Indian Forest Service Examination-2013

वियोज्य DETACHABLE

STATISTICS
Paper I

Time Allowed : Three Hours | Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions.

There are EIGHT questions in all out of which, FIVE are to be attempted.

Question no. **1** & **5** are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B.

Attempts of questions shall be counted in chronological order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the answer book must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Answers must be written in ENGLISH only.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

SECTION 'A'

1. Answer the following :

1.(a) A particular test for diabetes, when administered to people who have diabetes, can detect the disease correctly in 95% of the cases. When the test is administered to people who do NOT have diabetes, the test shows 'false positive' in 2% of the cases.

Assuming that 3% of the population has diabetes, answer the following :

1.(a) (i) What is the probability that a randomly selected person will be declared to be diabetic by the test ?

1.(a) (ii) If a person tests positive for diabetes, what is the probability that he is actually diabetic ? 8

1.(b) Two fair dice are thrown. If X is the sum of the numbers shown up, use Chebychev's inequality to get an upper bound for $P\{|X-7| \geqslant 3\}$. Also obtain the exact value of this probability. 8

1.(c) The distribution function of a bivariate random vector (X, Y) is

$$F(x, y) = \left(1-e^{-x}\right)\left(1-e^{-y}\right)\left[1+\alpha e^{-(x+y)}\right] \text{ for } x, y > 0 \text{ and } |\alpha| < 1.$$

1.(c) (i) Compute the marginal densities of X and Y.

1.(c) (ii) For what values of α are X and Y independent ? 8

1.(d) A random sample of size n is drawn from a distribution having p.d.f.

$f(x, \theta) = \theta \exp(-\theta x),\ x \geq 0, \theta > 0.$

1.(d) (i) What is the maximum likelihood estimator for $\frac{1}{\theta}$?

1.(d) (ii) Is this estimator consistent ? 8

1.(e) Let $X_1, X_2, ..., X_n$ be i.i.d. with each X_i being a Bernoulli (θ) random variable. Obtain the likelihood ratio test for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$. 8

2.(a) Let (X, Y) be a random vector.

2.(a) (i) Show that $E(XY) = E(X)\, E(Y)$ whenever X and Y are independent

2.(a) (ii) Give an example to show that the converse of (i) above in false 10

2.(b) Give an example to show that convergence in probability *does not* imply convergence almost surely. 10

2.(c) A random variable has characteristic function $\phi(t) = e^{-|t|}$. What is its density function ? 10

2.(d) The transition probability matrix of a Markov chain with state space {0, 1, 2, 3} is

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Is the Markov chain irreducible ? If the Markov chain starts at state 0, what is the probability that it is at state 3 after two units of time ? 10

3.(a) Give an example to show that a sufficient statistic need not be complete. 10

3.(b) Let $X_1, X_2, ..., X_n$ be i.i.d. Poisson (λ) random variables. Suppose the prior distribution of λ is Gamma (α, β). What is its posterior distribution ? 10

3.(c) Assume $X_1, X_2, ..., X_n$ is a random sample from a Poisson (λ) distribution. Examine whether the sample mean is UMVUE for λ. 10

3.(d) Let T be an unbiased estimator for θ. Is it always true that T^2 is NOT an unbiased estimator of θ^2 ? 10

4.(a) Let $X_1, X_2, ..., X_n$ be i.i.d. random variables from a $N(\mu, \sigma^2)$ population. Obtain the $100(1-\alpha)\%$ confidence interval

4.(a) (i) for μ when σ^2 is known

4.(a) (ii) for σ^2 when μ is known 10

4.(b) Suppose $X_1, X_2, ..., X_n$ are i.i.d. random variables following the uniform distribution on the interval $(\theta, \theta+1)$, $-\infty<\theta<\infty$.

Show that $T(X)=(X_{(1)}, X_{(n)})$ where $X_{(1)} = \min\{X_1,...,X_n\}$ and $X_{(n)} = \max\{X_1,...,X_n\}$ is a minimal sufficient statistic. 10

4.(c) The lifetime (in hours) of a sample of size 6 each of two different brands of batteries are given in the table below :

Brand A :	40	30	40	45	55	30
Brand B :	50	50	45	55	60	40

Using the Kolmogorov-Smirnov test statistic, examine whether the brands are different with respect to their lifetime distribution. $\left(D_{6,6,\cdot05} = \frac{2}{3}\right)$ 10

4.(d) Assume that the population follow a Poisson law with parameter λ, derive the SPRT for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1$ $(\lambda_1 > \lambda_0)$. 10

SECTION 'B'

5. Answer the following :

5.(a) Consider a Gauss-Markov linear model $(X, A\theta, \sigma^2 I)$, where $X = (X_1, X_2, X_3)'$, $\theta = (\theta_1, \theta_2, \theta_3)'$ and A (3×3) and $E(X_1) = \theta_1 + \theta_2 + \theta_3$, $E(X_2) = \theta_1 + \theta_3$, $E(X_3) = \theta_2$. Obtain a necessary and sufficient condition for estimability of the linear parametric function $l_1\theta_1 + l_2\theta_2 + l_3\theta_3$. In the case of estimability, obtain the BLUE and also the variance of the BLUE. 8

5.(b) Define Hotelling's T^2 statistic. Discuss in detail, the applications of the statistic in various testing problems. 8

5.(c) Describe the linear systematic sampling procedure. Obtain the variance of the sample mean in the presence of linear trend in a population of size $N(= n \times k)$ where n and k are positive integers. 8

5.(d) Two independent random samples of sizes n_1 and n_2 are drawn from two independent normal populations, $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ where σ^2 is common but unknown. Explain how would you construct a 95% confidence interval for the ratio $\frac{\mu_1}{\mu_2}$ (assume $\mu_2 \neq 0$). 8

5.(e) Write short notes on the following :

5.(e) (i) Discriminant Analysis

5.(e) (ii) Confounding in factorial experiments 8

6.(a) In a Gauss-Markov linear model, show that any solution to normal equations, minimizes the error sum of squares. Illustrate the result through a small numerical example. 10

6.(b) In cluster sampling, obtain an unbiased estimator of the population mean when the clusters (of unequal sizes) are selected using SRS. Compute also the variance of the estimator. 10

6.(c) In two stage sampling, obtain an unbiased estimator of the population mean if SRSWR is used at both stages. Also derive the variance of the estimator. 10

6.(d) Using missing plot technique, estimate the missing observation in a RBD. Carry out the analysis by including the estimated observation. 10

7.(a) Derive the characteristic function of a p-variate normal random vector. Hence obtain the first two moments of the random vector. 10

7.(b) Define stratified sampling. When is it useful ? Obtain the sizes of samples from various strata under optimum allocation. Find the corresponding variance of the sample mean. 10

7.(c) Discuss when the ratio estimator of the population mean under SRS is preferred to the conventional unbiased estimator. 10

7.(d) What is a factorial design ? Briefly sketch the analysis of a 2^3 factorial design. 10

8.(a) What are principal components ? Mention its uses. Explain how the principal components can be extracted from a dispersion matrix. 10

8.(b) If clusters are selected using SRSWR from a population of K clusters of size M each, then obtain the variance of an unbiased estimator of the population mean in terms of the population intraclass correlation coefficient. 10

8.(c) Define a BIBD. Give a layout of such a design and outline its analysis. 10

8.(d) Write short notes on the following :

8.(d) (i) Two stage sampling and two phase sampling – their merits and demerits.

8.(d) (ii) Hotelling's T^2 and Mahalanobis D^2 – their connections and applications. 10
