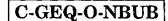


I.F.S. EXAM-2015





MATHEMATICS Paper - II

Time Allowed: Three Hours

Maximum Marks: 200

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions:

There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.

Questions no. 1 and 5 are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Answers must be written in **ENGLISH** only.

Unless otherwise mentioned, symbols and notations have their usual standard meanings. Assume suitable data, if necessary, and indicate the same clearly.

SECTION A

- Q1. (a) If in a group G there is an element a of order 360, what is the order of a^{220} ? Show that if G is a cyclic group of order n and m divides n, then G has a subgroup of order m.
 - (b) Let $\sum_{n=1}^{\infty} a_n$ be an absolutely convergent series of real numbers.

Suppose
$$\sum_{n=1}^{\infty} a_{2n} = \frac{9}{8}$$
 and $\sum_{n=0}^{\infty} a_{2n+1} = \frac{-3}{8}$. What is $\sum_{n=1}^{\infty} a_n$?

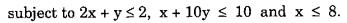
Justify your answer. (Majority of marks is for the correct justification).

(c) Let $u(x, y) = \cos x \sinh y$. Find the harmonic conjugate v(x, y) of u and express u(x, y) + i v(x, y) as a function of z = x + iy.



(d) Solve graphically:

Maximize z = 7x + 4y



(Draw your own graph without graph paper).



10

- If p is a prime number and e a positive integer, what are the elements 'a' **Q2.** (a) in the ring \mathbb{Z}_{p^e} of integers modulo p^e such that a^2 = a ? Hence (or otherwise) determine the elements in \mathbb{Z}_{35} such that $a^2 = a$. 14
 - Let X = (a, b]. Construct a continuous function $f: X \to \mathbb{R}$ (set of real (b) numbers) which is unbounded and not uniformly continuous on X. Would your function be uniformly continuous on $[a + \varepsilon, b]$, $a + \varepsilon < b$? 14 Why?
 - $\int \frac{z^2}{(z^2+1)(z-1)^2} dz, \quad \text{where r is the circle}$ Evaluate the integral (c) 12 |z| = 2.
- What is the maximum possible order of a permutation in S₈, the group Q3. (a) of permutations on the eight numbers {1, 2, 3, ..., 8}? Justify your 13 answer. (Majority of marks will be given for the justification).
 - Let $f_n(x) = \frac{x}{1 + nx^2}$ for all real x. Show that f_n converges uniformly to a (b) function f. What is f? Show that for $x \neq 0$, $f'_n(x) \rightarrow f'(x)$ but $f'_n(0)$ does not converge to f'(0). Show that the maximum value $|f_n(x)|$ can take is $\frac{1}{2\sqrt{n}}$. 13
 - A manufacturer wants to maximise his daily output of bulbs which are (c) made by two processes P_1 and P_2 . If x_1 is the output by process P_1 and x_2 is the output by process P_2 , then the total labour hours is given by $2x_1 + 3x_2$ and this cannot exceed 130, the total machine time is given by $3x_1 + 8x_2$ which cannot exceed 300 and the total raw material is given by $4x_1 + 2x_2$ and this cannot exceed 140. What should x_1 and x_2 be so that the total output $x_1 + x_2$ is maximum? Solve by the simplex method only. 14





Compute the double integral which will give the area of the region between the y-axis, the circle $(x-2)^2 + (y-4)^2 = z^2$ and the parabola $2y = x^2$. Compute the integral and find the area.

15

(b) Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$ by using contour integration and the residue theorem.

15

(c) Solve the following transportation problem:

10

	D_1	D_2	D^3	Supply
O ₁	5	3	6	20
O_2	4	7	9	40
Demand	15	22	23	60



SECTION B

Q5. (a) Store the value of -1 in hexadecimal in a 32-bit computer.



(b) Show that $\sum_{k=1}^{n} l_k(x) = 1$, where $l_k(x)$, k = 1 to n, are Lagrange's

fundamental polynomials.

10

- (c) Derive the Hamiltonian and equation of motion for a simple pendulum. 10
- (d) Find the solution of the equation $u_{xx} 3u_{xy} + u_{yy} = \sin(x 2y)$.

10

Q6. (a) Solve the following system of linear equations correct to two places by Gauss-Seidel method:

x + 4y + z = -1, 3x - y + z = 6, x + y + 2z = 4.

16

- (b) Solve the differential equation $u_x^2 u_y^2$ by variable separation method. 12
- (c) In a steady fluid flow, the velocity components are u = 2kx, v = 2ky and w = -4kz. Find the equation of a streamline passing through (1, 0, 1).
- Q7. (a) Solve the heat equation

$$\frac{\partial \mathbf{u}}{\partial \mathbf{t}} = \frac{\partial^2 \mathbf{u}}{\partial \mathbf{x}^2}, \ 0 < \mathbf{x} < 1, \ \mathbf{t} > 0$$

subject to the conditions u(0, t) = u(1, t) = 0 for t > 0 and $u(x, 0) = \sin \pi x$, 0 < x < 1.

14

- (b) Find the moment of inertia of a uniform mass M of a square shape with each side a about its one of the diagonals.

 12
 - Use the classical fourth order Runge-Kutta methods to find solutions at x = 0.1 and x = 0.2 of the differential equation $\frac{dy}{dx} = x + y$, y(0) = 1 with

step size h = 0.1.

14

Q8. (a) Write a BASIC program to compute the product of two matrices.

(b) Suppose $\overrightarrow{v} = (x - 4y)\hat{i} + (4x - y)\hat{j}$ represents a velocity field of an incompressible and irrotational flow. Find the stream function of the flow.

12

- (c) Solve the wave equation $\frac{\partial^2 \mathbf{u}}{\partial \mathbf{t}^2} = \mathbf{c}^2 \frac{\partial^2 \mathbf{u}}{\partial \mathbf{x}^2}$ for a string of length l fixed at both ends. The string is given initially a triangular deflection
 - $\mathbf{u}(\mathbf{x},\,0) = \begin{cases} \frac{2}{l}\,\mathbf{x}, & \text{if } 0 < \mathbf{x} < \frac{l}{2} \\ \frac{2}{l}\,(l-\mathbf{x}), & \text{if } \frac{l}{2} \leq \,\mathbf{x} < l \end{cases} \quad \text{with initial velocity} \quad \mathbf{u}_{\mathbf{t}}(\mathbf{x},\,0) = 0. \qquad 16$

(c)