I.E.S./I.S.S. Examination, 2026

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T.B.C. : SNCS–B–STS

Test Booklet Series

A

Serial No. 1001713

TEST BOOKLET

STATISTICS

Paper—II

Time Allowed : Two Hours

Maximum Marks : 200

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET *DOES NOT* HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. **Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Test Booklet Series A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the Answer Sheet liable for rejection.**
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. *DO NOT* write *anything else* on the Test Booklet.
4. This Test Booklet contains **80** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose *ONLY ONE* response for each item.
5. You have to mark all your responses *ONLY* on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. ***All*** items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator *only the Answer Sheet*. You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**
    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.
    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.
    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO

1. Let $x_1, x_2, ..., x_n$ be a random sample from a distribution for which mean μ and variance σ^2 exist. Then the consistent estimator of μ is

(a) $x_{(1)} = \min(x_1, x_2, ..., x_n)$

(b) $x_{(n)} = \max(x_1, x_2, ..., x_n)$

(c) $\frac{\sum_{i=1}^{n} x_i}{n}$

(d) $\frac{x_{(n)} + x_{(1)}}{2}$

2. Let $x_1, x_2, ..., x_n$ be a random sample from the p.d.f. $f(x) = \frac{\theta}{x^2}$; $0 < \theta \leq x < \infty$.

Which one of the following is a sufficient statistic for θ?

(a) Mean of $(x_1, x_2, ..., x_n)$

(b) Median of $(x_1, x_2, ..., x_n)$

(c) Minimum of $(x_1, x_2, ..., x_n)$

(d) Minimum of $(x_1^2, x_2^2, ..., x_n^2)$

3. Let $x_1, x_2, ..., x_n$ be a random sample from the distribution having p.d.f. $f(x; \theta, \beta) = \beta e^{-\beta(x-\theta)}$; $x \geq \theta$, $\beta > 0$. To test $H_0 : \beta = 2$, $\theta = 3$ against $H_1 : \beta = 4$, $\theta = 2$, the best critical region is given by

(a) $\bar{x} \geq \frac{1}{2}\left\{8 + \log 2 - \frac{\log k}{n}\right\}$

(b) $\bar{x} \geq \left\{1 + \frac{\log 2}{2} - \frac{\log k}{n}\right\}$

(c) $\bar{x} \leq \left\{1 + \log 2 - \frac{\log k}{2n}\right\}$

(d) $\bar{x} \leq \left\{1 + \frac{\log 2}{2} - \frac{\log k}{2n}\right\}$

where $k > 0$.

4. Let $x_1, x_2, ..., x_n$ be a random sample from $N(\mu, \sigma^2)$ distribution, the parameter μ is known. Which of the following statements is/are correct?

I. The Cramer-Rao lower bound to the variance of an unbiased estimator of σ is $\frac{2\theta^2}{n}$, where $\theta = \sigma^2$.

II. An unbiased estimator of σ is $\frac{\sum_{i=1}^{n} |x_i - \mu|}{n}$.

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

5. Let $x_1, x_2, ..., x_n$ be a random sample from the distribution having p.d.f. $f(x, \theta) = e^{-(x-\theta)}$; $x > \theta$. Further, $\left(x_{(1)} - \frac{\log 40}{n}, x_{(1)}\right)$ is $100\beta\%$ confidence interval for θ, where

$$x_{(1)} = \min\{x_1, x_2, ..., x_n\}$$

What is the value of β?

(a) 0·90

(b) 0·92

(c) 0·95

(d) 0·975

6. Let $x_1, x_2, ..., x_n$ be a random sample from $N(\theta, \sigma^2)$ distribution, σ^2 is known, θ is unknown. Which of the following statements is/are correct?

I. The MLE of $e^{2\theta}$ is $e^{2\bar{x}}$ with bias $e^{\frac{2\sigma^2}{n}} - 1$.

II. An unbiased estimator of $e^{2\theta}$ is $e^{2\bar{x} - \frac{2\sigma^2}{n}}$.

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

7. Let $x_1, x_2, ..., x_n$ be a random sample from the following probability distribution :

$$P(X=1) = \frac{2(1-\theta)}{2-\theta}, \quad P(X=2) = \frac{\theta}{2-\theta}; \quad \theta \in (0, 1)$$

The moment estimate of θ is

(a) $\bar{x}$

(b) $2(1-\bar{x})$

(c) $2\left(1-\frac{1}{\bar{x}}\right)$

(d) $\frac{2}{2-\bar{x}}$

8. Let $X \sim f(x, \theta)$. The most powerful test of size 0·10 for testing $H_0 : f(x, \theta) = 8x^7$, $0 < x < 1$ against $H_1 : f(x, \theta) = 4x^3$, $0 < x < 1$ has power

(a) $\sqrt{0 \cdot 10}$

(b) $\sqrt[4]{0 \cdot 10}$

(c) $\sqrt[6]{0 \cdot 10}$

(d) $\sqrt[8]{0 \cdot 10}$

9. Let $x_1, x_2, ..., x_n$ be a random sample from a distribution with p.d.f.

$$f(x, \theta) = \begin{cases} \frac{1}{\theta} e^{-(x/\theta)}; & 0 \le x < \infty \\ 0 \quad ; & \text{otherwise} \end{cases}$$

The sufficient estimator of θ is

(a) $\sum_{i=1}^{n} x_i^2$

(b) $\sum_{i=1}^{n} x_i + 1$

(c) $\sum_{i=1}^{n} x_i^2 + 1$

(d) $\sum_{i=1}^{n} x_i$

10. A random sample $x_1, x_2, ..., x_n$ is taken from a normal population with mean 0 and variance σ^2, that is, $X \sim N(0, \sigma^2)$. The minimum variance bound (MVB) estimator for σ^2 is

(a) $\frac{\sum_{i=1}^{n} x_i^2}{n}$

(b) $\frac{\sum_{i=1}^{n} x_i}{n}$

(c) $\frac{\sum_{i=1}^{n} x_i}{n+1}$

(d) $\frac{\sum_{i=1}^{n} x_i}{n-1}$

11. If 95% confidence interval for μ with $Z_{\alpha/2} = 1{\cdot}96$ is $(400, 407{\cdot}84)$ and the sample size is large, the value of the standard error of mean $\bar{X}$ will be

(a) 2

(b) 4

(c) 10

(d) 20

12. Let $x_1, x_2, ..., x_n$ be a random sample from a distribution with p.d.f. or p.m.f. $f(x, \theta)$, $\theta \in \Theta$. A statistic $T = (x_1, x_2, ..., x_n)$ is said to be sufficient for θ, iff the conditional distribution of X, given $T = t$, is

(a) dependent on θ

(b) independent of θ

(c) dependent on θ^2

(d) None of the above

13. Let $x_1, x_2, ..., x_n$ be a random sample from a distribution F_θ for which mean and variance exist. Let μ be the mean and σ^2 be the variance of the distribution. Then the unbiased estimator of σ^2 is

(a) $s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

(b) $s^2 = \frac{1}{n-2}\sum_{i=1}^{n}(x_i - \bar{x})^2$

(c) $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

(d) $s^2 = \frac{2}{n(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2$

14. Neyman-Pearson fundamental lemma provides the most powerful (MP) test of its size for testing

(a) simple null hypothesis against composite alternative

(b) composite null hypothesis against simple alternative

(c) simple null hypothesis against simple alternative

(d) composite null hypothesis against composite alternative

15. Let $x_1, x_2, ..., x_n$ be a random sample from a normal (μ, θ) and μ is known. Which of the following statements is/are correct?

I. $T = \sum_{i=1}^{n}(x_i - \mu)^2$ is sufficient statistic for θ.

II. $S_0^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$ is sufficient statistic for θ.

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

16. Let T_0 be an MVU estimator of $\gamma(\theta)$ and T_1 an unbiased estimator with efficiency $e_\theta < 1$. Which of the following statements is/are correct?

I. The unbiased linear combination of T_0 and T_1, i.e., $U = a_0T_0 + a_1T_1$; $a_1 \neq 0$, $a_0 + a_1 = 1$ is MVU.

II. The unbiased linear combination of T_0 and T_1, i.e., $V = a_0T_0 - a_1T_1$; $a_1 \neq 0$, $a_0 - a_1 = 1$ is MVU.

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

17. Let $x_1, x_2, ..., x_n$ be a random sample from $N(\mu, \sigma^2)$, the parameter μ being known. If the unknown $(\theta = \sigma^2)$ parameter is σ^2 with the parametric space $\Theta = \{\theta | 0 < \theta < \infty\}$, then $E_\theta\left(\frac{\partial^2 \log f_\theta(x)}{\partial\theta^2}\right)$ is equal to

(a) $-\frac{1}{2\theta^2}$

(b) $2\theta^2$

(c) $-\frac{n}{2\theta^2}$

(d) $\frac{n}{2\theta^2}$

18. Let $x_1, x_2, ..., x_n$ be a random sample from $N(\mu, \sigma^2)$, the parameter μ being known. The unknown $(\theta = \sigma^2)$ parameter is σ^2 with parametric space $\Theta = \{\theta | 0 < \theta < \infty\}$ and $S_0^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$.

Which of the following statements is/are correct?

I. $\text{Var}(S_0^2)$ is more than the CR lower bound.

II. $\text{Var}(S_0^2)$ coincides with the CR lower bound.

III. S_0^2 is an MVB estimator of θ.

Select the answer using the code given below.

(a) I only

(b) III only

(c) II and III

(d) I and III

19. A batch of 19 students is examined in three subjects—Statistics (x_1), Mathematics (x_2) and Computer Programming (x_3). It has been given that $r_{13\cdot2} = 0\cdot6$. It is expected that x_2 will affect the correlation between x_1 and x_3 (i.e., the partial correlation between x_1 and x_3 after eliminating the effect of x_2 is zero). The value of test statistic for testing $H_0 : \rho_{13\cdot2} = 0$ against $H_1 : \rho_{13\cdot2} \neq 0$ is

(a) $2\cdot5$

(b) 3

(c) 4

(d) $4\cdot5$

20. In order to test $H_0 : \theta = \theta_0$ against the alternative $H_1 : \theta \neq \theta_0$, the likelihood ratio to be used as the test statistic is given by

(a) $\dfrac{L(\theta_0)}{\max(L(\Theta))}$

(b) $\dfrac{\max(L(\Theta))}{L(\theta_0)}$

(c) $\min\left(\dfrac{L(\theta_0)}{L(\Theta)}\right)$

(d) $\min\left(\dfrac{L(\Theta)}{L(\theta_0)}\right)$

where Θ is the parametric space.

21. Let $x_1, x_2, \ldots, x_n$ be a random sample from $N(\mu, \sigma^2)$ distribution with μ known. Consider the following testing problems for $N(\mu, \sigma^2)$:

I. $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 < \sigma_0^2$

II. $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 > \sigma_0^2$

III. $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$

The UMP level α test exists for

(a) I and II

(b) III

(c) I but not for II

(d) II but not for I

22. Let $x_1, x_2, \ldots, x_n$ be a random sample from a Bernoulli population with parameter p, where $0 < p < 1$. Which of the following is/are sufficient statistic(s) for p?

I. $\dfrac{1}{n}\sum_{i=1}^{n} x_i$

II. $\dfrac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

III. $\sum_{i=1}^{n} x_i$

Select the correct answer using the code given below.

(a) I only

(b) III only

(c) I and II

(d) I and III

23. If T_1 is an MVUE of $\gamma(\theta)$, $\theta \in \Theta$ and T_2 is any other unbiased estimator of $\gamma(\theta)$ with efficiency $e < 1$, then which of the following is/are correct?

I. $T = (T_1 + T_2)/2$ is MVUE of $\gamma(\theta)$

II. $\text{Var}(T) = \text{Var}(T_1/4)$

III. T cannot be MVUE for $\gamma(\theta)$

IV. T is unbiased for $\gamma(\theta)$

Select the answer using the code given below.

(a) I

(b) IV only

(c) II and IV

(d) III and IV

24. If $X \sim \text{Exp}(\lambda)$, then based on a random sample of size n from this population, consider the following statements :

I. MLE and MME estimators of λ are equal to $\hat{\lambda} = \frac{1}{\bar{x}}$.

II. MLE and MME estimators of λ are equal to $\hat{\lambda} = \bar{x}$.

III. MLE and MME of λ are unequal.

Which of the statements given above is correct?

(a) I

(b) II

(c) III

(d) Neither I nor II nor III

25. If T_1 and T_2 are two unbiased estimators of $\gamma(\theta)$ having the same variance σ^2 and ρ is the correlation between them, then which one of the following is correct?

(a) $\rho = 2e - 1$, where e is the efficiency of each estimator

(b) $\rho \geq 2e - 1$, where e is the efficiency of each estimator

(c) $\rho = 2e_1 - e_2$, where e_1 is the efficiency of T_1 and e_2 is the efficiency of T_2

(d) $\rho = e_1 + e_2 - 1$, where e_1 is the efficiency of T_1 and e_2 is the efficiency of T_2

26. An estimator T_n is consistent for $\gamma(\theta)$ under which of the following?

I. T_n is unbiased for $\gamma(\theta)$

II. $V(T_n) \leq 1$

III. $T_n \xrightarrow{p} \gamma(\theta)$

IV. $\frac{T_n}{n} \xrightarrow{p} \gamma(\theta)$

Select the correct answer using the code given below.

(a) I and II

(b) II and III

(c) III only

(d) IV

27. Let $X \sim$ Bernoulli (P). Then the Fisher information based on a random sample of size one from this population is

(a) $P(1-P)$

(b) $1/(P - P^2)$

(c) $1 - P$

(d) $1/(1 - 2P + P^2)$

28. Let $x_1, x_2, ..., x_n$ be a random sample from a distribution with p.d.f.

$$f(x) = \begin{cases} \theta x^{\theta-1}; & 0 < x < 1,\ 0 < \theta < \infty \\ 0 \ ; & \text{otherwise} \end{cases}$$

Then a sufficient statistic for θ is

(a) $\sum_{i=1}^{n} x_i$

(b) $\sum_{i=1}^{n} x_i - n$

(c) $\prod_{i=1}^{n} x_i$

(d) $\prod_{i=1}^{n} \frac{1}{x_i} - n$

29. The most powerful test in testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ maximizes

(a) variance

(b) power

(c) type I error

(d) type II error

30. Consider a random sample of size n from a Bernoulli (p) population. To test $H_0 : p = 0{\cdot}4$ against $H_1 : p = 0{\cdot}6$, the most powerful test depends on

(a) $\sum_{i=1}^{n} x_i$

(b) $x_{(1)}$

(c) $\sum_{i=1}^{n} (x_i - \bar{x})^2$

(d) $\sum_{i=1}^{n} x_i^2$

31. If the SPRT of strength (α, β) and boundary points (A, B) terminates with probability 1, then which of the following is/are correct?

I. $A \leq \frac{1-\beta}{\alpha}$

II. $B \geq \frac{\beta}{1-\alpha}$

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

32. Under regularity conditions, the maximum likelihood estimators are asymptotically

I. unbiased

II. normal

III. efficient

Select the correct answer using the code given below.

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

33. Let $T = T(x_1, x_2, ..., x_n)$ be an estimator of $\gamma(\theta)$. Consider the following statements :

I. $E_\theta[T - \gamma(\theta)]^2 = E_\theta[T - E_\theta(T)]^2 + [E_\theta(T) - \gamma(\theta)]^2$

II. The bias of T is $E_\theta(T) - \gamma(\theta)$.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

34. Let $x_1, x_2, ..., x_n$ be a random sample from a population with probability density function $f(x; \theta) = \theta x^{\theta-1}$; $0 < x < 1$, $\theta > 0$. Which of the following statements are correct?

I. $\prod_{i=1}^{n} x_i$ is a sufficient estimator of θ.

II. $\sum_{i=1}^{n} x_i$ is a sufficient estimator of θ.

III. $-\frac{n}{\sum_{i=1}^{n} \log x_i}$ is a maximum likelihood estimator of θ.

Select the answer using the code given below.

(a) I and II only

(b) I and III only

(c) II and III only

(d) I, II and III

35. Let $V = V(x_1, x_2, ..., x_n)$ be an unbiased estimator of $\gamma(\theta)$ and T be a sufficient statistic for $\gamma(\theta)$. Define $\Phi(t) = E[V|T = t]$. Which of the following statements is/are correct?

I. $\Phi(t)$ is an unbiased estimator of $\gamma(\theta)$.

II. The variance of $\Phi(t)$ is less than or equal to variance of V.

III. $\Phi(t)$ is a function of sufficient statistic which is dependent on θ.

Select the answer using the code given below.

(a) I only

(b) II and III only

(c) I and II only

(d) I, II and III

36. Let $x_1, x_2, ..., x_n$ be a random sample from $N(\mu, \sigma^2)$; $-\infty < \mu < \infty$, $\sigma^2 > 0$, where both μ and σ^2 are unknown. Which of the following statements are correct?

I. $\hat{\mu} = \bar{x}$ and $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ maximize the likelihood function $L(\mu, \sigma^2; x)$.

II. $\hat{\mu} = \bar{x}$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ maximize the likelihood function $L(\mu, \sigma^2; x)$.

III. $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is an unbiased estimator of σ^2.

IV. $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is an unbiased estimator of σ^2.

Select the answer using the code given below.

(a) II and IV

(b) II and III

(c) I and III

(d) I and IV

37. Let T_1 and T_2 be two unbiased estimators for a parameter θ such that $E_\theta(T_i^2) < \infty$; $i = 1, 2$. If the efficiency of T_1 relative to T_2 is $e_\theta\left(\frac{T_1}{T_2}\right) = \frac{\text{Var}_\theta(T_2)}{\text{Var}_\theta(T_1)}$, then which one of the following is correct?

(a) $e_\theta\left(\frac{T_1}{T_2}\right) > 1$ implies T_1 is more efficient than T_2

(b) $e_\theta\left(\frac{T_1}{T_2}\right) < 1$ implies T_1 is more efficient than T_2

(c) $e_\theta\left(\frac{T_2}{T_1}\right) > 1$ implies T_1 is more efficient than T_2

(d) $e_\theta\left(\frac{T_2}{T_1}\right) < 1$ implies T_2 is more efficient than T_1

38. Let $x_1, x_2, ..., x_n$ be a random sample from

$$f(x; \alpha, \beta) = \begin{cases} \frac{1}{\beta - \alpha}, & \alpha < x < \beta \\ 0, & \text{otherwise} \end{cases}$$

Then the MLEs of α and β are respectively given by

(a) $\hat{\alpha} = \bar{x}$ and $\hat{\beta} = \bar{x} - 1$

(b) $\hat{\alpha} = \frac{1}{\bar{x}}$ and $\hat{\beta} = \frac{2}{\bar{x}}$

(c) $\hat{\alpha} = x_{(1)}$ and $\hat{\beta} = x_{(n)}$

(d) $\hat{\alpha} = x_1$ and $\hat{\beta} = x_n$

39. Consider the following :

I. Bootstrap

II. Jackknife

III. Cross-validation

IV. Stratified sampling

How many of the above are resampling methods?

(a) One

(b) Two

(c) Three

(d) All the four

40. The critical region W and consequently a test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ is unbiased if

(a) $P(x \in W|H_1) \geq P(x \in W|H_0)$

(b) $P(x \in W|H_0) \geq P(x \in W|H_1)$

(c) $P(x \in W|H_0) = P(x \in W|H_1)$

(d) $P(x \in W|H_1) > P(x \in W|H_0)$

41. Let $x_1, x_2, ..., x_n$ be independent random variables each having the distribution $P(x_i = k) = \frac{1}{N}$; $k = 1, 2, ..., N$; $N \in Z$, the set of positive integers. If $x_{(n)} = \max(x_i)$ and $x_{(1)} = \min(x_i)$, then the sufficient statistic for the parameter N is

(a) $x_{(1)}$

(b) $x_{(n)}$

(c) $\frac{x_{(1)}}{2}$

(d) None of the above

42. Let $x_1, x_2, ..., x_n$ constitute a random sample of size n from the probability density function $f(x; \theta) = \theta e^{-\theta x}$; $0 < x < \infty$, $\theta > 0$. Consider the following statements :

I. The estimator of θ with method of moments is $\bar{x}$.

II. Both maximum likelihood estimator and moment estimator of θ are same.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

43. Let $x_1, x_2, ..., x_n$ be independent and identically distributed $b(1, p)$, i.e., Bernoulli random variables with parameter p, $0 < p < 1$. Define $T = \sum_{i=1}^{n} x_i$. The UMVUE for $\Psi(p) = p(1-p)$ is given by

(a) $\dfrac{T(1-T)}{n(n-1)}$

(b) $\dfrac{T(n-T)}{n(n-1)}$

(c) $\dfrac{T}{n(n-1)}$

(d) $T(n-1-T)$

44. Consider the following statements :

I. If a sufficient statistic t for θ exists, then the maximum likelihood estimator will be a function of the sufficient statistic t.

II. If $\hat{\theta}$ is the maximum likelihood estimator of θ, then $\dfrac{\hat{\theta}}{1+\hat{\theta}}$ is the maximum likelihood estimator of $\dfrac{\theta}{1-\theta}$.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

45. Let $x_1, x_2, ..., x_n$ be a random sample from a distribution with probability density function $f(x; \theta) = \dfrac{1}{\pi} \cdot \dfrac{1}{1+(x-\theta)^2}$; $-\infty < x < \infty$, θ is real. Then the minimum variance bound estimator of θ

(a) is $\dfrac{1}{n}\sum_{i=1}^{n} x_i$

(b) is $\sum_{i=1}^{n} x_i$

(c) is $\dfrac{1}{n}\sum_{i=1}^{n} x_i^2$

(d) does not exist

46. Let $x_1, x_2, ..., x_n$ be a random sample from a distribution with probability density function

$$f(x; \alpha) = \frac{1}{\Gamma(\alpha)} x^{\alpha-1} e^{-x}; \quad 0 \le x < \infty, \quad \alpha > 0$$

Then the estimator of α obtained by method of moments is

(a) $\dfrac{\bar{x}}{2}$

(b) $\bar{x}$

(c) $\dfrac{1}{\bar{x}}$

(d) $2\bar{x}$

47. In an SPRT with n, α and β, which one of the following is correct?

(a) $n \to$ fixed constant, $\alpha \to$ fixed constant, $\beta \to$ fixed constant

(b) $n \to$ random variable, $\alpha \to$ fixed constant, $\beta \to$ fixed constant

(c) $n \to$ fixed constant, $\alpha \to$ random variable, $\beta \to$ random variable

(d) $n \to$ random variable, $\alpha \to$ random variable, $\beta \to$ fixed constant

where the symbols have their usual meanings.

48. Let $f(x;\theta) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\theta)^2}{2}}; \quad -\infty < x < \infty,$ $-\infty < \theta < \infty$. The sequential probability ratio test is used for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (> \theta_0)$. Then we reject H_0 if

(a) $\sum_{i=1}^{m} x_i \le \frac{1}{\theta_1 - \theta_0} \log\left(\frac{\beta}{1-\alpha}\right) + \frac{m(\theta_0 + \theta_1)}{2}$

(b) $\sum_{i=1}^{m} x_i \ge \frac{1}{\theta_1 - \theta_0} \log\left(\frac{1-\beta}{\alpha}\right) + \frac{m(\theta_0 + \theta_1)}{2}$

(c) $\sum_{i=1}^{m} x_i = \frac{1}{\theta_1 - \theta_0} \log\left(\frac{\beta}{1-\alpha}\right) - \frac{m(\theta_0 + \theta_1)}{2}$

(d) $\sum_{i=1}^{m} x_i \ge \frac{1}{\theta_1 - \theta_0} \log\left(\frac{1-\beta}{\alpha}\right) - \frac{m(\theta_0 + \theta_1)}{2}$

49. Consider the following statements :

I. Neyman-Pearson lemma gives a general method for testing simple null hypothesis against composite alternative hypothesis.

II. Likelihood ratio test gives a general method for testing simple or composite null hypothesis against simple or composite alternative hypothesis.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

50. For testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$, the 90% confidence interval for μ of normal distribution, when σ^2 is unknown, is (360, 400). Then the value of sample mean is given by

(a) 380

(b) 390

(c) 400

(d) 420

51. Let A be the matrix of 6 columns with rank 3 and y be a non-null vector. The number of linearly independent solutions of the consistent equation $Ax = y$ is

(a) 3

(b) 4

(c) 5

(d) 6

52. Consider the following statements under the linear model $\mathbf{Y} = X\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ and G be a generalized inverse of $X'X$:

I. XGX' is invariant to G.

II. XGX' is symmetric only when G is symmetric.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

53. Consider the following statements :

I. The rank of a generalized inverse of a matrix A does necessarily have the same rank as A.

II. The rank of a generalized inverse of a matrix A is the same as the rank of A, if and only if it is a reflexive inverse.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

54. Let Y_1, Y_2, Y_3 and Y_4 be uncorrelated observations with common unknown variance σ^2. Let

$$E(Y_1) = \beta_1 + \beta_2 + \beta_3 = E(Y_2)$$

$$E(Y_3) = \beta_1 - \beta_2 = E(Y_4)$$

where β_1, β_2 and β_3 are unknown parameters. Define $e_1 = \frac{1}{\sqrt{2}}(Y_1 - Y_2)$ and $e_2 = \frac{1}{\sqrt{2}}(Y_3 - Y_4)$. An unbiased estimate of σ^2 is

(a) $\frac{1}{2}(e_1^2 - e_2^2)$

(b) $\frac{1}{2}(e_1^2 + e_2^2)$

(c) $\frac{1}{4}(e_1^2 + e_2^2)$

(d) $e_1^2 + e_2^2$

55. Consider Gauss-Markov linear model as $E(Y_1) = \beta_1 + \beta_2$, $E(Y_2) = \beta_1 - \alpha\beta_2$, $E(Y_3) = 2\beta_1 - \beta_2$. For what value of α, BLUEs of β_1 and β_2 (i.e., $\hat{\beta}_1$ and $\hat{\beta}_2$) are uncorrelated?

(a) -2

(b) -1

(c) 1

(d) 2

56. Let Y_1, Y_2 and Y_3 be uncorrelated observations with common variance and expectations given by $E(Y_1) = \beta_1$, $E(Y_2) = \beta_2$ and $E(Y_3) = \beta_1 + \beta_2$, where β_1 and β_2 are unknown parameters. The BLUE of $(\beta_1 + \beta_2)$ is

(a) Y_3

(b) $Y_1 + Y_2$

(c) $\dfrac{Y_1 + Y_2 + 2Y_3}{3}$

(d) $\dfrac{Y_1 + Y_2 + Y_3}{3}$

57. Model–I : $Y = \beta_0 + \beta_1 X$ and Model–II : $Y = \beta_0 + \beta_1 X + \beta_2 X^2$ are fitted for the data set (X_i, Y_i); $i = 1, 2, ..., n$.

Denote $\hat{\beta}_0$ and $\hat{\beta}_1$ as least square estimates of β_0, β_1 from Model–I and β_0^*, β_1^*, β_2^* be the least square estimates from Model–II.

Define $P = \sum_{i=1}^{n} (Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i)^2$ and $Q = \sum_{i=1}^{n} (Y_i - \beta_0^* - \beta_1^* X_i - \beta_2^* X_i^2)^2$.

Consider the following statements :

I. $P \geq Q$

II. It can happen that $Q = 0$ but $P > 0$.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

58. Using the ordinary least squares method, Student–I fits the model $\hat{Y}_i = \hat{\alpha}_0 + \hat{\alpha}_1 X_i$ (that is Y on X) and Student–II fits the model $\hat{X}_i = \hat{\beta}_0 + \hat{\beta}_1 Y_i$ (that is X on Y) for the given data set (Y_i, X_i); $i = 1, 2, ..., n$. Which one of the following pairs is a possible value of $(\hat{\alpha}_1, \hat{\beta}_1)$?

(a) $(-0 \cdot 3,\ 0 \cdot 6)$

(b) $(0 \cdot 7,\ 3 \cdot 5)$

(c) $(0 \cdot 5,\ -0 \cdot 5)$

(d) $(-1 \cdot 5,\ -0 \cdot 3)$

59. Let $\lambda'\beta$ be an estimable function of β in the model $\mathbf{Y} = X\beta + \varepsilon$, where $E(\mathbf{Y}) = X\beta$ and X is an $n \times p$ matrix of rank $k < p \leq n$. Let $\hat{\beta}$ be any solution to the normal equations $X'X\hat{\beta} = X'\mathbf{Y}$ and let $\mathbf{r}$ be any solution to $X'X\mathbf{r} = \lambda$. Consider the following statements for the two estimators $\lambda'\hat{\beta}$ and $\mathbf{r}X'\mathbf{Y}$:

I. $\lambda'\hat{\beta}$ is equal to $\mathbf{r}'X'\mathbf{Y}$ for any $\hat{\beta}$ or $\mathbf{r}$.

II. $\lambda'\hat{\beta}$ and $\mathbf{r}'X'\mathbf{Y}$ are invariant to the choice of any $\hat{\beta}$ or $\mathbf{r}$.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

60. In the Gauss-Markov linear model $\mathbf{Y}_{n\times 1} = X_{n\times k}\beta_{k\times 1} + \varepsilon_{n\times 1}$, $E(\varepsilon) = \mathbf{0}$ with rank $r(X) = r < k$, the set

$$\{l \in \mathbb{R}^n : E(l'\mathbf{Y}) \equiv 0\}$$

is

(a) not a subspace

(b) a subspace of $\mathbb{R}^n$ with dimension $k - r$

(c) a subspace of $\mathbb{R}^n$ with dimension $n - k$

(d) a subspace of $\mathbb{R}^n$ with dimension $n - r$

61. Which of the following divisions are parts of the National Statistical Office of the Ministry of Statistics and Programme Implementation?

I. National Accounts Division (NAD)

II. Field Operations Division (FOD)

III. Infrastructure and Project Monitoring Division (IPMD)

IV. Data Informatics and Innovation Division (DIID)

Select the correct answer using the code given below.

(a) I, II and III

(b) I, II and IV

(c) I, III and IV

(d) II, III and IV

62. Which one of the following is ***not*** an objective of the International Comparison Programme (ICP)?

(a) To produce purchasing power parities (PPPs)

(b) To produce comparable price level indexes (PLIs) of economies

(c) To monitor exchange rate of different currencies

(d) To convert measures of Gross Domestic Product (GDP) of different economies into a common currency

63. Consider the following statements :

I. The Ministry of Statistics and Programme Implementation is releasing the Consumer Price Index (CPI) with revised Base 2024 = 100.

II. The Ministry of Statistics and Programme Implementation is releasing the National Accounts Estimates with revised Base period 2023–2024.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

64. Which of the following are the new initiatives taken for the Census 2027?

I. Data will be collected using mobile applications.

II. Option to self-enumerate will be provided to the public.

III. Caste enumeration will be included.

Select the correct answer using the code given below.

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

65. Which of the following are correct regarding 'Swachh Survekshan'?

I. It is the world's largest urban cleanliness survey.

II. It is conducted every five years.

III. It is conducted by the Ministry of Statistics and Programme Implementation.

IV. It is conducted by the Ministry of Housing and Urban Affairs.

Select the answer using the code given below.

(a) I and IV only

(b) II and III only

(c) I, II and III

(d) I, II and IV

66. Which of the following statements are correct regarding 'Agriculture Census'?

I. The latest Agriculture Census has 2021–2022 as reference period.

II. It is conducted every five years.

III. It is conducted by the Ministry of Statistics and Programme Implementation.

IV. It is conducted by the Ministry of Agriculture and Farmers' Welfare.

Select the answer using the code given below.

(a) I and IV only

(b) II and III only

(c) I, II and III

(d) I, II and IV

67. Which of the following components has the highest share in the expenditure component of Gross Domestic Product (revised base)?

(a) Gross Fixed Capital Formation (GFCF)

(b) Government Final Consumption Expenditure (GFCE)

(c) Private Final Consumption Expenditure (PFCE)

(d) Exports (X)

68. Why are official statistics considered 'public good'?

I. They are financed from general tax revenue.

II. Their use by one person does not affect the use by others.

III. They are used by public officials but private persons are denied access to them.

Select the correct answer using the code given below.

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

69. Consider the following pairs :

	Index	*Released by*
I.	CPI-IW	Labour Bureau, Ministry of Labour and Employment
II.	CPI-AL/RL	National Sample Survey Office, Ministry of Statistics and Programme Implementation
III.	CPI-U, R, Combined	National Statistical Office, Ministry of Statistics and Programme Implementation
IV.	WPI	Office of the Economic Advisor, Ministry of Commerce and Industry

How many of the pairs given above are correctly matched?

(a) One

(b) Two

(c) Three

(d) All the four

70. Consider the following statements regarding the National Statistical Commission (NSC) :

I. NSC is statutory body created through an Act passed by the Parliament in 2003.

II. NSC has a part-time Chairperson, part-time Members and an ex officio Member.

III. The Chief Statistician of India (CSI) and Secretary to the Government of India in the Ministry of Statistics and Programme Implementation is the Secretary to NSC.

Which of the statements given above is/are correct?

(a) I and II

(b) II and III

(c) III only

(d) II only

71. As per the constitutional provisions in India, the subject matter of collection of statistics comes under the

I. Union List
II. State List
III. Concurrent List

Which of the above is/are correct?

(a) I only
(b) II only
(c) I and III only
(d) I, II and III

72. The Sustainable Development Goals (SDGs) framework has been developed by which one of the following agencies?

(a) The United Nations to track various socio-economic indicators for gender awareness
(b) The United Nations to measure progress across nations in respect of designated goals, targets and indicators by 2030
(c) The World Health Organization to carry out surveys and census for improving quality of life
(d) The World Health Organization to measure progress across nations in respect of designated goals, targets and indicators by 2030

73. Which of the following are included in the secondary sector in the estimation of GDP?

I. Manufacturing
II. Construction
III. Forestry and Fishing
IV. Mining and Quarrying

Select the correct answer using the code given below.

(a) I and III
(b) I and II
(c) I and IV
(d) II and III

74. Which of the following is/are correctly defined?

I. Neonatal Mortality Rate (NMR) is defined as

$$NMR = \frac{\text{Number of infant deaths of less than 29 days during the year}}{\text{Number of live births during the year}} \times 1000$$

II. Post-Neonatal Mortality Rate (PNMR) is defined as

$$PNMR = \frac{\text{Number of infant deaths of less than one year during the year}}{\text{Number of live births during the year}} \times 1000$$

Select the answer using the code given below.

(a) I only
(b) II only
(c) Both I and II
(d) Neither I nor II

75. Which of the following are correct regarding the Population Census 2027?

I. It will be the 15th Census in the country.
II. It will be the 8th after the Independence.
III. The reference date will be 1st March, 2027.
IV. The reference date will be 1st February, 2027.

Select the answer using the code given below.

(a) I and II
(b) I and III
(c) II and III
(d) II and IV

76. In general, inflation is calculated by which one of the following?

(a) Producer Price Index

(b) Consumer Price Index

(c) Wholesale Price Index

(d) A Weighted Aggregative Price Index

77. Which one of the following divisions under the Ministry of Statistics and Programme Implementation deals with eSAKSHI portal?

(a) Administrative Statistics and Policy Division (ASPD)

(b) Household Survey Division (HSD)

(c) Data Informatics and Innovation Division (DIID)

(d) Members of Parliament Local Area Development Scheme (MPLADS) Division

78. Consider the following data :

Commodity	*Base period price* (in ₹)	*Current period price* (in ₹)	*Percentage change*
P	20	40	100
Q	50	60	20
R	40	50	25
S	20	30	50

What will be the simple aggregative price index for the above data?

(a) 138·50

(b) 140·00

(c) 141·25

(d) 145·75

79. It is given that $\sum p_1 q_0 = 2070$, $\sum p_0 q_0 = 1660$, $\sum p_0 q_1 = 1470$ and $\sum p_1 q_1 = 1790$. The Fisher's Price Index is close to

(a) 115

(b) 119

(c) 123

(d) 127

80. In which one of the following price index numbers, weights are the arithmetic mean of the base year quantities and current year quantities?

(a) Walsch Price Index

(b) Dorbish-Bowley Price Index

(c) Fisher's Ideal Index

(d) Marshall-Edgeworth Price Index

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