GEO-PHYSICS

Paper – I

Time Allowed : Three Hours | Maximum Marks : 200

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **TEN** questions divided under **TWO** sections.*

*Candidate has to attempt **SIX** questions in all.*

*Questions No. **1** and **6** are **compulsory**.*

*Out of the remaining **EIGHT** questions, **FOUR** questions are to be attempted choosing **TWO** from each section.*

The number of marks carried by a question / part is indicated against it.

Neat sketches may be drawn to illustrate answers, wherever required. These shall be drawn in the space provided for answering the question itself.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

*Answers must be written in **ENGLISH** only.*

SECTION A

Q1. (a) An earthquake causes an average of 2·5 m strike-slip displacement over an 80 km long, 23 km deep portion of a transform fault. Assuming that the rock along the fault has an average rupture strength of 175 kPa, estimate the seismic moment (M_0) and moment magnitude (M_w) of the earthquake. *10*

(b) Derive the expression for electric potential due to a point current source placed on the surface of a homogeneous medium. Explain with a neat sketch of current flow and equipotential lines. *10*

(c) Explain different sources of heat on the Earth's surface. Show the variation of heat flow with depth within the internal structure of the Earth. *10*

(d) Define Singular Value Decomposition (SVD). Verify the SVD problem for the given rectangular matrix :

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ *3+7*

Q2. (a) (i) Define Gauss divergence theorem and explain its significance.

(ii) The region $r \leq l$ in spherical coordinates has an electric field intensity

$$\overline{E} = \frac{\rho r}{3\varepsilon}\, \overline{a}_r$$

where 'l' is the radius of the spherical volume and $\overline{a}_r$ is the unit vector.

Prove the divergence theorem for the $\overline{E}$ field. *5+5*

(b) Discuss ray theory for horizontally stratified earth and explain how to get velocity of fairly thick layers and series of thin layers. *10*

(c) Using Maxwell's equations, relating the electric and magnetic field vectors, derive the expression for skin depth associated with electromagnetic induction phenomenon. *10*

(Note : Consider the inducing magnetic field is of the form, $H = H_0\, e^{i\omega t}$)

Q3. (a) Estimate the surface rupture length, rupture area and maximum surface displacements for earthquake of $M_w = 6{\cdot}0$, assuming the earthquake has occurred along strike-slip fault. *10*

(b) (i) Define sensitivity matrix in inverse problems and describe the methodology to determine it.

(ii) Explain the Benioff – Wadati zone with a neat sketch. In what type of tectonic setting is it observed ? *5+5*

(c) Discuss the procedure for linearization of the non-linear problem in geophysical inverse theory. *10*

Q4. (a) (i) A sinusoidal EM wave is propagating in free space with a period of 2×10^{-6} sec. Calculate the phase difference in electric fields at two points at distances of 1200 m and 1500 m from the source.

(ii) An infinitely long conducting thick cylindrical shell of inner radius r_1 and outer radius r_2 has its axis along z-axis. z-axis carries a line charge density of λ/unit length. Find the electric field at points in 3 regions :

(I) $\rho < r_1$

(II) $r_1 < \rho < r_2$

(III) $\rho > r_2$

where ρ is the distance of field point from z-axis. *5+5*

(b) Draw p-wave velocity, s-wave velocity and density variation with depth from the surface of the Earth with a neat sketch diagram. Explain major discontinuities with these physical parameters. *10*

(c) Which seismic phases are observed in shadow zones ? Explain their travel path with appropriate diagram. *10*

Q5. (a) When the transmitter (Tx) and receiver (Rx) placed on the surface of the Earth are separated by a large distance, show that the magnetic field measured at Rx is much smaller and weaker, compared to that at Tx. *10*

(b) Compute the p-wave and s-wave velocities of a material whose specific gravity is $7{\cdot}85 \times 10^3$ kg/m^3 and Young's modulus is $2{\cdot}8 \times 10^5$ MPa and shear modulus is $7{\cdot}9 \times 10^4$ MPa. *10*

(c) (i) Define Königsberger's ratio (Q). Write the examples of the classification of rocks for Q >> 1 and Q << 1.

(ii) Distinguish between absolute and relative gravity. Briefly explain the methodology of absolute gravity measurements. *5+5*

SECTION B

Q6. (a) Show that the integral

$$\int_A^B P\,dx + Q\,dy,$$

where $P = 3x^2y$, $Q = x^3 + 1$, is independent of path joining points A and B. Find the value of integral when A has coordinates (1, 2) and the point B is (4, 5). *10*

(b) Use Simpson's rule to find the numerical value of $\int_1^5 x^2dx$ by taking $\Delta x = 1$. *10*

(c) (i) Write down the Maxwell's equation in the interior of a perfect waveguide.

(ii) What are 'TE' wave, 'TM' wave and 'TEM' wave ? *4+6*

(d) (i) Briefly discuss L_1 and L_2 band in GPS communication system.

(ii) Define C/A-code and P-code in GPS system. Make a comparison of C/A-code and P-code.

(iii) Name India's first satellite launched for GPS communication. *3+6+1*

Q7. (a) Use the method of contour integration in complex plane to show that

$$\int_0^\infty \frac{dx}{(x^2+1)(x^2+9)(x^2+16)} = \frac{\pi}{420}.$$ *10*

(b) A dipole rotating with angular frequency ω is given by

$$\vec{p} = p_0\,[\hat{x}\cos\omega t + \hat{y}\sin\omega t]$$

$\hat{x}$, $\hat{y}$ are unit vectors along the X and Y axes respectively. Find the potentials $\vec{A}$, ϕ due to the rotating dipole. You may use the following formula for dipole p_0 along z-axis :

$$\phi(x, y, z, t) = \frac{-p_0\omega}{4\pi\varepsilon_0 c}\left(\frac{z}{x^2+y^2+z^2}\right)\sin[\omega(t - rk)]$$

$$A(x, y, z, t) = \frac{-\mu_0\omega p_0}{4\pi r}\sin\left[\omega\left(t - \frac{r}{c}\right)\right]\frac{z}{r}$$ *10*

(c) (i) Explain with diagram the height profile of atmosphere with respect to atmospheric density, pressure, velocity of sound and temperature.

(ii) Find the transmitting range for an antenna of height 100 metres. *7+3*

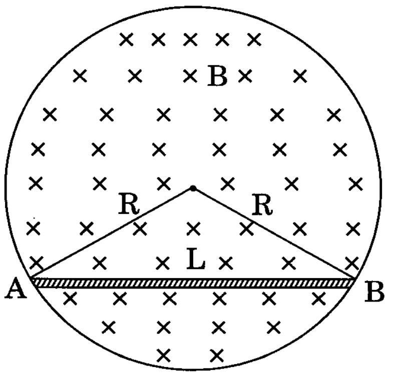
Q8. (a) Use Laplace transform to solve the differential equation

$$\frac{dy}{dx} + 2y = \cos x$$ *10*

(b) Explain the laws of electromagnetic induction. A conducting rod of length L is kept in a magnetic field which fills a cylindrical volume of radius R. It is given that the magnetic field is uniform and is increasing with time

$$B = B_0 t$$

and points along the axis of cylinder. Find the potential difference between the ends of the rod. *10*

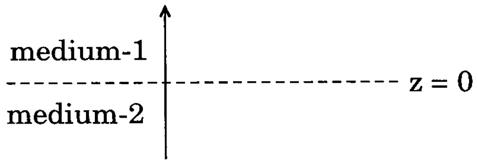


Magnetic field is uniform and points into the plane of paper.

(c) Consider the atmospheric pressure and temperature at sea level are P_0 and T_0 respectively. Assume that the temperature decreases uniformly with increase in height at a rate of τ K/km. Derive the relationship for the height 'z' of a pressure surface P. *10*

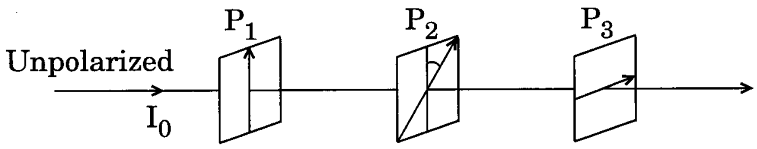
(Note : Consider the air as ideal gas)

Q9. (a) Two infinitely extended homogeneous isotopic dielectric media (medium-1 and medium-2 with dielectric constant $\frac{\varepsilon_1}{\varepsilon_0} = 2$ and $\frac{\varepsilon_2}{\varepsilon_0} = 5$, respectively) meet at the z = 0 plane as shown in the figure. A uniform electric field is given by $\vec{E}_1 = 2\hat{i} - 3\hat{j} + 5\hat{k}$. The interface separating the two media is charge free. Calculate the electric displacement vector in the medium-2. *10*



(b) (i) What are the uses of a polaroid ?

(ii) Consider three polarizers P_1, P_2 and P_3 placed along an axis as shown in the figure.



The pass axes of P_1 and P_3 are at right angles to each other while the pass axis of P_2 makes an angle θ with that of P_1. A beam of unpolarized light of intensity I_0 is incident on P_1 as shown. Calculate the intensity of light emerging from P_3. *2+8*

(c) Find the Green's function $G(x, x_0)$ satisfying the differential equation

$$\frac{d^2G(x, x_0)}{dx^2} = \delta(x - x_0)$$

and obeying the boundary conditions

$$G(-a, x_0) = 0; \; G(a, x_0) = 0.$$ *10*

Q10. (a) Starting from Maxwell's equations in free space, define scalar and vector potentials $\phi(\vec{r}, t)$, $\vec{A}(\vec{r}, t)$. Show that $\phi'(\vec{r}, t)$ and $\vec{A}'(\vec{r}, t)$ related by

$$\phi'(\vec{r}, t) = \phi(\vec{r}, t) - \Lambda(\vec{r}, t)$$

$$\vec{A}'(\vec{r}, t) = \vec{A}(\vec{r}, t) + \nabla\Lambda(\vec{r}, t)$$

give rise to same fields as ϕ and $\vec{A}$. Here $\Lambda(\vec{r}, t)$ is an arbitrary function of space time. *10*

(b) (i) Write down the differences between ground wave and sky wave.

(ii) What is VSAT communication system ? Draw the basic building block of a VSAT system. *4+6*

(c) (i) Write down the Lorentz gauge transformations of scalar potential ϕ and vector potential $\vec{A}$.

(ii) A plane electromagnetic wave is propagating in a lossless dielectric medium. The electric field is given by

$$\vec{E}(x, y, z, t) = E_0(\hat{x} + A\hat{z}) \exp [ik_0\{-ct + (x + \sqrt{3}\,z)\}]$$

where c is the velocity of light in vacuum, E_0, A and k_0 are constants and $\hat{x}$ and $\hat{z}$ are unit vectors along the x and z axes. Calculate the relative dielectric constant of the medium ε_r and constant A. *2+8*