Indian Forest Services (Main)
Examination-2025

DJSM–B–STSC

STATISTICS

Paper – II

*Time Allowed : **Three** Hours*

*Maximum Marks : **200***

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

*Answers must be written in **ENGLISH** only.*

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

SECTION-A

Q1. (a) Define hazard function. Obtain the survival function of the following hazard function :

$$h(t) = \frac{1}{(\alpha_0 + \alpha_1 t)(\beta_0 + \beta_1 t)} \, ; \; t > 0, \;\; \alpha_0, \beta, \alpha_1, \beta_1 > 0.$$ *8*

(b) The diameter of one end of a drive-shaft is required within 1140 ± 10. The control charts for $\overline{\overline{X}}$ and R-charts are initiated. After 30 sub-groups of 5 shafts each have been examined,

$$\Sigma \overline{X} = 34{,}290 \;\text{ and }\; \Sigma R = 330$$

(i) Establish the mean μ' and standard deviation σ' of the process assuming that it is in statistical control.

(ii) Determine the 3-sigma limits of $\overline{X}$ and R-charts.

(iii) Determine the natural specification limits of the process.

(Given that $d_2 = 2{\cdot}326$, $A_2 = 0{\cdot}58$, $D_3 = 0$ and $D_4 = 2{\cdot}11$) *3+3+2=8*

(c) Four teachers A, B, C and D are capable of teaching any of the four different courses a, b, c and d. The course preparation time (in hours) taken by four teachers are given in the table below. Each teacher is assigned only one course. How are the courses assigned to teachers so as to minimize the total course preparation hours ? *8*

Teachers \ Courses	a	b	c	d
A	7	15	14	12
B	20	9	19	13
C	18	19	21	16
D	9	20	18	14

(d) Two players A and B, without showing to each other, put their coin on the table. A wins ₹ 8 when both coins show head and ₹ 1 when both coins show tail. B wins ₹ 3 if upper faces of the coins do not match.

(i) Write pay-off matrix for both players.

(ii) Determine optimum strategies for both players and the value of the game. 8

(e) A woman buys three type of oils, A, B and C. She never buys the same oil on successive weeks. If she buys type A oil one week, then the next week she buys type B oil. However, if she buys either type B or type C oil, then the next week she is three times as likely to buy type A oil as the other types. Obtain transition probability matrix and determine steady-state probabilities of buying three types of oils. 8

Q2. (a) (i) Explain and bring out the distinction between Acceptance Quality Level (AQL) and Average Outgoing Quality Limit (AOQL).

(ii) Describe the method of double sampling plan. Derive its Operating Characteristic (OC), Average Sample Number (ASN) and Average Total Inspection (ATI). 15

(b) Suppose that a given individual in a population has a survival time which is exponential with hazard rate θ. Each individual's hazard rate θ is potentially different and is sampled from a gamma distribution with density function :

$$f(\theta) = \frac{\lambda^{\beta}\, \theta^{\beta-1}\, e^{-\lambda\theta}}{\Gamma(\beta)}, \quad \begin{matrix} \theta > 0, \\ \lambda, \beta > 0 \end{matrix}$$

Let X be the life length of a randomly chosen member of this population :

(i) Find the survival function of X.

(ii) Find the hazard rate of X. What is the rough shape of the hazard function ? 10

(c) Explain parallel systems and their reliability function. Consider a system, consisting of n components such that the failure of the i^{th} component occurs in accordance with a Poisson process of fixed intensity. Compute the reliability and expected life of the system under parallel system structures. 15

Q3. (a) Solve the following transportation problem in which cell entries represent unit costs :

	W_1	W_2	W_3	W_4	Available
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Requirement	5	8	7	14	

Also, find minimum transportation cost. 15

(b) Use simplex procedure to solve the following linear programming problem :

Maximize $(z) = 2x_1 - 3x_2 + x_3$

subject to constraints :

$$3x_1 + 6x_2 + x_3 \leq 6$$

$$4x_1 + 2x_2 + x_3 \leq 4$$

$$x_1 - x_2 + x_3 \leq 3$$

$$x_1, x_2, x_3 \geq 0.$$ 15

(c) An item used in the manufacture of surgical equipment has an annual usage of 24,000 units, which costs ₹ 1·25 per unit. Placing each order costs ₹ 22·50 and the carrying cost is 5·4% per year of the average inventory.

Find the economic lot size and inventory cost (including material cost).

Should the company accept the offer made by the supplier of a discount of 5% on the cost price, on a single order of 24,000 units ? 10

Q4. (a) (i) A random sample of size n is drawn from a Normal Population $N(\mu' + k\sigma', \sigma'^2)$. Then show that

$$p_n = P\left[\text{Sample mean} > \mu' + 3\frac{\sigma'}{\sqrt{n}}\right]$$

$$= 1 - \Phi(3 - k\sqrt{n})$$

where $\Phi(\cdot)$ is the c.d.f. of Standard Normal Distribution.

(ii) If r^{th} sample mean is the first to exceed $\left(\mu' + 3\frac{\sigma'}{\sqrt{n}}\right)$, then show that $E(r) = \frac{1}{p_n}$. *15*

(b) Determine the optimal sequence of performing 5 jobs on 4 machines. For each job machines are required in the order of A, B, C and D and the process timings are as follows :

	Machine			
Job	A	B	C	D
1	8	3	4	7
2	9	2	5	5
3	6	4	5	8
4	12	5	1	9
5	7	1	2	3

Also, find minimum total elapsed time. *15*

(c) Explain censored and truncated experiment. Describe life testing censored and truncated experiments for exponential models. *10*

SECTION B

Q5. (a) What is stationary time series ? Explain Augmented Dickey-Fuller test and interpretation of the test results. 8

(b) Explain the utility of index number of industrial production. Describe in detail, a method of its construction. 8

(c) Describe the functions of Central Statistical Organisation. 8

(d) Describe the different methods of scaling of test scores in several tests, stating the necessary assumptions in each method. 8

(e) Why is population projection important for development ? Describe the mathematical method and component method for population projection. 8

Q6. (a) Discuss the identifiability of the following model assuming

$$\sum = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix}$$

$$y_1 + \gamma_{11} x_1 = u_1$$

$$\beta_{21} y_1 + y_2 + \gamma_{21} x_1 = u_2$$

$$\beta_{31} y_1 + \beta_{32} y_2 + y_3 + \gamma_{31} x_1 = u_3.$$ 15

(b) (i) Explain Dimensional Invariance test in the case of price and quantity. 5

(ii) Calculate Fisher's Ideal Index Number from the following data and verify whether it satisfies the mathematical tests for formula error. 10

Commodity	Base Year		Current Year	
	Price per unit	Expenditure (₹)	Price per unit	Expenditure (₹)
A	2	40	5	75
B	4	16	8	40
C	1	10	2	24
D	5	25	10	60

(c) Discuss the collection of trade statistics in India with special reference to inland and internal trade of India. 10

Q7. (a) What do you mean by measure of morbidity ? Explain the problems that occur while morbidity rates are constructed.

Define Morbidity Incidence Rate (MIR) and Morbidity Prevalence Rate (MPR). When would MPR approximate MIR ? *15*

(b) Find the equivalence curve of y and x if both x and y have exponential distributions with parameters θ_1 and θ_2, respectively.

Further, if x and y are both normally distributed with means μ_x and μ_y and variances σ_x^2 and σ_y^2 respectively, then show that the equivalence curve method reduces to z scaling. *15*

(c) The following table gives the mean and standard deviation of scores and the raw scores of two students in three examinations A_1, A_2 and A_3. Compare the overall performance of two students. Write your conclusion. *10*

Examination	Mean	S.D.	Raw score of student I	Raw score of student II
A_1	150	25·4	190	160
A_2	35	10·2	30	55
A_3	95	13·5	140	110

Q8. (a) Consider the following data on monthly expenditures for clothing and on incomes of 10 families.

Assume that the variance of the disturbance term is proportional to the square of expenditure on clothing. Obtain the approximate transformation so as to make the resulting disturbance term homoscedastic. Compare the regression equation of the transformed data with that of the original data. *15*

Expenditure on clothing (₹) :	20	25	30	40	45	50	55	60	65
Income (₹) :	1500	2000	3000	3500	4500	4500	5500	6000	5500

(b) The l_x column of a certain life table for ages 0, 1, 2, 3, ..., 100 is given by the series 100, 99, 98, 97, ..., 0. In usual notations, calculate the following values :

$$e_0^0, T_5, L_7, q_{10}, p_{20}, e_{25} \text{ and } d_{70}.$$ *15*

(c) Define various measures of fertility. Compute GRR and NRR for the following data : 10

Age	Female population (100)	Total no. of female live births (100)	Survival rate (per 100,000)
15 – 19	157680	4642	58056
20 – 24	147524	14433	55880
25 – 29	124300	14068	52971
30 – 34	105875	8429	48953
35 – 39	89365	4046	44164
40 – 44	77797	2168	39145
45 – 49	61171	698	34189