Indian Forest Services (Main) Examination-2025

DJSM–B–MATH

MATHEMATICS

PAPER—II

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are EIGHT questions in all, out of which FIVE are to be attempted.

Question Nos. **1** and **5** are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Assume suitable data, if necessary, and indicate the same clearly.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Answers must be written in ENGLISH only.

SECTION—A

1. *(a)* If H and K are finite subgroups of a group and $HK = \{hk \mid h \in H,\ k \in K\}$, prove that $|HK| = \dfrac{|H||K|}{|H \cap K|}$. 8

(b) Let S be a non-empty subset of $\mathbb{R}$, bounded below, and $T = \{-x : x \in S\}$. Prove that the set T is bounded above and $\sup T = -\inf S$. 8

(c) Prove that the series $1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \ldots$ converges to $\frac{3}{2}\log 2$. 8

(d) Find the image of $|z| < 1$ under the bilinear transformation which maps $z = 1,\ i,\ -1$ onto $w = i,\ 0,\ -i$ respectively. 8

(e) The forest department aims to afforest up to 100 hectares with teak and pine to maximize CO_2 absorption, planting at least 10 hectares of each, and no more than 60 hectares of teak. The table below provides the CO_2 absorption and resource requirements :

Tree	CO_2 *Absorption* (tons/ha/year)	*Labour* (hours/ha/week)	*Water* (litres/ha/week)
Teak	20	40	200
Pine	15	20	150

The available resources are 3200 labour hours and 16000 litres of water per week. Formulate this as a linear programming problem. 8

2. *(a)* If p is an odd prime, prove that there is no group that has exactly p elements of order p. 10

(b) Let $f_n(x) = nx(1-x)^n$, $x \in [0, 1]$, $n \in \mathbb{N}$. Show that *(i)* the sequence $\{f_n\}$ converges to a function f on $[0, 1]$ and *(ii)* f is integrable on $[0, 1]$ and $\lim_{n \to \infty} \int_0^1 f_n = \int_0^1 f$, but still the convergence of the sequence $\{f_n\}$ is not uniform. 5+10

(c) Evaluate the integral $\int_0^\infty \sin(x^2)\,dx$ using the method of contour integration. 15

3. *(a)* Evaluate

$$\iint_E \frac{\sqrt{a^2b^2 - b^2x^2 - a^2y^2}}{\sqrt{a^2b^2 + b^2x^2 + a^2y^2}}\,dx\,dy$$

the field of integration E being the positive quadrant of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. 10

(b) Show that in the ring $\mathbb{Z} \times \mathbb{Z}$, (i) the ideal $S = \{(a, 0) : a \in \mathbb{Z}\}$ is a prime ideal but not maximal and (ii) the ideal $T = \{(m, n) \in \mathbb{Z} \times \mathbb{Z} : 3 | n\}$ is a maximal ideal. 10+5

(c) Solve the following linear programming problem by simplex method :

$$\text{Maximize } Z = 3x_1 + 2x_2 + 3x_3$$

subject to

$$x_1 + x_2 + x_3 \leq 4$$
$$2x_1 + x_2 + 3x_3 \leq 10$$
$$x_1 + 2x_2 + x_3 \leq 6$$
$$x_1,\ x_2,\ x_3 \geq 0$$

Is the solution unique? If not, find all the optimal solutions. 15

4. (a) Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0 \in \mathbb{Z}[x]$, and suppose that p is prime such that $p \nmid a_n$, $p | a_{n-1}, \ldots, p | a_0$ and $p^2 \nmid a_0$. Prove that $f(x)$ is irreducible over Q. 10

(b) Let $f(z) = \dfrac{z}{(z-1)(z+2)}$. Find the Laurent series expansion of $f(z)$ in the following regions :

(i) $1 < |z| < 2$

(ii) $|z| > 2$ 8+7

(c) A company has 5 workers A, B, C, D, E and 4 jobs I, II, III, IV. The profit (in rupees) that each worker earns from completing each job is given in the table below :

Worker \ Job	*I*	*II*	*III*	*IV*
A	17	20	18	21
B	19	16	17	20
C	16	15	20	18
D	18	17	15	16
E	14	19	13	15

Assign each job to exactly one worker in such a way that the profit is maximized. 15

SECTION—B

5. (a) Eliminate the arbitrary function F from the given equation $F\left(\dfrac{xy}{z}, \dfrac{x-y}{z}\right) = 0$ and find the corresponding partial differential equation. 8

(b) Find the interval in which the root of the equation $xe^x = 1$ lies between 0 and 1, obtained by using three iterations of bisection method. 8

(c) Perform the operations (i) $(+42)+(-13)$ and (ii) $(-42)-(-13)$ in binary using signed 2's complement representation for negative numbers in 8-bit system. Give the final answer in decimal. 4+4

(d) Derive the Lagrange's equation for a particle of mass m that slides on a frictionless wire hanging as a cycloid given by the equations $x = a(\theta - \sin\theta)$, $y = a(1+\cos\theta)$. Further, if $\cos\frac{\theta}{2} = f$ (say), then transform the Lagrange's equation to the form $\frac{d^2 f}{dt^2} + \frac{g}{4a} f = 0$, where t symbolizes time. 8

(e) Prove that the equation of motion of a homogeneous inviscid liquid moving under forces arising from a potential V may be written in the form

$$\frac{\partial \vec{q}}{\partial t} - \vec{q} \times \vec{\zeta} = -\vec{\nabla}\left(\frac{p}{\rho} + \frac{1}{2}\vec{q}^{\,2} + V\right)$$

where $\vec{q}$, t, $\vec{\zeta}$, p, ρ respectively stand for velocity vector, time, vorticity vector, pressure, density, and $\vec{\nabla}()$ the gradient operator.

If the velocity $\vec{q}$, referred to cylindrical polar coordinates (r, θ, z), is given by

$$\vec{q} = \begin{cases} \left[0, \frac{1}{2}\omega r, 0\right] & (0 \le r \le a) \\ \left[0, \frac{1}{2}\frac{\omega a^2}{r}, 0\right] & (r \ge a) \end{cases}$$

where ω is a constant, prove that the vorticity is given by

$$\vec{\zeta} = \begin{cases} [0, 0, \omega] & (0 \le r \le a) \\ [0, 0, 0] & (r \ge a) \end{cases}$$

8

6. (a) Find a complete integral of the equation $p^2 x + q^2 y = z$, where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. 10

(b) Write down the algorithm for finding the integral $I = \int_a^b f(x)\,dx$ using Simpson's $\frac{1}{3}$rd rule. Hence, find $I = \int_0^1 (4x - 3x^2)\,dx$ taking 10 intervals. Also, find the relative error. 7+8

(c) A uniform rod of length $2a$ and mass m can rotate freely about a fixed end. What is the least angular velocity required to start with, from the lowest position so as to reach the top in order to make a complete revolution? Also, find out the time consumed. 15

7. *(a)* In an examination, the number of candidates who secured marks between certain limits was as follows :

Marks	:	0–19	20–39	40–59	60–89	90–99
No. of candidates	:	41	62	65	50	17

Estimate the number of candidates getting marks less than 50. 10

(b) Find the general solution of the partial differential equation

$$(4D^2 - 4DD' + D'^2)z = 8\log(x+2y) + \cos 2x \cos y$$

where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$. 15

(c) A long infinite cylinder of radius a is placed in a uniform stream such that its axis lies perpendicular to the stream. Besides, a circulation round the cylinder is produced by a uniform line vortex through the origin. If the uniform stream velocity is $-U\hat{i}$ and the circulation is $2\pi k$, then find out the complex velocity potential. Show, by using the theorem of Blasius, that the cylinder experiences an uplifting force. 15

8. *(a)* A simple source, of strength m, is fixed at the origin O in a uniform stream of incompressible fluid moving with velocity $U\hat{i}$. Show that the velocity potential ϕ at any point P of the stream is $\frac{m}{r} - Ur\cos\theta$, where $OP = r$ and θ is the angle $\overrightarrow{OP}$ makes with the direction $\hat{i}$. Find the differential equation of the streamlines and show that they lie on the surfaces $Ur^2 \sin^2\theta - 2m\cos\theta = \text{constant}$. 10

(b) Let $u(x, t)$ be the solution of the wave equation

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, \ 0 < x < \pi, \ t > 0$$

with the initial conditions $u(x, 0) = \sin x + \sin 2x + \sin 3x$; $\frac{\partial u}{\partial t}(x, 0) = 0, \ 0 < x < \pi$ and the boundary condition $u(0, t) = u(\pi, t) = 0, \ t \geq 0$. Find the value of $u\left(\frac{\pi}{2}, \pi\right)$. 15

(c) Using Runge-Kutta method of fourth order, find y at $x = 0 \cdot 2$, given that $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 0$ and $h = 0 \cdot 1$, correct to four decimal places. 15

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