GENERAL ECONOMICS

PAPER—I

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions.

There are THIRTEEN questions divided under THREE Sections.
The ONLY question in Section—A is compulsory.
In Section—B, FIVE out of SEVEN questions are to be attempted.
In Section—C, THREE out of FIVE questions are to be attempted.
Candidates should attempt questions/parts as per the instructions given in the Sections.
The number of marks carried by a question/part is indicated against it.
Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.
Any page or portion of the page left blank in the Question-cum-Answer (QCA) Booklet must be clearly struck off.
Candidates are required to write clear, legible and concise answers.
Answers must be written in ENGLISH only.
1. Answer all the following seven parts:

(a) In a two-commodity framework, the marginal rate of substitution is everywhere equal to 2. The prices of the two goods are equal. Draw a diagram to identify the utility maximizing equilibrium.

(b) The cost-minimizing demand for labor is

\[ L = \frac{Q}{50} \sqrt{\frac{r}{w}} \]

and that for capital is

\[ K = \frac{Q}{50} \sqrt{\frac{w}{r}} \]

where \( w \) and \( r \) denote wage and price of capital respectively. Find the production function.

(c) Explain the principle of average cost pricing in the context of a natural monopoly.

(d) Find a monopolist’s demand function for labor when the labor market is perfectly competitive.

(e) Explain the concept of external economies in the context of marginal social benefits and marginal social costs.

(f) Suppose that the Leontief input-output coefficient matrix is

\[ A = \begin{bmatrix} 0.1 & 0.4 \\ 0.2 & 0.5 \end{bmatrix} \]

and the final demand vector is \( \begin{bmatrix} 1 \\ 1 \end{bmatrix} \). Find the total direct and indirect requirement of the second input to satisfy the final demand.

(g) Show that in the regression model \( Y_i = \alpha + \beta X_i + U_i; \ i = 1, 2, \ldots, n, \) the covariance between the regressor and the error term is zero under ordinary least squares method of estimation.
SECTION—B

Answer any five of the following seven questions: 18 × 5 = 90

2. (a) An individual has the utility function \( U = XY \) and her budget equation is \( 10X + 10Y = 1000 \). Find the maximum utility that she can attain. 6

(b) If the price of good \( X \) decreases to 5, find the compensating variation in income in order to maintain her level of satisfaction in part (a). 6

(c) An individual buys two goods \( X \) and \( Y \) at prices \( P_X \) and \( P_Y \). Check whether her behavior satisfies the Weak Axiom of Revealed Preference, given the following information:
   - When \((P_X, P_Y) = (1, 2), (X, Y) = (1, 2)\)
   - When \((P_X, P_Y) = (2, 1), (X, Y) = (2, 1)\) 6

3. (a) Show that \( q = \gamma [\delta L^{-\alpha} + (1 - \delta)K^{-\alpha}]^{-\frac{1}{\alpha}} \) is a production function that represents the average of two inputs \( L \) and \( K \) for different values of \( \alpha \), given that \( \gamma > 0 \) and \( 0 < \delta < 1 \). 12

(b) Find the marginal rate of technical substitution for the production function given in part (a). 6

4. (a) A firm with market power faces the demand curve given by \( P = 100 - 3Q + 4\sqrt{A} \), where \( P \), \( Q \) and \( A \) denote price, quantity and expenditure on advertising respectively. The total cost is given as \( C = 4Q^2 + 10Q + A \). Find the firm’s profit-maximizing price. 8

(b) Suppose that the demand and supply functions in a market are given as
   \( Q_D = 100 - P \) and \( Q_S = 200 - 5P \)
   Analyze whether the equilibrium is Walrasian stable or Marshallian stable. 10

5. (a) Consider a profit-maximizing monopolist who is also a monopsonist and uses a single-variable input. Determine the equilibrium price of the input. 9

(b) In the context of part (a), explain the concept of monopsonistic exploitation of labor using a suitable diagram. 9
6. (a) Explain the conditions for Pareto optimality in exchange and production in a model where there are two consumers and two goods, each good being produced with labor and capital.

(b) In a competitive market, the demand function is given as $Q_D = 24 - P$, while the marginal cost of production is $MC = 2 + Q$. There is a negative externality giving rise to the total externality cost $C = -2Q + \frac{Q^2}{2}$. Compare the private and social optimum quantity and price.

7. Consider the following data on two variables:

<table>
<thead>
<tr>
<th>$X$</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>25</td>
<td>16</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
</tr>
</tbody>
</table>

(a) Find the product-moment correlation coefficient between $X$ and $Y$.

(b) Can you develop a suitable linear regression model to explain $Y$ with the help of $X$?

8. (a) Define Gini coefficient with the help of Lorenz curve and show that Gini is defined as

\[ Gini = 1 - 2 \times \text{Area below Lorenz curve} \]

(b) How is Gini affected if $\theta$ amount of income is added to all the persons in an income distribution?

(c) Show that Gini is distribution insensitive.

SECTION—C

Answer any three out of the following five questions: 25x3=75

9. (a) Write down the Slutsky equation in elasticity form and prove that the ordinary demand curve will have a greater demand elasticity than the compensated demand curve for a normal commodity. How does your result change if the commodity becomes inferior?
(b) A price-discriminating monopolist produces its output with a total cost (TC) function given by TC = 5Q + 20 and sells its output in two markets which are completely separated from each other. The demand curves for the two markets are

\[ Q_1 = 55 - R_1 \quad \text{and} \quad Q_2 = 70 - 2P_2 \]

Given this information, answer the following:

(i) What quantity should the monopolist sell in each market?
(ii) What price should be charged in each market?
(iii) Explain the result in terms of price elasticity of demand in the two markets.

10. (a) What are the characteristics of a public good?

(b) The demand for a public good is given as \( Q = 100 - R_1 \) and \( Q = 200 - P_2 \) for two individuals 1 and 2. Find the optimal provision of the public good if the marginal cost of producing it is (i) 240 and (ii) 50. In each case, draw diagrams to explain your answer.

(c) What is the Coase theorem? A factory pollutes a river where fishermen operate. A pollution treatment plant costs Rs 2 lakhs to install while the damage to the fishermen is Rs 1 lakh if the pollution treatment plant is not installed. Demonstrate the results of the Coase theorem when—

(i) the factory has the right to pollute;
(ii) the fishermen have the right to clean river water.

11. (a) Consider the multiple regression model \( Y_i = \alpha + \beta_2 X_{2i} + \beta_3 X_{3i} + U_i \); \( i = 1, 2, \ldots, n \), where \( U_i \)'s are independent and normally distributed with mean zero and variance \( \sigma^2 \). Also consider the auxiliary regressions

\[ X_{2i} = \hat{\alpha} + \hat{\beta} X_{3i} + \hat{V}_{2i} \]
\[ X_{3i} = \hat{\alpha} + \hat{\beta} X_{2i} + \hat{V}_{3i} \]

where \( \hat{V}_2 \) and \( \hat{V}_3 \) are error terms. Show that \( \hat{\beta}_2 \), the ordinary least squares estimate of \( \beta_2 \), can be interpreted as a simple regression of \( Y \) on \( \hat{V}_2 \).
(b) Consider the regression \( Y_i = \gamma + \delta_2 \hat{V}_{2i} + \delta_3 \hat{V}_{3i} + W_i \), where \( \hat{V}_2 \) and \( \hat{V}_3 \) are as defined in part (a). Find the relation between the ordinary least squares estimate \( \hat{\delta}_2 \) and the estimate \( \hat{\beta}_2 \) as defined in part (a).

(c) (i) In the regression model of part (a), what will be the variance of the ordinary least squares estimate \( \hat{\beta}_2 \)?

(ii) Develop the \( F \)-test statistic for the goodness of fit of the regression model of part (a).

12. (a) Suppose that the relation to be estimated is \( Y_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + U_i \) and \( X_1 \) and \( X_2 \) are related with the exact relation \( X_2 = kX_1 \), where \( k \) is an arbitrary constant. Show that the estimates of the coefficients are indeterminate and standard errors of these estimates become infinitely large.

(b) Assume three explanatory variables like \( X_1, X_2 \) and \( X_3 \), which are found to be highly collinear. The following three principal components (PC) and their corresponding eigenvalues (\( \lambda \)) are reported as

\[
\begin{align*}
PC_1 &= a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\
PC_2 &= a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\
PC_3 &= a_{31}x_1 + a_{32}x_2 + a_{33}x_3
\end{align*}
\]

where \( a_{ij} \) be the factor loading of the \( i \)-th principal component of the \( j \)-th factor, and \( \lambda_1, \lambda_2 \) and \( \lambda_3 \) are the eigenvalues of first, second and third principal components respectively. How are factor loadings (\( a_{ij} \)) related to eigenvalue in each PC? What is the statistical meaning of the square of \( a_{ij} \) in each PC? Do you think that \( \lambda_1 > \lambda_2 > \lambda_3 \)? What is the sum of \( \lambda \)'s?

(c) How do you use principal component analysis to tackle the problem of multicollinearity in regression analysis?

13. (a) The demand function \( Q_1 = 50 - p_1 \) intersects another linear demand function \( Q_2 \) at \( p = 10 \). The elasticity of demand for \( Q_2 \) is six times larger than that of \( Q_1 \) at that point. Find out demand function for \( Q_2 \).

(b) For monopolist, the demand law is \( p = (6 - X)^2 \) and the marginal cost is \( 14 + X \). Find consumer's surplus.
A firm is producing two goods $A$ and $B$. It has two factories that jointly produce the two goods in the following quantities (per hour):

<table>
<thead>
<tr>
<th></th>
<th>Factory 1</th>
<th>Factory 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good $A$</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Good $B$</td>
<td>25</td>
<td>25</td>
</tr>
</tbody>
</table>

The firm receives an order for 300 units of $A$ and 500 units of $B$. The costs of operating the two factories are $10,000$ and $8,000$ per hour. Formulate the linear programming problem of minimizing the total cost of meeting this order. Also find the minimum cost.