart for variables and compare it with the traditional Shewhart control chart. What is a V-mask ? *8*

(c) Why is the use of artificial variables necessary sometimes to solve a LPP ? Outline the M-technique, using artificial variables. *8*

(d) Given a failure censored sample $x_1 < x_2 < \dots < x_r$ from $N(\mu, \sigma^2)$ distribution and the number n of items put to test ($r < n$), derive the MLE of reliability function when both μ and σ^2 are unknown. *8*

(e) Define the OC curve of a sampling inspection plan. How does it differ from the OC curve of a control chart ? What is an 'ideal' OC curve for sampling inspection ? *8*

Q.2. (a) Explain the concept of dominance while solving two-person zero-sum games. In this context, define the mixed strategies for the players.

Solve the game specified by the following pay-off matrix : (B pays A)

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 9 & 3 & 1 & 8 & 0 \\ 6 & 5 & 4 & 6 & 7 \\ 2 & 4 & 3 & 3 & 8 \\ 5 & 6 & 2 & 2 & 1 \end{bmatrix} \end{array}$$

15

(b) State and prove the Chapman – Kolmogorov equation for higher order transition probabilities. Explain the utility of this basic result. *10*

(c) Define the terms :

(i) Reliability function

(ii) Hazard function

(iii) Reliability of a series system

(iv) Reliability of a parallel system

The mean life of a component is 100 hrs.

How many components will be needed, assuming a constant hazard model, in order to build a parallel system with mean life of 200 hrs ? *15*

Q.3. (a) Sixteen boxes of electronic switches, each containing 20 units, were randomly selected and inspected. The result is summarized below :

Box No.	1	2	3	4	5	6	7	8
No. of defects	12	15	9	14	18	26	8	6

Box No.	9	10	11	12	13	14	15	16
No. of defects	11	12	16	13	19	18	14	21

Compute the 3-sigma control limits for a chart that is appropriate here. Justify your choice of the chart and also draw suitable conclusions, after plotting the chart. *15*

(b) Define the renewal density and renewal function of a renewal process.

If N_t denotes the number of renewals upto time t and S_{N_t} the waiting time for N_t, then prove that

$$P\left[\lim_{t\to\infty} \frac{N_t}{t}\right] = \mu$$

where μ denotes the average time between renewals. *10*

(c) Distinguish between group and item replacement situations.

A computer has 20,000 resistors. When a resistor fails, it is replaced. The cost of replacing a resistor is ₹ 1. If all the resistors are replaced at the same time, the cost per resistor is reduced to ₹ 0·40. The percentage surviving at the end of month t and the probability of failure during the month are recorded below :

Month t :	0	1	2	3	4	5	6
Percentage surviving :	100	96	90	65	35	20	0
Probability of failure :	—	0·04	0·06	0·25	0·30	0·15	0·20

Derive an optimum replacement plan, showing the steps clearly. *15*

Q.4. (a) Explain redundancy in reliability studies. Evaluate the reliability function of a maintained system with Hot Standby Redundancy when redundant unit fails with the same rate. *15*

(b) Define an M/G/1 queuing system and explain in brief the characteristics of such a system.

A group of engineers has two terminals to help in their computations. The average computing job needs 20 minutes of terminal time and each engineer requires some computations, about once in every half an hour. Assuming exponential distribution for inter-service time and the group to have six engineers, compute the average number of engineers waiting to use a terminal. Clearly show the working. *10*

(c) Mention the role of software in statistical computation, with special mention of SPSS. What are the modules available in the current version of SPSS ? State the limitations of this software in relation to STATA and SYSTAT. *15*

SECTION B

Q.5. Answer the following : *8×5=40*

(a) Define Fisher's 'Ideal' index number for prices. Show that it satisfies Time and Factor reversal tests. *8*

(b) Outline Leslie's matrix method for population projection. *8*

(c) Explain auto-correlation in time series data. Describe Durbin – Watson test procedure. *8*

(d) How is validity of test scores determined in psychometry ? Illustrate. *8*

(e) Describe King's method for constructing an abridged life table. What are the applications of life tables ? *8*

Q.6. (a) Define the classical linear regression model, stating the assumptions. If the disturbances are independently and normally distributed, show that OLS and ML methods lead to identical estimators for regression coefficients. *15*

(b) Define a consumer price index number and state its uses. Outline any two standard methods for computing this index and compare them. *10*

(c) Write explanatory notes on

(i) Functions of the NSSO

(ii) Secular trend measurement

(iii) ARIMA models *15*

Q.7. (a) Explain identification problem in a system of simultaneous equations. Derive the rank and order conditions for identifiability. Are these both sufficient conditions ? *15*

(b) Describe the heteroscedasticity problem and its consequences in econometric studies. Outline any two methods to tackle this problem. *15*

(c) Describe a logistic model and its features. Examine its suitability for human population growth analysis. *10*

Q.8. (a) Describe the stable population model. Show that in a stationary population the birth rate is the reciprocal of the expectation of life at birth. What is an age-sex pyramid ? How is this useful ? *15*

(b) Explain as to how one can use the abridged life-table to construct a complete life-table. Interpret the columns of the resulting table. *10*

(c) Write explanatory notes on

(i) Box – Jenkins method

(ii) Periodogram analysis

(iii) Multicollinearity in econometric analysis *15*