GEOPHYSICS

PAPER—I

Time Allowed : Three Hours	Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are **TEN** questions divided under **TWO** Sections.

Candidate has to attempt **SIX** questions in all.

Question Nos. **1** and **6** are **compulsory**.

Out of the remaining **EIGHT** questions, **FOUR** questions are to be attempted choosing **TWO** from each Section.

The number of marks carried by a question/part is indicated against it.

Neat sketches may be drawn to illustrate answers, wherever required. These shall be drawn in the space provided for answering the question itself.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

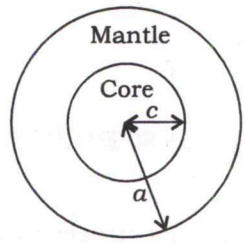
Answers must be written in **ENGLISH** only.

SECTION—A

1. *(a)* Formulate a linear inverse problem to determine ρ_c (average density of core) and ρ_m (average density of mantle) from the two data points d_1 and d_2 as per the details given below :

d_1 : Mass of the Earth

d_2 : Moment of inertia



c : Radius of core

a : Radius of mantle

Also write the expression for least square solution of the inverse problem. (Assume the Earth as a spherical body) 10

(b) *(i)* Explain, briefly, seismic body waves using suitable sketches. 5

(ii) Differentiate between compressional waves and transverse waves. 5

(c) What is natural period of seismometer? At critical damping, the response of seismometer to a periodic disturbing signal with frequency ω is given by

$$u = \frac{A\omega^2}{(\omega_0^2 + \omega)} \cos(\omega t - \Delta)$$

Using this equation, define how long-period seismometer becomes displacement meter and short-period seismometer becomes accelerometer. Also give the frequency range of seismic signals recorded by displacement seismometer and accelerometer. 10

(d) *(i)* Explain divergence theorem. 4

(ii) Solve the following using divergence theorem :

If $\vec{F} = xy\hat{i} + y^2 z\hat{j} + z^2\hat{k}$, evaluate $\iint_S (\vec{F} \cdot \hat{n})\, dS$, where S is the unit cube defined by $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$. 6

2. *(a)* What is 'a priori' information? Assume that the average value of three model parameters (m_1, m_2, m_3) of a linear inverse problem is given 'a priori' information. Write this information in the form $\underline{F}\ \underline{m}=\underline{h}$ ($\underline{m}$ is model parameter vector). Write an expression for least square solution of a linear inverse problem without and with 'a priori' information. 10

(b) What do you understand by the reference spheroid and the geoid? Explain with the help of suitable sketch for the situations—

(i) as wrapping of the geoid for local mass;

(ii) large-scale wrapping. 10

(c) *(i)* Give the geometric interpretation of Snell's law. How can Fermat's principle be used in deriving Snell's law? 8

(ii) Give ray curvature due to increasing and decreasing velocity with depth. 2

3. *(a)* *(i)* Describe, briefly, Biot-Savart law with the help of suitable diagram. 5

(ii) Explain vector potential of a current element as drawn for above *(i)*. 5

(b) Describe the orbit of a planet or comet in the solar system as an ellipse with the Sun at one of its focal points. Write down the condition under which the object will escape from the solar system. 8+2=10

(c) What is generalized inverse $\underline{G}^{-g}$? Write the expressions for generalized inverse ($\underline{G}^{-g}$) for least square solution, minimum length solution and damped least square solution of a linear inverse problem $\underline{d}=\underline{G}\,\underline{m}$ ($\underline{d}$: data vector, $\underline{G}$: data kernel and $\underline{m}$: model parameter vector). 10

4. *(a)* *(i)* Show that the Laplace equation $\nabla^2 V=0$ satisfies for current flow in a homogeneous medium, where V is electrical potential.

(ii) Solve the Laplace equation when single current electrode is kept at a depth in the homogeneous and isotropic media. 10

(b) What is ray parameter in seismology? Derive the travel time equation for spherical Earth

$$T = P\Delta + 2\int_{r_t}^{r_0} \frac{\sqrt{\xi^2 - P^2}}{r}\, dr$$

$$\Delta = 2P\int_{r_t}^{r_0} \frac{dr}{r\sqrt{\xi^2 - P^2}}$$

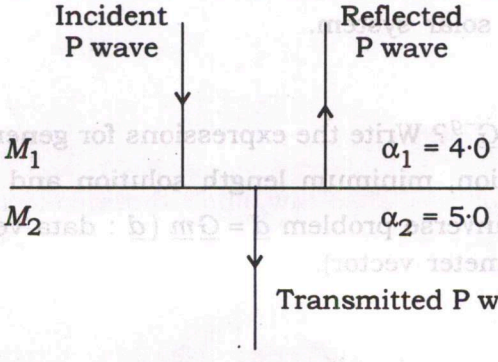
where $\xi = \frac{r}{v}$, r_t is radius to the turning point of the ray, $P = \frac{r \sin i}{v(r)}$ and r_0 is the radius of the sphere. 10

(c) Write an expression for Singular Value Decomposition (SVD) of a matrix. Describe the SVD analysis of a least square solution of a linear inverse problem. How does SVD analysis help to identify an ill-posed inverse problem? 10

5. (a) (i) Try to draw a suitable diagram illustrating the difference between the Earth's geographic, geomagnetic and magnetic poles and equator. 5

(ii) The palaeomagnetic latitude from the magnetic measurement was made on basalt flow at present at 47° N, 20° E. The angle of inclination of the remanent magnetization of this basalt is 30°. Compute the magnetic latitude at this site at the time the basalt was magnetized. 5

(b) (i) For normal incidence P wave, give the expression of reflection and transmission coefficient. Compute the reflection coefficient (RC) and transmission coefficient (TC), energy reflected (E_r) and energy transmitted (E_t). M_1 and M_2 are medium one and medium two : 6



(ii) Describe *b* value in seismology and the equation used to calculate *b* value from earthquake data. 4

(c) Explain the gravity effect of a sphere at a point with the help of suitable sketch. Compute depth to the centre of the spherical body, z and mass of the anomalous body, M from the given data as $x_{1/2} = 25$ m and $g_{max} = 0 \cdot 5$ mgal. 10

SECTION—B

6. *(a)* Using Cayley-Hamilton theorem matrix, find the eigenvalues of the matrix

$$B = A^4 - 3A^3 + 3A^2 - 2A + 8I$$

where $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$. 10

(b) What is Cherenkov radiation? Derive the condition for emission of Cherenkov radiation from a charged particle passing through a medium. 10

(c) Derive the condition for static equilibrium for plasma as a conducting fluid. 10

(d) When radio waves propagate through the ionosphere, show that the induced current developed due to propagation of wave is inductive in nature. 10

7. *(a)* Given, $z(x, 0) = x^2$ and $z(1, y) = \cos y$. Find the solution of

$$\frac{\partial^2 z(x, y)}{\partial x \partial y} = x^2 y$$

10

(b) If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ and I is a 2×2 identity matrix, then what will be the value of e^A? 10

(c) Show that the Fourier transform of a Gaussian function $f(x) = Ae^{-\beta x^2}$ is also a Gaussian function, where both A and β are constants. 10

8. *(a)* Prove that for a moving conductor loop in a time-varying magnetic field

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t} + \nabla \times (\vec{u} \times \vec{B})$$

10

(b) Show that the radiation emitted by a charged particle (Bremsstrahlung) is independent of acceleration or deceleration. 10

(c) For an aluminium airplane, using Gauss' law, show that no electric field exists inside the plane. 10

9. *(a)* Describe the physical significance of Maxwell's equations in differential and integral forms. Explain Faraday's law and its consequences using differential form of equations. 10

(b) Prove that for an electromagnetic wave, the sum of reflection coefficient and transmission coefficient at normal incidence is equal to unity. 10

(c) If two plane waves travelling with speed c at angles θ_1 and θ_2, compare their net velocities in a rectangular waveguide. 10

10. *(a)* *(i)* Explain the term 'fading' in radio wave propagation and state the factors responsible for fading. 5

(ii) Discuss in brief how fading can be minimized for radio wave propagation. 5

(b) *(i)* Explain the terms 'virtual height', 'skip distance' and 'maximum usable frequency' as applied to sky wave propagation. 6

(ii) For the Earth, the atmosphere consists of 22% O_2 and 78% N_2. What is the scale height of the Earth atmosphere?

(Given, $g = 9 \cdot 81 \text{ m/s}^2$, $T = 290 \text{ K}$ and Boltzmann constant $k = 1 \cdot 38 \times 10^{-23}$ joule per kelvin) 4

(c) *(i)* What is satellite communication? Distinguish between active and passive satellite communication systems. 4

(ii) Calculate the value of frequency at which an electromagnetic wave must propagate through the D-region having refractive index 0·5 and electron density of $3 \cdot 24 \times 10^4$ electrons/m^3. 6

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