I. F.S. EXAM-2015

C-GEQ-O-NBUB

MATHEMATICS
Paper – II

Time Allowed : Three Hours

Maximum Marks : 200

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. **1** and **5** are compulsory. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

*Answers must be written in **ENGLISH** only.*

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

SECTION A

Q1. (a) If in a group G there is an element a of order 360, what is the order of a^{220} ? Show that if G is a cyclic group of order n and m divides n, then G has a subgroup of order m. *10*

(b) Let $\sum_{n=1}^{\infty} a_n$ be an absolutely convergent series of real numbers.

Suppose $\sum_{n=1}^{\infty} a_{2n} = \frac{9}{8}$ and $\sum_{n=0}^{\infty} a_{2n+1} = \frac{-3}{8}$. What is $\sum_{n=1}^{\infty} a_n$?

Justify your answer. (Majority of marks is for the correct justification). *8*

(c) Let u(x, y) = cos x sinh y. Find the harmonic conjugate v(x, y) of u and express u(x, y) + i v(x, y) as a function of z = x + iy. *12*

(d) Solve graphically :

Maximize $z = 7x + 4y$

subject to $2x + y \leq 2$, $x + 10y \leq 10$ and $x \leq 8$.

(Draw your own graph without graph paper). *10*

Q2. (a) If p is a prime number and e a positive integer, what are the elements 'a' in the ring $\mathbb{Z}_{p^e}$ of integers modulo p^e such that $a^2 = a$? Hence (or otherwise) determine the elements in $\mathbb{Z}_{35}$ such that $a^2 = a$. *14*

(b) Let $X = (a, b]$. Construct a continuous function $f : X \rightarrow \mathbb{R}$ (set of real numbers) which is unbounded and not uniformly continuous on X. Would your function be uniformly continuous on $[a + \varepsilon, b]$, $a + \varepsilon < b$? Why ? *14*

(c) Evaluate the integral $\int_r \frac{z^2}{(z^2 + 1)(z - 1)^2} \, dz$, where r is the circle $|z| = 2$. *12*

Q3. (a) What is the maximum possible order of a permutation in S_8, the group of permutations on the eight numbers {1, 2, 3, ... , 8} ? Justify your answer. (Majority of marks will be given for the justification). *13*

(b) Let $f_n(x) = \frac{x}{1 + nx^2}$ for all real x. Show that f_n converges uniformly to a function f. What is f ? Show that for $x \neq 0$, $f_n'(x) \rightarrow f'(x)$ but $f_n'(0)$ does not converge to $f'(0)$. Show that the maximum value $|f_n(x)|$ can take is $\frac{1}{2\sqrt{n}}$. *13*

(c) A manufacturer wants to maximise his daily output of bulbs which are made by two processes P_1 and P_2. If x_1 is the output by process P_1 and x_2 is the output by process P_2, then the total labour hours is given by $2x_1 + 3x_2$ and this cannot exceed 130, the total machine time is given by $3x_1 + 8x_2$ which cannot exceed 300 and the total raw material is given by $4x_1 + 2x_2$ and this cannot exceed 140. What should x_1 and x_2 be so that the total output $x_1 + x_2$ is maximum ? Solve by the simplex method only. *14*

Q4. (a) Compute the double integral which will give the area of the region between the y-axis, the circle $(x-2)^2 + (y-4)^2 = z^2$ and the parabola $2y = x^2$. Compute the integral and find the area. *15*

(b) Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$ by using contour integration and the residue theorem. *15*

(c) Solve the following transportation problem : *10*

	D_1	D_2	D_3	Supply
O_1	5	3	6	20
O_2	4	7	9	40
Demand	15	22	23	60

SECTION B

Q5. (a) Store the value of -1 in hexadecimal in a 32-bit computer. 10

(b) Show that $\sum_{k=1}^{n} l_k(x) = 1$, where $l_k(x)$, $k = 1$ to n, are Lagrange's fundamental polynomials. 10

(c) Derive the Hamiltonian and equation of motion for a simple pendulum. 10

(d) Find the solution of the equation $u_{xx} - 3u_{xy} + u_{yy} = \sin(x - 2y)$. 10

Q6. (a) Solve the following system of linear equations correct to two places by Gauss-Seidel method :

$$x + 4y + z = -1, \; 3x - y + z = 6, \; x + y + 2z = 4.$$ 16

(b) Solve the differential equation $u_x^2 - u_y^2$ by variable separation method. 12

(c) In a steady fluid flow, the velocity components are $u = 2kx$, $v = 2ky$ and $w = -4kz$. Find the equation of a streamline passing through $(1, 0, 1)$. 12

Q7. (a) Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \; 0 < x < 1, \; t > 0$$

subject to the conditions $u(0, t) = u(1, t) = 0$ for $t > 0$ and $u(x, 0) = \sin \pi x$, $0 < x < 1$. 14

(b) Find the moment of inertia of a uniform mass M of a square shape with each side a about its one of the diagonals. 12

(c) Use the classical fourth order Runge-Kutta methods to find solutions at $x = 0{\cdot}1$ and $x = 0{\cdot}2$ of the differential equation $\frac{dy}{dx} = x + y$, $y(0) = 1$ with step size $h = 0{\cdot}1$. 14

Q8. (a) Write a BASIC program to compute the product of two matrices. 12

(b) Suppose $\vec{v} = (x - 4y)\hat{i} + (4x - y)\hat{j}$ represents a velocity field of an incompressible and irrotational flow. Find the stream function of the flow. 12

(c) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ for a string of length l fixed at both ends. The string is given initially a triangular deflection

$$u(x, 0) = \begin{cases} \frac{2}{l} x, & \text{if } 0 < x < \frac{l}{2} \\ \frac{2}{l}(l - x), & \text{if } \frac{l}{2} \leq x < l \end{cases}$$

with initial velocity $u_t(x, 0) = 0$. 16