

## PHYSICS

## Paper - I

Time Allowed : **Three Hours**Maximum Marks : **200**

## Question Paper Specific Instructions

**Please read each of the following instructions carefully before attempting questions :**

There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.

Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Answers must be written in **ENGLISH** only.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Neat sketches may be drawn, wherever required.

**Useful Constants :**

Electron charge ( $e$ )	$= 1.602 \times 10^{-19} \text{ C}$
Electron rest mass ( $m_e$ )	$= 9.109 \times 10^{-31} \text{ kg}$
Proton mass ( $m_p$ )	$= 1.672 \times 10^{-27} \text{ kg}$
Vacuum permittivity ( $\epsilon_0$ )	$= 8.854 \times 10^{-12} \text{ farad/m}$
Vacuum permeability ( $\mu_0$ )	$= 1.257 \times 10^{-6} \text{ henry/m}$
Velocity of light in free space ( $c$ )	$= 3 \times 10^8 \text{ m/s}$
Boltzmann constant ( $k$ )	$= 1.380 \times 10^{-23} \text{ J/K}$
Electron volt (eV)	$= 1.602 \times 10^{-19} \text{ J}$
Planck constant ( $h$ )	$= 6.626 \times 10^{-34} \text{ Js}$
Stefan constant ( $\sigma$ )	$= 5.67 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$
Avogadro number ( $N$ )	$= 6.022 \times 10^{26} \text{ kmol}^{-1}$
Gas constant ( $R$ )	$= 8.31 \times 10^3 \text{ J kmol}^{-1} \text{ K}^{-1}$
exp (1)	$= 2.718$

## SECTION A

### Q1. Answer the following :

8×5=40

- (a) A bead slides on a wire in the shape of a cycloid described by the equations

$$x = a (\theta - \sin \theta)$$

$$y = a (1 + \cos \theta) \text{ with } 0 \leq \theta \leq 2\pi.$$

Find the Lagrangian and equation of motion.

8

- (b) Show that the relativistic invariance laws of conservation of momentum lead to the concepts of variation of mass with velocity and mass energy equivalence.

8

- (c) A parallel beam of light of wavelength  $5890 \text{ \AA}$  is incident at an angle of  $30^\circ$  on a plane transmission grating with 15000 lines/inch. Find the highest order of spectrum that can be observed.

8

- (d) Discuss absorption loss in an optical fibre comparing and contrasting the intrinsic and extrinsic absorption mechanisms.

8

- (e) Although the principle of operation of a basic LASER is based upon two energy levels, why does one need a 3-level or a 4-level scheme to achieve satisfactory lasing ? Explain your answer with special reference to a Ruby-laser.

8

- Q2. (a) Discuss the mechanics of a system of point particles with special emphasis on the conservation theorems. How can we extend the results to a system with continuous mass distribution ?

10

- (b) (i) State and prove Hamilton's principle and use it to prove that the shortest distance between two points in space is a straight line joining them.

- (ii) Use Hamiltonian mechanics to find the differential equation for planetary motion, moving under force  $f(r) = -\frac{k}{r^2}$  and prove that the areal velocity is constant.

8+7=15

- (c) (i) What is Holography ?

- (ii) Show with simple diagrams, how a hologram is written and read using a laser.

- (iii) Mention some important applications of holography.

3+8+4=15

- Q3.** (a) State the fundamental postulates of Einstein's special theory of relativity. Deduce Lorentz transformation equation and discuss how this accounts for the phenomenon of length contraction. 10
- (b) Discuss the properties of Cornu spiral. Show that the spiral can be used to obtain the intensity distribution in the Fresnel's diffraction pattern due to a straight edge. 10
- (c) (i) Using the concept of spontaneous and stimulated emission of radiation, obtain the relation between Einstein's A and B coefficients.
- (ii) What is the physical significance of Einstein's A coefficient ?
- (iii) Justify why lasing action is much more difficult at X-ray frequency than in case of infrared frequency spectrum. 10+5+5=20
- Q4.** (a) (i) In a Michelson's interferometer, 100 fringes cross the field of view when the movable mirror is displaced through  $2.894 \times 10^{-3}$  cm. Calculate the wavelength of the monochromatic source of light.
- (ii) A shift of 200 fringes is observed when the movable mirror of a Fabry-Pérot interferometer is shifted by 0.0298 mm. Calculate the wavelength of the incident radiation. 8+7=15
- (b) State and explain Fermat's principle of extremum path and use the same to deduce the laws of reflection and refraction of light. 10
- (c) (i) Explain the reason for pulse broadening due to intermodal and material dispersion. Deduce the relation of pulse broadening for intermodal dispersion in optical fiber.
- (ii) A step index fiber in air has a numerical aperture of 0.16, a core refractive index of 1.45 and a core diameter of 60  $\mu\text{m}$ . Determine the normalized frequency for the fiber when light at a wavelength of 0.8  $\mu\text{m}$  is transmitted. Also estimate the number of guided modes propagating in the fiber. 10+5=15



## SECTION B

Q

**Q5. Answer the following :**

**8×5=40**

- (a) In a one-dimensional device, the charge density is given by

$$\rho_V = \rho_0 \frac{x}{a}.$$

If  $E = 0$  at  $x = 0$  and  $V = 0$  at  $x = a$ ,

find  $V$  and  $E$  using Laplace equation of electrostatics.

8

- (b) State and explain the Biot-Savart law. Derive an expression for the magnetic field at a point due to an infinitely long straight current carrying conductor.

8

- (c) Write down the four Maxwell's equations and explain the contribution of Maxwell in the development of these equations.

8

- (d) Prove the thermodynamic relation :

$$\left( \frac{\partial S}{\partial V} \right)_T = \left( \frac{\partial P}{\partial T} \right)_V$$

and hence show that

$$\frac{dP}{dt} = \frac{L}{T(V_2 - V_1)};$$

all the terms have their usual meanings.

8

- (e) Describe neutron star on the basis of Fermi-Dirac statistics and obtain the condition of critical mass for a neutron star.

8

Q

- Q6. (a)** Use the method of electric images to find the electric field on the surface of a grounded conducting sphere.

10

- (b) (i) State Faraday's law of electromagnetic induction and prove that it can be expressed in the following vector form :

$$\text{Curl } \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

with  $\vec{E}$  and  $\vec{B}$  being the electric and magnetic fields.

- (ii) A coil of 10 turns has dimension  $9 \text{ cm} \times 7 \text{ cm}$ . It rotates at the rate of  $15\pi \text{ rad/sec}$  in a uniform field whose flux density is  $0.6 \text{ weber/m}^2$ . What is the maximum e.m.f. induced in the coil ?

10+5=15

- (c) How does one explain the observed spectrum of black-body radiation using Planck's quantum hypothesis ? State and obtain Wien's displacement law. Also explain the important features of this law. 15

**Q7.** (a) (i) Using Maxwell's equations, obtain the relation

$$\frac{1}{c} \frac{\partial}{\partial t} \left( \frac{E^2 + B^2}{2} \right) + \nabla \cdot (\vec{E} \times \vec{B}) = 0$$

- (ii) What is Poynting vector ? Deduce Poynting theorem for the flow of energy in an electromagnetic field. 5+10=15
- (b) Discuss the reflection and refraction of plane electromagnetic waves at plane dielectric boundaries for normal incidence and also find the reflection and transmission coefficients. 15
- (c) What do you understand by spontaneous magnetization below Curie temperature ? Explain with an appropriate diagram, the occurrence of a hysteresis loop in a ferromagnetic material. 10

**Q8.** (a) State Maxwell's distribution law of molecular speeds. Draw and explain a curve between  $n(c)$  and  $c$  in a gas at a given temperature  $T$ , where  $n(c) dc$  is the number of molecules having speed between  $c$  and  $c + dc$ . Discuss the effect of  $T$  and mass  $m$  of the molecule on the nature of the curve. 15

- (b) (i) Define and explain the significance of the quality factor of an electrical machine.
- (ii) Discuss in brief, the working principle of a transformer. 5+5=10
- (c) Derive the mathematical expression for the total energy of a degenerate Fermi gas at a temperature  $T$  and calculate the specific heat of the Fermi gas at this temperature. 15

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