Indian Forest Services (Main) Examination-2025

DJSM–U–STSC

STATISTICS

PAPER—I

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are EIGHT questions in all, out of which FIVE are to be attempted.

Question Nos. **1** and **5** are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

Answers must be written in ENGLISH only.

SECTION—A

1. *(a)* There are sometimes errors in transmission of coded messages. Morse code uses 'dots' and 'dashes' which generally happen in the proportion 3 : 4. Assume that there is interference on transmission line and a received dot is mistakenly taken as a dash with probability $\frac{1}{6}$ and vice versa. Find *(i)* the probability that a dot has been received and *(ii)* the probability that a dot was sent given that dot was received. 4+4=8

(b) *(i)* Check whether

$$F(x) = \begin{cases} \dfrac{e^x}{2}, & x < 0 \\ \dfrac{1}{2}, & 0 \le x < 1 \\ 1 - \dfrac{e^{1-x}}{2}, & x \ge 1 \end{cases}$$

is the cumulative distribution function of a random variable X.

(ii) For $i = 1, 2, ..., n$, let X_i's be independent and identically distributed as $N(\mu, \sigma^2)$. Show that weak law of large numbers holds. 4+4=8

(c) Let $X_1, X_2, ..., X_n$ be a random sample from $U(0, \theta)$, $\theta \in \Theta = (0, \infty)$. Let $X_{(n)} = \max\{X_1, X_2, ..., X_n\}$ and $Y_n = \frac{2}{n}\sum_{i=1}^{n} X_i$.

(i) Show that $X_{(n)}$ is consistent estimator of θ.

(ii) Check the estimator Y_n for consistency and unbiasedness for the parameter θ. 4+4=8

(d) Let $X_1, X_2, ..., X_n$ be a random sample from exponential distribution with mean θ.

(i) Obtain the maximum likelihood estimator of θ.

(ii) Obtain the asymptotic distribution of the maximum likelihood estimator that you have obtained in *(i)* above. 4+4=8

(e) Let X be a random variable whose p.m.f. under H_0 and H_1 is given by

x	1	2	3	4	5	6	7
$f(x \mid H_0)$	0·01	0·01	0·01	0·01	0·01	0·01	0·94
$f(x \mid H_1)$	0·06	0·05	0·04	0·03	0·02	0·01	0·79

(i) Find the most powerful test for H_0 versus H_1 with size 0·04. Compute the probability of Type II error for this test.

(ii) Write down the most powerful test for H_0 versus H_1 with size 0·10. Compute the power of your test. 4+4=8

2. *(a)* Let

$$f(x) = \begin{cases} \dfrac{x+1}{2}, & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

be the probability density function (p.d.f.) of a continuous random variable X. Let $Y = X^2$. Find *(i)* the probability density function of Y and check that it is a valid p.d.f., *(ii)* $E(Y)$ and *(iii)* $V(Y)$. 4+3+3=10

(b) *(i)* Let $\{X_n\}$ be a sequence of random variables such that

$$P(X_n = 0) = 1 - \left(\frac{1}{2}\right)^n \text{ and}$$

$$P(X_n = 1) = \left(\frac{1}{2}\right)^n, \; n = 1, 2, \ldots$$

Then check whether $X_n \to X$ in probability, almost everywhere and in second mean, where X is a random variable taking value '0' with probability 1.

(ii) State Glivenko-Cantelli theorem.

(iii) Let $\{A_n\}$ be an independent sequence of events such that

$$\sum_{n=1}^{\infty} P(A_n) = \infty$$

Then show that $P\left(\limsup_{n} A_n\right) = P(A_n \text{ i.o.}) = 1.$ 6+4+5=15

(c) *(i)* Let $X_1, X_2, \ldots, X_n$ be a random sample from normal distribution with mean θ and variance σ^2, where σ^2 is known. Obtain the uniformly most powerful size α test for testing $H_0 : \theta \le \theta_0$ versus $H_1 : \theta > \theta_0$. Using the test, write down $(1-\alpha)$ level one-sided confidence interval for θ.

(ii) State the purpose of Wilcoxon-Mann-Whitney test and write down the test statistic. State the asymptotic distribution of the test statistic. 8+7=15

3. *(a)* For two continuous random variables X and Y, the joint probability density function of (X, Y) is given as

$$f(x, y) = \begin{cases} c(x+2y), & 0 < x < 2, \; 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find *(i)* the value of c, *(ii)* the marginal probability density functions (p.d.f.) of X and Y, *(iii)* the joint cumulative distribution function (c.d.f.) of X and Y, *(iv)* $\text{cov}(X, Y)$ (covariance of X and Y) and *(v)* the correlation between X and Y, and comment upon the type of relationship. 2+4+4+4+6=20

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from Poisson distribution with mean λ. Suppose prior distribution of λ over $(0, \infty)$ is given by $\pi(\lambda) = \begin{cases} e^{-\lambda}, & \lambda > 0 \\ 0, & \text{otherwise} \end{cases}$.

Find the Bayes estimator of λ under quadratic error loss function. Is the prior distribution conjugate? Justify your answer. 10

(c) A random sample of size 10 from a continuous distribution over [0, 1] is as below :

0·73, 0·25, 0·48, 0·68, 0·79, 0·89, 0·61, 0·96, 0·54, 0·41

Check whether the data were randomly drawn from uniform distribution over [0, 1], using Kolmogorov-Smirnov goodness of fit test. (The critical value for $\alpha = 0{\cdot}05$ is 0·409) 10

4. (a) (i) Let X and Y be discrete random variables with the probability mass function given as

		X = 1	X = 2	X = 3
Y	2	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$
	3	$\frac{1}{6}$	0	$\frac{1}{6}$
	4	0	$\frac{1}{3}$	0

Check whether X and Y are independent random variables.

(ii) Let X follow binomial distribution with parameters n and p. Find the characteristic function (c.f.) of X. Using the c.f., find $E(X)$ and $V(X)$. 5+5=10

(b) (i) Let $\{X_n \mid n \geq 0\}$ be a Markov Chain (MC) with state space {0, 1, 2}. The transition probability matrix (t.p.m.) is given as

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Check whether the given MC is irreducible. Find the period of each state. Identify the recurrent states. 1+2+2=5

(ii) Let X be the number of ice blocks in a drink with probability mass function (p.m.f.)

x	3	4	5	6
$P(X = x)$	0·4	0·3	0·1	0·2

For a sample of 49 drinks, find $P(\overline{X} < 4 \cdot 5)$.

(Refer the tables enclosed) 5

STANDARD NORMAL DISTRIBUTION

Table Values Represent AREA to the LEFT of the Z score

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997

STANDARD NORMAL DISTRIBUTION

Table Values Represent AREA to the LEFT of the Z score

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
–3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003
–3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
–3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
–3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
–3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
–3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
–3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
–3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
–3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
–3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
–2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
–2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
–2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
–2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
–2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
–2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
–2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
–2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
–2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
–2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
–1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
–1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
–1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
–1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
–1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
–1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
–1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08691	0.08534	0.08379	0.08226
–1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
–1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
–1.0	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
–0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
–0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
–0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
–0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
–0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
–0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
–0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
–0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
–0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
–0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414

(c) Define one-parameter exponential family of distributions and verify whether the following distributions belong to the one-parameter exponential family of distributions :

(i) $P(X = x|\lambda) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!(1-e^{-\lambda})}; & x = 1, 2, \ldots \\ & \lambda > 0 \\ 0 \quad ; & \text{otherwise} \end{cases}$

(ii) $f_X(x|\mu) = \begin{cases} \dfrac{1}{2}e^{-(x-\mu)/2}; & x > \mu \\ 0 \quad ; & \text{otherwise} \end{cases}$

(iii) $f_X(x|\theta) = \begin{cases} 1 & \text{if } \theta < x < \theta + 1 \\ 0 \quad ; & \text{otherwise} \end{cases}$

(iv) $f_X(x|\sigma) = \begin{cases} \dfrac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}(x-5)^2/\sigma^2}; & -\infty < x < \infty \\ & \sigma > 0 \\ 0 \quad ; & \text{otherwise} \end{cases}$

5+5+5+5=20

SECTION—B

5. (a) Compute the Mahalanobis distance of a point (3, 5) from the sample mean vector $\begin{bmatrix} 4 \\ 6\cdot 4 \end{bmatrix}$ with the sample variance matrix $\begin{bmatrix} 2\cdot 5 & 3\cdot 25 \\ 3\cdot 25 & 4\cdot 7 \end{bmatrix}$. 8

(b) For the purpose of simple linear regression, the following data set (X, Y) is taken :

X	8	6	9	10	12
Y	10	5	18	7	10

Study the sensitivity of the outlier (9, 18) in the given data and interpret. 8

(c) A sample of 50 students is to be drawn from a population of 500 students from two colleges A and B. The means and standard deviations of their scores are given below :

College	*Total number of students*	*Mean*	*Standard deviation*
A	300	40	10
B	200	50	40

How would you draw the samples using proportional allocation? Hence, obtain the variance of estimate of the population mean and compare its efficiency with simple random sampling without replacement. 8

(d) The following data were obtained from an experiment involving four tropical feeds A, B, C and D tried on 20 chicks. All the 20 chicks were treated alike in all respects except the feeding treatments :

Feed	*Gain in weight* (in gm)				
A	55	49	42	21	52
B	61	112	30	89	63
C	42	97	81	95	92
D	169	137	169	85	154

Test whether all the feeds are equally effective at $\alpha = 0 \cdot 05$ level of significance. (Refer the table enclosed at the end) 8

(e) Suppose the random variables X_1, X_2, X_3 have covariance matrix

$$\Sigma = \begin{bmatrix} 2 & 0 \cdot 5 & -1 \\ 0 \cdot 5 & 1 & 0 \cdot 5 \\ -1 & 0 \cdot 5 & 2 \end{bmatrix}$$

Find the first principal component vector and its variance. 8

6. (a) Let X_1, X_2, X_3 be uncorrelated random variables having common variance σ^2, and $E(X_1) = \theta_1 + 2\theta_2$, $E(X_2) = 2\theta_1 + \theta_2$ and $E(X_3) = 2\theta_1 + 4\theta_2$.

(i) Obtain the normal equations and the least square estimates $\hat{\theta}_1$ and $\hat{\theta}_2$.

(ii) Find $V(\hat{\theta}_1)$, $V(\hat{\theta}_2)$ and $\text{cov}(\hat{\theta}_1, \hat{\theta}_2)$. 15

(b) (i) Let X be a vector random variable with the following covariance matrix :

$$\begin{bmatrix} 1 & \theta & \frac{\theta}{2} \\ \theta & 1 & -\frac{\theta}{2} \\ \frac{\theta}{2} & -\frac{\theta}{2} & 1 \end{bmatrix}$$

$$Y_1 = X_1 + 2X_2 + X_3 \text{ and } Y_2 = 3X_2 - X_3$$

For what value of θ are Y_1 and Y_2 uncorrelated? Also, find the variance of Y_1. 5

(ii) X and Y are random vectors with $\text{cov}(X) = \Sigma_{11}$, $\text{cov}(Y) = \Sigma_{22}$ and $\text{cov}(X, Y) = \Sigma$ such that

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} = \begin{bmatrix} 1 & 0\cdot 4 & 0\cdot 5 & 0\cdot 6 \\ 0\cdot 4 & 1 & 0\cdot 3 & 0\cdot 4 \\ 0\cdot 5 & 0\cdot 3 & 1 & 0\cdot 2 \\ 0\cdot 6 & 0\cdot 4 & 0\cdot 2 & 1 \end{bmatrix}$$

with $\Sigma_{11}^{-\frac{1}{2}} = \begin{bmatrix} 1\cdot 0681 & -0\cdot 2229 \\ -0\cdot 2229 & 1\cdot 0681 \end{bmatrix}$ and $\Sigma_{22}^{-1} = \begin{bmatrix} 1\cdot 0417 & -0\cdot 2083 \\ -0\cdot 2083 & 1\cdot 0417 \end{bmatrix}$. Find the first canonical correlation. 5

(c) Suppose X is a p-variate multivariate normal random vector and Y is also a p-variate multivariate normal random vector. X and Y are independent with $E(X) = \mu_x$ and $E(Y) = \mu_y$ and with $V(X) = V(Y) = \Sigma$. If n_1 observations are obtained from X and n_2 observations are obtained from Y, derive the test statistic for testing $H_0 : \mu_x = \mu_y$ against $H_1 : \mu_x \neq \mu_y$. Mention the distribution of the test statistic under null hypothesis. 15

7. (a) Find out the missing entries denoted by * in the following ANOVA table of a Latin square design :

Source of variation	*d.f.*	*Sum of squares* (SS)	*Mean sum of squares*	F_{cal}
Rows	3	259·54	*	*
Columns	*	*	*	*
Treatments	*	1372·12	*	*
Error	6	*	26·06	
Total	*	1943·07		

Stating the hypotheses, give your conclusions at $\alpha = 0\cdot 05$ level of significance. Also, calculate the relative efficiency of the above design (LSD) with respect to RBD and CRD.
(Refer the table enclosed at the end) 15

(b) (i) Show that systematic sampling is more precise than simple random sampling if the variance within the systematic samples is more than the total variation in the population. 10

(ii) When do we make use of ratio estimator? Also, state the conditions under which the ratio estimate is optimum. 5

(c) Differentiate between partial and complete confounding, and write the ANOVA of a partially confounded 2^3-experiment. 10

8. *(a)* The following is a sample of (X, Y, Z) of size 5 :

$$(2, 6, 3), (3, 6, 4), (4, 8, 7), (5, 10, 9), (6, 10, 7)$$

Find the partial correlation $r_{xy \cdot z}$ and multiple correlation coefficient $R_{y \cdot xz}$. 15

(b) The surface finish of products produced in a machine shop is suspected to be affected by a factor 'Operator' and another factor 'Shift'. The data of this experiment with two replications in different treatment combinations are given below :

		Shift (B)	
		1	*2*
Operator (A)	*1*	65	20
		70	40
	2	30	50
		35	40

Test whether Operators, Shifts or their joint effects are significantly different at $\alpha = 0 \cdot 05$.
(Refer the table enclosed at the end) 10

(c) *(i)* In which situations is cluster sampling preferred? Obtain the variance of sample mean of simple random sample (without replacement) of n clusters from a population of N clusters each having M elements. 10

(ii) What are non-sampling errors? Write in brief about the measures to control these errors. 5

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F Distribution Table

	$\alpha = 0.05$																		
	d.f.N.																		
d.f.D.	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***12***	***15***	***20***	***24***	***30***	***40***	***60***	***120***	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.52	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00