वियोज्य DETACHABLE

GEOPHYSICS

Paper I

Time Allowed : Three Hours	Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions.

There are **TEN** questions divided under **TWO** Sections.

Candidate has to attempt **SIX** questions in all.

Questions No. **1** and **6** are ***compulsory***.

Out of the remaining **EIGHT** questions, **FOUR** questions are to be attempted choosing **TWO** from each Section.

The number of marks carried by a question/part is indicated against it.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the answer-book must be clearly struck off.

Answers must be written in **ENGLISH** only.

Neat sketches may be drawn to illustrate answers, wherever required.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

SECTION 'A'

1.(a) How many half-lives must elapse before the activity of a radioactive isotope decreases to 2% of its initial value? How long is this time for ^{14}C (carbon 14 isotope), which has a decay rate of $1{\cdot}21\times10^{-4}$ yr^{-1}? 10

1.(b) The moment of the recent Indonesian earthquake has been estimated to be about $1{\cdot}0\times10^{23}$ Nm. Assume that the fault is horizontal, where slip area is 100 km wide and 1300 km long, and the shear modulus $\mu = 3{\cdot}0\times10^{10}$ N/m^2, calculate the average displacement on the fault. 10

1.(c) Discuss about the origin of the geomagnetic field. 10

1.(d) The travel time t_{ij} measured for a seismic refraction survey over a two-layered Earth structure with velocities v_0 and v_1 $(v_1 > v_0)$ follows the relation

$$t_{ij} = \frac{x_{ij}}{v_1} + P_i + q_j$$

where x_{ij} is the horizontal distance between shots (s_1 & s_2) and Receivers (R_1, R_2 & R_3), P_i & q_j are delay times associated with the ith shot and jth receiver positions. Formulate the problem in a standard $d = Gm$ matrix equation, where d is the data, G is the kernel and m is the model parameter matrix. 10

2.(a) (i) How is the Earth's core significant to the life on earth? Explain on the basis of Geophysics and other scientific observations.

(ii) Define horizontal with respect to equipotential Earth's surface through an illustrative figure. Also explain the work done by the force $\vec{F}$, causing a displacement $d\vec{r}$ in a direction making an angle θ between the two vectors $\vec{F}$ & $d\vec{r}$. 5+5=10

2.(b) Assume that the horizontal component of the 2-D stress tensors are

$$\tau = \begin{bmatrix} \tau_{xx} & \tau_{xy} \\ \tau_{yx} & \tau_{yy} \end{bmatrix} = \begin{bmatrix} -30 & -20 \\ -20 & -40 \end{bmatrix} \text{MPa}$$

(where MPa stands for Mega Pascal)

Calculate the normal and shear stresses on a fault that strikes 15° East of North. 10

2.(c) Calculate the magnitude (M_d) of the dipole field and its location (θ_d, ϕ_d) on the Earth's surface for epoch IGRF 2000, where θ_d is the colatitude and ϕ_d is longitude with respect to Greenwich meridian. [Use Gauss coefficients for IGRF 2000 : $g_1^0 = -29556{\cdot}8\ nT,\ g_1^1 = -1671{\cdot}7\ nT,\ h_1^1 = 5079{\cdot}8\ nT$.] 10

3.(a) Derive the expression for unit covariance matrix using the generalised inverse matrix $\left[G_g^{-1}\right]$ and discuss its relative significance over the data and model resolution matrices. 10

3.(b) The gravity anomaly (g_z) profile over a long horizontal cylinder of radius R and density ρ along a line on the surface perpendicular to its axis is given as :

$x(m)$	$g_z(m\ gal)$
0	12·00
2	11·30
4	9·65
6	8·00
8	6·60
10	5·25
12	4·20
14	3·45
16	2·75
18	2·15
20	1·80

Determine depth of the centre of cylinder in terms of R & ρ from one-third $x_{1/3}$ on gravity profile, where $x_{1/3}$ is the distance at which g_z has fallen off to $\frac{1}{3}$rd of its maximum value. [Use G = gravitational constant] 10

3.(c) Assume harmonic P-waves are travelling through a solid with velocity $\alpha = 10$ km/sec. If the maximum strain is 10^{-8}, what is the maximum particle displacement for waves with period of : (i) 1 sec; (ii) 10 sec; (iii) 100 sec ? 4+3+3=10

4.(a) (i) State whether an earthquake can affect the orientation of the Earth's rotation axis. Give your answer with proper justification.

(ii) When is a long period seismometer used ? Justify as to why the broadband seismometer are preferred for most of the seismological studies. 5+5=10

4.(b) Obtain the forward time centered space (FTCS) finite difference approximation for the transient 1-D heat. Conduction in a layer of thickness d and expressed as

$$\frac{\partial H}{\partial t} = \beta \frac{\partial^2 H}{\partial z^2}, \quad \begin{array}{l} 0 \le z \le d, \\ t > 0 \end{array}$$

where β is the thermal diffusivity. The boundary and initial conditions are : $H(0, t) = H_0$, $H(d, t) = H_d$ and $H(z, 0) = f_0(z)$. Also discuss the criteria for the stability of the FTCS solution. 10

4.(c) What are the geophysical constraints provided by the Gauss's Law for magnetic field due to the localised magnetic sources buried in the subsurface. Assume the lateral extent of the magnetic survey to be large compared to the size of the magnetic sources. 10

5.(a) (i) What is the P/S (Primary/Secondary or α/β) velocity ratio for a rock with a Poisson's ratio of 0·35 ? Explain the nature of physical property of the rock on the basis of Poisson's ratio and P/S velocity value determined.

(ii) Write the continuity equation for mantle mass displacement in Eulerian and Lagrangian form. Explain all the variables with physical significance. 5+5=10

5.(b) (i) Draw the following seismic arrival phases by the help of simple (free hand sketch) ray tracing :

PcP; ScS; SS; SKS; SKIKS; PKiKP

(ii) Find the exact origin (including seconds) of an earthquake that occurred on 7th August 1999 at 3 hr 40 minutes, where following P-wave and S-wave arrival times are given below :

$t_P(seconds)$	$t_S(seconds)$
38·0	39·0
38·7	40·3
39·0	41·0
39·6	42·1
41·0	44·1

(Linear free hand plot is required) 5+5=10

5.(c) Derive the underdetermined inverse problem for M unknowns and N 'a priori' or 'constraints' using Lagrangian multipliers. 10

SECTION 'B'

6.(a) Show that the conditions for a thermodynamic system to be in equilibrium with a heat reservoir at a constant temperature T in terms of changes in the Helmholtz function F and the Gibbs function G are

$(dF)_{T,\,V} \leqslant 0$ at constant volume

$(dG)_{T,\,P} \leqslant 0$ at constant pressure

Give a physical interpretation to the above inequalities. 10

6.(b) Using Cauchy's residue theorem evaluate the integral

$$\int_0^{\pi} \frac{d\theta}{a - b\cos\theta}$$

where a and b are real and $a > b > 0$. 10

6.(c) Write down the Abraham-Lorentz formula in connection with radiation reaction force. Discuss the implications of this formula by using a suitable example. 4+6=10

6.(d) Write down the hydrostatic equation, explicitly mentioning the symbols' meanings. Discuss its solution. 5+5=10

7.(a) Solve the heat equation for temperature variation function $u(x, y, t)$ for a rectangular plate of dimension (a, b) for the boundary condition

$u(0, y, t) = u(a, y, t) = 0 \qquad 0 \leqslant y \leqslant b, \quad t > 0$

$u(x, 0, t) = u(x, b, t) = 0 \qquad 0 \leqslant x \leqslant a, \quad t > 0$

and the initial condition

$u(x, y, 0) = f(x, y) \qquad 0 \leqslant x \leqslant a, \; 0 \leqslant y \leqslant b$

and $u = 0$ at $t = \infty$

Assume $b \leqslant a$, so that heat flows through the plate only along x-direction. Obtain the steady state solution if $u(0) = 0$ and $v(a) = 100$. 15

7.(b) The generating function for the Hermite polynomials is given as

$$S(\xi, s) \equiv e^{\xi^2 - (s-\xi)^2} \equiv \sum_{n=0}^{\infty} \frac{H_n(\xi)}{n!} s^n$$

Using above relation evaluate the integral

$$\int_{-\infty}^{\infty} H_n(\xi) H_m(\xi)\, e^{-\xi^2} d\xi$$

10

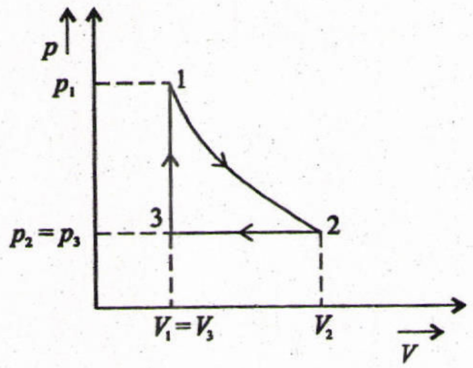
7.(c) Find the Laplace transform of $f(t) = (\cos 3t)^2$. 5

8.(a) Consider a system of two particles each of which can occupy 3 different levels with energies 0, $\in$ and $2\in$. Write the partition function for the system if the particles are (i) distinguishable, (ii) indistinguishable (fermions). Find the entropy and the average energy of the system in part (ii). 10

8.(b) (i) Sketch the distribution functions of FD, BE and MB statistics as a function of E in units of kT. Obtain a condition under which MB distribution can be used. Assume that the de Broglie wavelength λ of the particles is much less than the average separation d between them.

(ii) Calculate the ratio of the number of H-atoms in the first excited state to the ground state at the temperatures 300°K and 6000°K. 6+4=10

8.(c)



Suppose one mole of an ideal gas is subjected to the cyclic process shown above.

$1 \rightleftharpoons 2$ is an isothermal expansion/compression

$2 \rightleftharpoons 3$ is an isobaric expansion/compression

$3 \rightleftharpoons 1$ is an isochoric heating/cooling

Calculate ΔU, ΔQ and ΔW along each of the paths $1 \rightleftharpoons 2$, $2 \rightleftharpoons 3$ and $3 \rightleftharpoons 1$. What is the change in internal energy for the entire cyclic process ? 10

9.(a) Starting from the expression for the electrostatic potential

$$\phi(\vec{r}) = \frac{1}{4\pi \in_0} \int_V \frac{\rho(\vec{r}_0) dV_0}{|\vec{r} - \vec{r}_0|}$$

Obtain Poisson's equation. Symbols have their usual meanings. 10

9.(b) Write down Maxwell's equations assuming that no dielectric or magnetic material is present. In the following justify your answer.

(i) If the system is space inverted, i.e., $x \rightarrow x' = -x$, what happens to the charge density and current density, ρ and $\vec{j}$ and to $\vec{E}$ and $\vec{B}$?

(ii) If the system is time reversed, i.e., $t \rightarrow t' = -t$ what happens to $\rho, \vec{j}, \vec{E}$ and $\vec{B}$? 4+3+3=10

9.(c) A metallic ring of cross-section $2 \cdot 5$ cm^2, mean radius 40 cm and relative permeability 1500 is wound uniformly with 3000 turns of wire. If a current of $1 \cdot 6$ A passes through the wire, find the mean $\vec{B}$ and the magnetisation in the ring. 10

10.(a) Three conducting planes, intersecting at right angles to each other, are kept at a potential ϕ_0. Determine the potential at a point in space, if the total charge on a plane of area α be Q. 10

10.(b) What prompted the discovery of the Earth's ionosphere ? Define the different layers or regions of ionosphere with the help of suitable diagram(s). Discuss their characteristics. 2+5+3=10

10.(c) (i) Discuss the working principle of satellite navigation system.

(ii) Explain briefly the different types of errors in GPS measurements. 5+5=10
