I.F.S. EXAM-(M) 2018

FSI-P-MTH

MATHEMATICS

Paper – I

Time Allowed : ***Three*** *Hours*

Maximum Marks : ***200***

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

There are ***EIGHT*** *questions in all, out of which* ***FIVE*** *are to be attempted.*

Questions no. ***1*** *and* ***5*** *are* **compulsory**. *Out of the remaining* ***SIX*** *questions,* ***THREE*** *are to be attempted selecting at least* ***ONE*** *question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Answers must be written in ***ENGLISH*** *only.*

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

SECTION A

Q1. (a) Show that the maximum rectangle inscribed in a circle is a square. *8*

(b) Given that Adj A = $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and det A = 2. Find the matrix A. *8*

(c) If $f : [a, b] \to R$ be continuous in [a, b] and derivable in (a, b), where $0 < a < b$, show that for $c \in (a, b)$

$$f(b) - f(a) = cf'(c) \log (b/a).$$ *8*

(d) Find the equations of the tangent planes to the ellipsoid

$$2x^2 + 6y^2 + 3z^2 = 27$$

which pass through the line

$$x - y - z = 0 = x - y + 2z - 9.$$ *8*

(e) Prove that the eigenvalues of a Hermitian matrix are all real. *8*

Q2. (a) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is $x^2 + y^2 = 4$, $z = 2$. *10*

(b) Show that the matrices

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 2 \\ 3 & 2 & 0 \end{bmatrix}$$ are congruent. *10*

(c) If ϕ and ψ be two functions derivable in [a, b] and $\phi(x)\,\psi'(x) - \psi(x)\,\phi'(x) > 0$ for any x in this interval, then show that between two consecutive roots of $\phi(x) = 0$ in [a, b], there lies exactly one root of $\psi(x) = 0$. *10*

(d) Show that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$, $\alpha_3 = (0, -3, 2)$ form a basis for R^3. Express each of the standard basis vectors as a linear combination of $\alpha_1, \alpha_2, \alpha_3$. *10*

Q3. (a) Find the equation of the tangent plane that can be drawn to the sphere

$$x^2 + y^2 + z^2 - 2x + 6y + 2z + 8 = 0,$$

through the straight line

$$3x - 4y - 8 = 0 = y - 3z + 2.$$ 10

(b) If $f = f(u, v)$, where $u = e^x \cos y$ and $v = e^x \sin y$, show that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = (u^2 + v^2)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right).$$ 10

(c) Let $T : V_2(R) \rightarrow V_2(R)$ be a linear transformation defined by $T(a, b) = (a, a + b)$. Find the matrix of T, taking $\{e_1, e_2\}$ as a basis for the domain and $\{(1, 1), (1, -1)\}$ as a basis for the range. 10

(d) Evaluate $\iint_R (x^2 + xy)\, dx\, dy$ over the region R bounded by $xy = 1$, $y = 0$, $y = x$ and $x = 2$. 10

Q4. (a) Find the equations of the straight lines in which the plane $2x + y - z = 0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0$. Find the angle between the two straight lines. 10

(b) Show that the functions $u = x + y + z$, $v = xy + yz + zx$ and $w = x^3 + y^3 + z^3 - 3xyz$ are dependent and find the relation between them. 10

(c) Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$. 10

(d) If $(n + 1)$ vectors $\alpha_1, \alpha_2, \ldots, \alpha_n, \alpha$ form a linearly dependent set, then show that the vector α is a linear combination of $\alpha_1, \alpha_2, \ldots, \alpha_n$; provided $\alpha_1, \alpha_2, \ldots, \alpha_n$ form a linearly independent set. 10

SECTION B

Q5. (a) Find the complementary function and particular integral for the equation

$$\frac{d^2y}{dx^2} - y = xe^x + \cos^2 x$$

and hence the general solution of the equation. *8*

(b) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \log x \ (x > 0)$ by the method of variation of parameters. *8*

(c) If the velocities in a simple harmonic motion at distances a, b and c from a fixed point on the straight line which is not the centre of force, are u, v and w respectively, show that the periodic time T is given by

$$\frac{4\pi^2}{T^2}(b-c)(c-a)(a-b) = \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}.$$ *8*

(d) From a semi-circle whose diameter is in the surface of a liquid, a circle is cut out, whose diameter is the vertical radius of the semi-circle. Find the depth of the centre of pressure of the remainder part. *8*

(e) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and f(r) is differentiable, show that

$\text{div}[f(r)\vec{r}] = rf'(r) + 3f(r).$

Hence or otherwise show that $\text{div}\left(\frac{\vec{r}}{r^3}\right) = 0.$ *8*

Q6. (a) Solve the differential equation $(y^2 + 2x^2y)\,dx + (2x^3 - xy)\,dy = 0.$ *10*

(b) Let T_1 and T_2 be the periods of vertical oscillations of two different weights suspended by an elastic string, and C_1 and C_2 are the statical extensions due to these weights and g is the acceleration due to gravity. Show that $g = \frac{4\pi^2(C_1 - C_2)}{T_1^2 - T_2^2}$. *15*

(c) Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force. Hence, find the scalar potential. Also find the work done in moving a particle of unit mass in the force field from (1, – 2, 1) to (3, 1, 4). *15*

Q7. (a) The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is

$$\mu \log \frac{1 + (1 + \mu^2)^{\frac{1}{2}}}{\mu}$$

where μ is the coefficient of friction. *10*

(b) Solve : *10*

$$\frac{dy}{dx} = \frac{4x + 6y + 5}{3y + 2x + 4}$$

(c) A frame ABC consists of three light rods, of which AB, AC are each of length a, BC of length $\frac{3}{2}$a, freely jointed together. It rests with BC horizontal, A below BC and the rods AB, AC over two smooth pegs E and F, in the same horizontal line, at a distance 2b apart. A weight W is suspended from A. Find the thrust in the rod BC. *10*

(d) Let α be a unit-speed curve in R^3 with constant curvature and zero torsion. Show that α is (part of) a circle. *10*

Q8. (a) A solid hemisphere floating in a liquid is completely immersed with a point of the rim joined to a fixed point by means of a string. Find the inclination of the base to the vertical and tension of the string. *15*

(b) A snowball of radius r(t) melts at a uniform rate. If half of the mass of the snowball melts in one hour, how much time will it take for the entire mass of the snowball to melt, correct to two decimal places ? Conditions remain unchanged for the entire process. *15*

(c) For a curve lying on a sphere of radius a and such that the torsion is never 0, show that

$$\left(\frac{1}{\kappa}\right)^2 + \left(\frac{\kappa'}{\kappa^2 \tau}\right)^2 = a^2.$$ *10*