

STATISTICS

PAPER—I

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

**Please read each of the following instructions carefully
before attempting questions**

There are EIGHT questions in all, out of which FIVE are to be attempted.

Question Nos. 1 and 5 are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Answers must be written in ENGLISH only.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

SECTION—A

1. (a) The lifetime of a mobile charger (in hours) has the normal distribution with mean $(\mu) = 100$ and variance $(\sigma^2) = 400$.

- (i) What is the probability that the mobile charger lasts at least 125 hours?
(ii) If the mobile charger has already lasted for 105 hours, what is the conditional probability that it will last another 20 hours?

(Normal Distribution Table is given in Page Nos. 10 and 11)

4+4=8

- (b) (i) State Lindeberg condition for non-identically distributed independent variables to hold central limit theorem (CLT).

4

- (ii) Examine whether CLT holds for the sequence $\{X_n\}$, where

$$P\left\{X_n = \pm\left(\frac{1}{2^n}\right)\right\} = \frac{1}{2}$$

4

- (c) Suppose $X_1, X_2, \dots, X_n$ are independently identically distributed (iid) observations from a location parameter family with cumulative distribution function $F(x - \theta)$, $-\infty < \theta < \infty$. Show that $R = X_{(n)} - X_{(1)}$ is ancillary statistic, where $X_{(n)} = \max_i \{X_i\}$ and $X_{(1)} = \min_i \{X_i\}$.

8

- (d) Consider the problem of testing $H_0 : \theta = 1$ versus $H_1 : \theta = \frac{1}{2}$, where θ is the mean of a Poisson random variable. Let X and Y be a random sample from Poisson (θ) distribution. Consider the following test procedure :

Reject H_0 if $X = 1$ or $(Y = 1 \text{ and } X + Y \leq 2)$, otherwise accept H_0

Determine the probability of type I and type II errors.

8

- (e) In an ecological study of the feeding behaviour of birds, the number of hops between flights is counted for several birds :

No. of hops	Observed frequency
1	48
2	31
3	20
4	9
5	6
6	5
7	4
8	2
9	1
10	1
11	2
12	1
Total	130

Assuming that the data are generated by a geometric (p) model and take a uniform prior for p , what is the posterior distribution of parameter p ? What are the mean and the standard deviation of the posterior distribution?

4+2+2=8

2. (a) (i) Suppose that the transition probability matrix of a Markov chain model is given by

$$P = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$$

Compute n -step transition probability matrix (P^n).

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- (ii) A random variable X has mean (μ) = 40 and standard deviation (σ) = 2. Apply Chebyshev's inequality to estimate $P(25 \leq X \leq 55)$.

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- (b) Let $X_1, X_2, X_3, \dots$ be a sequence of independent random variables, where the probability mass function (PMF) of the random variables $X_n, n = 1, 2, 3, \dots$ is given by

$$P\left(X_n = \pm \frac{\sqrt{n}}{2}\right) = \frac{1}{2}$$

Does the law of large numbers hold for this sequence of random variables? 10

- (c) (i) Let X follow binomial $b(4, \theta)$ distribution, $0 < \theta < 1$. To test the hypothesis $H_0 : \frac{1}{3} \leq \theta \leq \frac{1}{2}$ versus $H_1 : \theta < \frac{1}{3}$ or $\theta > \frac{1}{2}$ with size 0.3, the test function is based on the following procedure :

Reject H_0 with probability γ_1 , if $X = 1$

Reject H_0 with probability γ_2 , if $X = 3$

Determine the constants γ_1 and γ_2 . Compute power of the test when $\theta = 0.2$.

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- (ii) Let $X_1, X_2, \dots, X_n$ be independently identically distributed (iid) random variables from exponential distribution with mean $\theta > 0$. Define

$$g(X) = \frac{X_n}{X_1 + X_2 + \dots + X_n}$$

Find $E_\theta[g(X)]$.

5

3. (a) Two random variables X and Y have the following joint probability density function :

$$f(x, y) = \begin{cases} k(4 - x - y); & 0 < x < 2, 0 < y < 2 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Find the following :

- (i) Constant k
- (ii) Marginal density function of X and Y
- (iii) $E(X|Y = 1)$
- (iv) $V(X|Y = 1)$
- (v) $\text{cov}(X, Y)$

20

- (b) A study is run to evaluate the effectiveness of a physical exercise in reducing systolic blood pressure (SBP) in 15 patients. SBPs are measured before joining the physical exercise and after six weeks of joining the physical exercise. The data are shown below :

Patient	SBP before exercise	SBP after exercise
1	127	120
2	134	136
3	140	132
4	132	134
5	129	128
6	130	137
7	122	126
8	127	107
9	129	132
10	138	142
11	137	128
12	134	130
13	141	134
14	138	132
15	133	125

Use Wilcoxon signed-rank test to test the difference in SBP after participating in physical exercise as compared to before at 5% level of significance.

[Given that $W_{(15, 0.05)} = 25$, $W_{(14, 0.05)} = 21$ for two-tailed test]

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- (c) (i) A 24-hour advance prediction of a day's high temperature is 'unbiased' if the long-term average of the error in prediction (true high temperature minus predicted high temperature) is zero. The errors in predictions (x) made by one meteorological station for 20 randomly selected days were recorded. The results were

$$\sum_{i=1}^{20} x_i = -15, \quad \sum_{i=1}^{20} x_i^2 = 35$$

Assume normal distribution of errors and test the null hypothesis that the predictions are unbiased versus the alternative that they are biased at 1% level of significance. Verify whether the decision would be the same at 5% and 10% level of significance.

[Given that $t_{(19, 0.005)} = 2.861$, $t_{(19, 0.025)} = 2.093$, $t_{(19, 0.05)} = 1.729$, where $P(t_n > t_{(n, \alpha)}) = \alpha$] 3+1+1=5

- (ii) Distinguish between Wald's SPRT and the test based on Neyman-Pearson theory to test simple null versus simple alternative hypotheses.

5

4. (a) Let X and Y be independent Poisson variables with $V(X + Y) = 9$ and

$$P(X = 3 | X + Y = 6) = \frac{5}{54}$$

Obtain $E(Y)$.

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- (b) Suppose X and Y are independent identically distributed as exponential variates with mean = 1. Obtain the characteristic function of X and $X - Y$. Hence, deduce the distribution of $Z = X - Y$.

10

- (c) Suppose that X is a discrete random variable with

$$P(X = 0) = \frac{2\theta}{3}, P(X = 1) = \frac{\theta}{3}, P(X = 2) = \frac{2(1-\theta)}{3} \text{ and } P(X = 3) = \frac{1-\theta}{3}$$

where $\theta \in [0, 1]$ is the parameter. The following 10 independent observations were taken from such distribution :

$$\{3, 0, 2, 1, 3, 2, 1, 0, 2, 1\}$$

- (i) Find the moment estimate of θ .
(ii) Find the approximate standard error for your estimate.
(iii) What is the maximum likelihood estimate (MLE) of θ ?
(iv) What is the approximate standard error of the MLE of θ ? 5+5+5+5=20

SECTION—B

5. (a) For the model $Y = \beta_0 + \beta_1 X + \varepsilon$, $\varepsilon \sim N(0, \sigma^2)$, a regression line is fitted on the basis of n paired observations $((x_i, y_i), i = 1, \dots, n)$. The fitted line is $y = b_0 + b_1 x$, where

$$\text{var}(b_0) = \frac{\sigma^2 \sum_{i=1}^n x_i^2}{\sum_{i=1}^n (x_i - \bar{x})^2}, \quad \text{var}(b_1) = \frac{\sigma^2}{\sum_{i=1}^n (x_i - \bar{x})^2} \quad \text{and} \quad \text{cov}(b_0, b_1) = -\frac{\sigma^2 \bar{x}}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

For a given $x = x_0$, one can predict the value of a new observation Y_0 as $\hat{Y}_0 = b_0 + b_1 x_0$. Find an expression for $\text{var}(\hat{Y}_0 - Y_0)$ and compare it with $\text{var}(\hat{Y}_0)$.

Find c_n , the standard deviation of $\frac{\hat{Y}_0 - Y_0}{\sigma}$.

8

- (b) Consider a p -dimensional random vector $\underline{X}$ with mean $\underline{\mu}$ and variance-covariance matrix Σ . Let $(\underline{u}_1, \dots, \underline{u}_p)$ be an orthonormal system of eigenvectors of Σ with corresponding eigenvalues $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_p$. Denote $U = (\underline{u}_1, \dots, \underline{u}_p)$. Let $\underline{Y} = (Y_1, \dots, Y_p)'$ be the vector of principal components of X . Then for any pair $(i, j) \in \{1, \dots, p\}$, show that

$$\text{cov}(X_i, Y_j) = u_{ij} \lambda_j$$

$$\rho(X_i, Y_j) = \frac{u_{ij} \sqrt{\lambda_j}}{\sqrt{\text{var}(X_i)}}$$

where $u_{ij} = (u_j)_i = (i, j)\text{th element of } U$.

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- (c) From the following population based on SRSWOR scheme with $n = 3$, verify that sample mean $\bar{y}$ and sample variance s^2 are unbiased for population mean $\bar{Y}$ and variance S^2 . Also show that the sampling variance of $\bar{y}$ agrees with the

expression for $V(\bar{y}) = \frac{N-n}{N} \cdot \frac{S^2}{n}$:

i	Y_i
1	5
2	8
3	3
4	11
5	9

8

- (d) Define main effects and interaction effects in a 2^2 factorial experiment. Consider an experiment with two factors, reactant concentration (A) and catalyst (B). Let the two levels of factor A be 15 percent (a_0) and 25 percent (a_1) concentration. The two levels of factor B are 2 pounds (b_1) and 1 pound (b_0) of the catalyst. The order in which the runs are made is random. The data obtained are as follows :

Treatment combination	Replication			Total
	I	II	III	
$a_0 b_0$	28	25	27	80
$a_1 b_0$	36	32	32	100
$a_0 b_1$	18	19	23	60
$a_1 b_1$	31	30	29	90

Obtain the best estimates of main effects and interaction effects. Test for the significance of main effects and interaction effects. Set up the ANOVA table.

8

(e) Observations $Y_1, \dots, Y_n$ are described by the model $Y_i = \beta x_i^2 + \varepsilon_i$, where $x_1, \dots, x_n$ are fixed constants and $\varepsilon_1, \dots, \varepsilon_n$ are iid Normal $(0, \sigma^2)$. Find the following :

(i) Least squares estimate of β

(ii) Maximum likelihood estimate of β

(iii) Best unbiased estimate of β

8

6. (a) The sample mean vector and covariance matrix, computed on the basis of a random sample of size 20 from a bivariate normal population $N_2(\mu, \Sigma)$, are given below :

$$\bar{x} = \begin{pmatrix} 10 \\ 20 \end{pmatrix}, \quad S = \begin{pmatrix} 40 & -50 \\ -50 & 100 \end{pmatrix}$$

(i) Evaluate the test statistic for testing $H_0 : \mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} = \begin{pmatrix} 7 \\ 11 \end{pmatrix}$.

(ii) Test H_0 at 5% level of significance. What conclusion can be reached?

(iii) Find the simultaneous confidence interval for μ_1 .

$$\begin{aligned} (F_{1, 19, 0.05} = 4.38 \quad F_{1, 19, 0.01} = 8.18 \\ F_{2, 18, 0.05} = 3.55 \quad F_{2, 18, 0.01} = 6.01) \end{aligned}$$

15

(b) What is orthogonal factor model in factor analysis? Write down an orthogonal factor model with m common factors together with the covariance structure.

Consider three standardized random variables Z_1, Z_2 and Z_3 with a single factor ($m = 1$) :

$$\begin{aligned} Z_1 &= 0.9F_1 + \varepsilon_1 \\ Z_2 &= 0.7F_1 + \varepsilon_2 \\ Z_3 &= 0.5F_1 + \varepsilon_3 \end{aligned}$$

where $\text{var}(F_1) = 1$, $\text{cov}(\varepsilon, F_1) = 0$ and

$$\Psi = \text{cov}(\varepsilon) = \begin{pmatrix} 0.19 & 0 & 0 \\ 0 & 0.51 & 0 \\ 0 & 0 & 0.75 \end{pmatrix}$$

Obtain the covariance matrix.

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(c) Suppose an analyst uses the model $y_i = \beta_0^* + \beta_1^* x_i + \varepsilon_i^*$ instead of the true model $y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 + \varepsilon_i$ with $\text{cov}(\underline{y}) = \sigma^2 I$, $\underline{y} = (y_1, \dots, y_7)'$.

(i) Obtain $E(\hat{\beta}_0^*)$ and $E(\hat{\beta}_1^*)$ (in terms of true model parameters), if the observations are taken at $x = -3, -2, -1, 0, 1, 2, 3$. Note here that $\hat{\beta}_0^*$ and $\hat{\beta}_1^*$ are the least square estimates of β_0^* and β_1^* .

(ii) Find $E(s_1^2)$ for the same set of values of x , where s_1^2 is the sample variance for the true model.

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7. (a) (i) For the following design, write the C-matrix (information matrix of the design) and obtain independent estimable treatment contrasts :

$$\begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 2 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 4 \\ 1 \\ 1 \end{pmatrix}$$

The number denotes the treatments. Also examine whether it is a connected design.

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- (ii) Define confounding. Suppose that $2^4 = 16$ treatments cannot be run using one batch of raw materials. The experimenter can run eight treatment combinations from a single batch of materials. A 2^4 confounded in two blocks is run. Construct a design with two blocks of eight observations each with ABCD confounded.

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- (b) (i) Describe the probability proportional to size sampling with replacement scheme. Obtain an unbiased estimator for the population total along with its sampling variance under this scheme.

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- (ii) Estimate the gain in efficiency due to PPS sampling compared to SRS sampling based on SRS sample.

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- (c) In sampling with unequal probabilities, without replacement a sample of size 2 is drawn. The first unit is drawn with PPS and the second unit with PPS of remaining units. Show that Yates, Grundy and Sen's variance estimator is always positive for this sampling system.

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8. (a) (i) Let $\alpha'\alpha$ is a vector of $(v - t)$ independent estimable treatment contrasts from an incomplete block design. Obtain the test statistic for testing the hypothesis $H_0 : \alpha'\alpha = 0$.

7

- (ii) Obtain the best estimate of treatment effect and derive the test statistic for testing the linear hypothesis of equality of treatment effects given yield from a balanced incomplete block design.

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- (b) Describe two-stage sampling. Obtain the estimator of population total when SRSWOR is used at both the stages.

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- (c) Show that under the usual assumptions for one-way ANOVA model $y_{ij} = \mu_i + \varepsilon_{ij}$, $j = 1, \dots, n_i$, $i = 1, \dots, k$ with $E(\varepsilon_{ij}) = 0$, $\text{var}(\varepsilon_{ij}) = \sigma^2$, $\text{cov}(\varepsilon_{ij}, \varepsilon_{i'j'}) = 0$, $i \neq i'$, $j \neq j'$, ε_{ij} 's are iid $N(0, \sigma^2) \forall i, j$

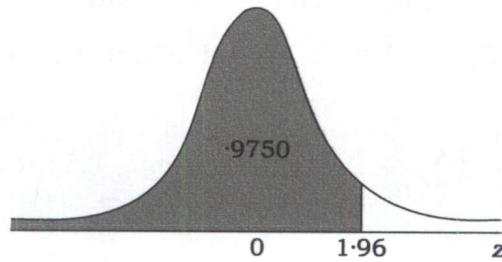
$$P \left[\sum_{i=1}^k c_i \bar{y}_{i.} - A_0 \sqrt{S_p^2 \sum_{i=1}^k \frac{c_i^2}{n_i}} \leq \sum_{i=1}^k c_i \mu_i \leq \sum_{i=1}^k c_i \bar{y}_{i.} + A_0 \sqrt{S_p^2 \sum_{i=1}^k \frac{c_i^2}{n_i}} \right] = 1 - \alpha$$

simultaneously for all $c = (c_1, \dots, c_k)$, where

$$A_0 = \sqrt{KF_{k, \sum_{i=1}^k (n_i - 1), \alpha}}, \quad S_p^2 = \frac{1}{\sum_{i=1}^k (n_i - 1)} \sum_{i=1}^k \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.})^2$$

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Normal Distribution Table



z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	0.00	z
-3.80	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	-3.80
-3.70	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	-3.70
-3.60	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002	-3.60
-3.50	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	-3.50
-3.40	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	-3.40
-3.30	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005	-3.30
-3.20	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007	-3.20
-3.10	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010	-3.10
-3.00	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013	-3.00
-2.90	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019	-2.90
-2.80	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026	-2.80
-2.70	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035	-2.70
-2.60	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047	-2.60
-2.50	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062	-2.50
-2.40	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082	-2.40
-2.30	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107	-2.30
-2.20	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139	-2.20
-2.10	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179	-2.10
-2.00	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228	-2.00
-1.90	.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287	-1.90
-1.80	.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359	-1.80
-1.70	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446	-1.70
-1.60	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548	-1.60
-1.50	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668	-1.50
-1.40	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808	-1.40
-1.30	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968	-1.30
-1.20	.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151	-1.20
-1.10	.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357	-1.10
-1.00	.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587	-1.00
-0.90	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841	-0.90
-0.80	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119	-0.80
-0.70	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.2420	-0.70
-0.60	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743	-0.60
-0.50	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085	-0.50
-0.40	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446	-0.40
-0.30	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821	-0.30
-0.20	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207	-0.20
-0.10	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602	-0.10
0.00	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000	0.00

Normal Distribution Table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.00	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	0.00
0.10	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	0.10
0.20	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	0.20
0.30	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	0.30
0.40	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	0.40
0.50	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	0.50
0.60	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	0.60
0.70	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852	0.70
0.80	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	0.80
0.90	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	0.90
1.00	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	1.00
1.10	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	1.10
1.20	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	1.20
1.30	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	1.30
1.40	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1.40
1.50	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1.50
1.60	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1.60
1.70	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1.70
1.80	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1.80
1.90	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1.90
2.00	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	2.00
2.10	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	2.10
2.20	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	2.20
2.30	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	2.30
2.40	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	2.40
2.50	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	2.50
2.60	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	2.60
2.70	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	2.70
2.80	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	2.80
2.90	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	2.90
3.00	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	3.00
3.10	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	3.10
3.20	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	3.20
3.30	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	3.30
3.40	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	3.40
3.50	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	3.50
3.60	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	3.60
3.70	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	3.70
3.80	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	3.80

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