

PHYSICS

PAPER—II

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

**Please read each of the following instructions carefully
before attempting questions**

There are EIGHT questions in all, out of which FIVE are to be attempted.

Question Nos. 1 and 5 are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Answers must be written in ENGLISH only.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

Neat sketches may be drawn, wherever required.

Useful Constants :

Mass of proton	=	1.673×10^{-27} kg
Mass of neutron	=	1.675×10^{-27} kg
Mass of electron	=	9.11×10^{-31} kg
Planck constant	=	6.626×10^{-34} J s
Boltzmann constant	=	1.380×10^{-23} J K ⁻¹
Bohr magneton (μ_B)	=	9.273×10^{-24} A m ²
Nuclear magneton (μ_N)	=	5.051×10^{-27} J T ⁻¹ (A m ²)
Electronic charge	=	1.602×10^{-19} C
Atomic mass unit (u)	=	1.660×10^{-27} kg
	=	931 MeV
g_s^P	=	$5.5855 \mu_N$
$m(n)$	=	1.00866 u
$m({}_{6}^{12}\text{C})$	=	12.00000 u
$m({}_{1}^2\text{H})$	=	2.014022 u
$m({}_{8}^{16}\text{O})$	=	15.999 u
\hbar	=	1.05×10^{-34} J s
$\hbar c$	=	197 eV nm
$m(p)$	=	1.00727 u
$m({}_{2}^4\text{He})$	=	4.002603 u
$m({}_{38}^{87}\text{Sr})$	=	86.908893 u
$m({}_{1}^3\text{H})$	=	3.0160500 u
ϵ_0	=	8.85×10^{-12} F m ⁻¹

SECTION—A

1. (a) Determine the size of the hydrogen atom using uncertainty principle. Given that the potential energy of electron $V = \frac{-e^2}{4\pi\epsilon_0 a}$, where a is the distance of the electron from the nucleus. 8
- (b) Calculate the group and phase velocities for the wave packet corresponding to a relativistic particle. 8
- (c) Calculate $[J_x^2, J_y]$, $[J_z^2, J_y]$ and $[J^2, J_y]$, and then show that $\langle J, m | J_x^2 | J, m \rangle = \langle J, m | J_y^2 | J, m \rangle$ 8
- (d) Calculate the possible angles between \vec{L} and \vec{S} for a d -electron in one-electron atom. 8
- (e) The force constant of the bond in CO molecule is 1900 N m^{-1} . Calculate the energy of the lowest vibrational level. The reduced mass of CO molecule is $1.14 \times 10^{-26} \text{ kg}$. Given $h = 6.63 \times 10^{-34} \text{ J s}$ and $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$. 8

2. (a) Consider a particle of mass m and charge q moving under the influence of a one-dimensional harmonic oscillator potential. Assume that it is placed in a constant electric field E . The Hamiltonian of this particle is therefore given by

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 - qEx$$

Derive the energy expression and wave function of the n th excited state. 15

- (b) Find the energy levels of a spin $S = \frac{3}{2}$ particle whose Hamiltonian is given by

$$H = \frac{\alpha}{\hbar^2} (S_x^2 + S_y^2 - 2S_z^2) - \frac{\beta}{\hbar} S_z$$

where α, β are constants. Are these levels degenerate? 10

- (c) Explain the formation of molecular hydrogen in the interstellar medium. 15

3. (a) A beam of hydrogen atoms in a Stern-Gerlach experiment obtained from an oven heated to a temperature of 400 K passes through a magnetic field of length 1 m and having a gradient of 10 T/m perpendicular to the beam. Calculate the transverse deflection of an atom of the beam at a point where the beam leaves the field. The value of Bohr magneton μ_B is $9.27 \times 10^{-24} \text{ A m}^2$ and the Boltzmann constant k is $1.38 \times 10^{-23} \text{ J/K}$. 15

- (b) (i) Describe in brief the Raman effect. 5
- (ii) In the vibrational Raman spectrum of HF, the Raman lines are observed at wavelengths 2670 Å and 3430 Å. Find the fundamental vibrational frequency of the molecule. 10
- (c) Discuss the rotational fine structure of electronic bands of molecule. 10

4. (a) Consider a particle of mass m moving in the potential

$$V(x) = \begin{cases} +\infty & ; x \leq 0 \\ \frac{1}{2} m\omega^2 x^2 & ; x > 0 \end{cases}$$

Estimate the ground state energy of this particle using the WKB method. 15

- (b) Consider a particle of mass m moving freely between $x=0$ and $x=a$ inside an infinite square well potential. Calculate the expectation values $\langle x \rangle_n$, $\langle p \rangle_n$, $\langle x^2 \rangle_n$ and $\langle p^2 \rangle_n$, and compare them with their classical counterparts. 15

- (c) Derive the degeneracy of a harmonic oscillator $g_n = \frac{1}{2}(n+1)(n+2)$. 10

SECTION—B

5. (a) A sample contains 4 mg of ^{210}Bi . If the half-life of radioactive ^{210}Bi is 5 days and the average energy of the β -particles emitted is 0.34 MeV, then at what rate does the sample emit energy? 8

- (b) In the case of deuterons, answer the following :

(i) What percentage of the time do the nucleons of a deuteron spend within the range of the nuclear forces? 4

(ii) From a study of the radial part of the Schrödinger equation of a deuteron for any angular momentum l , what do we conclude? 4

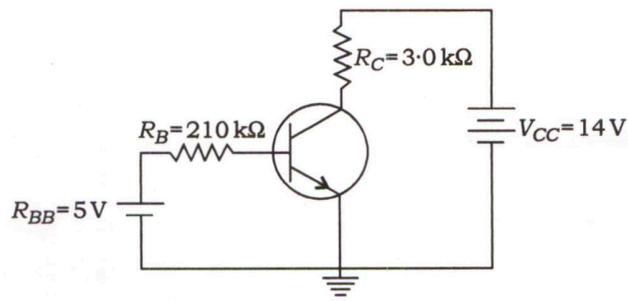
- (c) Write the decay reactions of the neutral pions (π^0) and the charged pions (π^+ , π^-). 8

- (d) Consider the planes with indices (1 0 0) and (0 0 1); the lattice is f.c.c., and the indices refer to the conventional cubic cell. What are the indices of these planes when referred to the primitive axes : $\vec{a}_1 = \frac{a}{2}(\hat{x} + \hat{y})$, $\vec{a}_2 = \frac{a}{2}(\hat{y} + \hat{z})$ and

$\vec{a}_3 = \frac{a}{2}(\hat{z} + \hat{x})$? 8

- (e) (i) Prove that $\overline{AB} + \overline{A} + AB = 0$. 4
- (ii) Draw a simplified logic circuit for the following Boolean expression : 4
- $$Y = (A + B)\overline{AB}$$
6. (a) A radionuclide (N_1) decays to a radioactive daughter (N_2) that subsequently decays to a third daughter (N_3). If at $t = 0$, the initial concentrations of N_1 , N_2 and N_3 are N_1^0 , 0 and 0 respectively, and the decay constants are λ_1 (from $N_1 \rightarrow N_2$) and λ_2 (from $N_2 \rightarrow N_3$), then derive the expression for N_2 at time t . Given that the daughter N_3 is stable. 15
- (b) (i) Apply the meson theory of nuclear forces to write the interaction between protons, neutrons, proton to neutron and neutron to proton. 4
- (ii) Assume that the range of the interaction of nuclear force and of meson is 2×10^{-15} m. Estimate the mass of the meson. 6
- (c) State the characteristics of an ideal Op-Amp. Explain the use of an Op-Amp as a summing amplifier and a differentiator. 15
7. (a) Calculate the speed and radius of α -particles of energy 5.998 MeV emitted into a magnetic field $B = 10^4$ Wb m⁻². Given that the mass of proton is 1.673×10^{-27} kg. 15
- (b) Given that for intrinsic Si at $T = 300$ K, the band gap is 1.1 eV and the intrinsic carrier concentration is $n_i = 1.5 \times 10^{10}$ cm⁻³.
- (i) Calculate the intrinsic carrier concentration at $T = 450$ K. Assume that the band gap at $T = 450$ K is 1.08 eV. 6
- (ii) Recalculate the intrinsic carrier concentration at $T = 450$ K by assuming that the effective electron and hole masses in the respective bands decrease by 0.5%. 4
- (c) State the Barkhausen criterion for sustained oscillations of a feedback circuit. Explain the operation of a Hartley oscillator using proper circuit diagram. 15
8. (a) What were the difficulties faced by the initial theory of β -decay? How did Pauli eliminate the difficulties? What were the expected properties of the new particle proposed by Pauli? 15
- (b) The density of a liquid monovalent metal near absolute zero temperature is given to be 0.081 g cm⁻³. Calculate the Fermi energy ϵ_F , the electron velocity v_F at the Fermi surface and the Fermi temperature T_F . 10

- (c) The $n-p-n$ transistor given below has $\beta = 100$, $I_{CO} = 12 \text{ nA}$ and $V_{BE} = 0.8 \text{ V}$. Determine the transistor currents and the region of operation of the transistor. What happens if R_C is indefinitely increased?



15

