INSTRUCTIONS
Please read each of the following instructions carefully before attempting questions.
There are EIGHT questions divided under TWO sections.
Candidate has to attempt SIX questions in all.
Question No. 1 and 5 are compulsory.
Out of the remaining SIX questions, FOUR questions are to be attempted choosing TWO from each section.
The number of marks carried by a question/part is indicated against it.
All parts and sub-parts of a question are to be attempted together in the answer book.
Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.
Any page or portion of the page left blank in the answer book must be clearly struck off.
Answers must be written in ENGLISH only.
Neat sketches may be drawn to illustrate answers, wherever required.
Unless otherwise mentioned, symbols and notations have their usual standard meanings.
Assume suitable data, if necessary and indicate the same clearly.
Section – A

1. (a) (i) Draw the characteristic gamma-ray spectra of K, U, Th and show their energy peaks.

(ii) What are the places, in India, where uranium and thorium ores are found? 5

(b) (i) Discuss briefly the use of spinner magnetometer for finding residual component of magnetization.

(ii) Discuss briefly the ferrimagnetism in minerals, and give at least one example. When a ferrimagnetic material will behave as paramagnetic material? 5

(c) (i) Some of the high density oceanic materials are having high velocity, substantiate your answer with suitable reason.

(ii) What is the principle of designing an inverse filter to remove the water reverberations due to first reflector? 5

(d) What are the three adjustments made in Bouguer gravity of land survey compared to the Bouguer gravity at an ocean floor site? 5

(e) (i) What do you mean by a digital filter?

(ii) Define Discrete Fourier Transform (DFT) and its Inverse Discrete Fourier Transform (IDFT) for a given discrete time series. 5

A-GSE-P-HQC 2 (Contd.)
(f) What is z-transform? Explain the z-transform for the sample values of a wavelet or sequence of time series. 5

(g) Discuss the energy sources and radiation principles involved in electromagnetic remote sensing techniques. 5

(h) (i) What will be the colour of water seen on a photograph taken in a wavelength between 0.75 μm - 1.3 μm? What is the spatial resolution of IRS and Quickbird sensors?

(ii) If length between two photographs is to be fixed as 0.85 km and speed of an aeroplane is 170 km/hr, what should be the time interval between the opening of the shutter? Find out how many photographs will be snapped along a profile line in 9 hours. 5

2. (a) (i) Explain deconvolution.

(ii) Compute the inverse filter for the normalized dipole (1, 0.5) and show the result of deconvolution of the signal \( X = (4, 2, 0.5, 1, -1.5) \) using above computed inverse filter response. 10

(b) (i) Write down the expression of Fourier Transform for a 2-dimensional Bouguer gravity anomaly.
(ii) Explain the role of low-pass and high-pass filters for the separation of region and residual anomalies.

(iii) Show the example of 2-dimensional maps of Bouguer, Regional and Residual anomalies.

(c) (i) What do you mean by Auto-correlation?

(ii) Find out the Auto-correlation of finite length wavelet, $X = (1, 4, 3, 1)$ and plot the same.

3. (a) Define photogrammetry. Discuss briefly the importance of photogrammetry.

(b) Derive an expression for parallax equation.

(c) A tall building manifests a parallax difference of 13.6 mm between its top and its base on photographs taken from an altitude of 4000 metres above ground. If the average photobase is 108 mm, calculate the height of building.

4. (a) Describe the working principle of a LaCaste-Romberg gravimeter specially designed for shipboard gravity survey.

(b) What does the ocean floor magnetic measurements indicate in terms of ocean floor dynamics?
(c) At a station 200 metres away from outcropping dyke, the spectrometric readings for thorium and uranium are found to be as 25 and 36 counts/min. The constants \( k_1 \) and \( k_2 \) for thorium and uranium channels are 0.6 and 0.13, respectively; and the stripping constants \( (S) \) for thorium \( \gamma \)-rays in uranium channel is one. Find the thorium and uranium contents in parts per million.

(d) What is that factor on which the strength of the measured magnetic field depends in absorption cell magnetometer? Find out the gyro-magnetic ratio of electron for measuring the earth’s magnetic field of 50000 gammas with precessional frequency as 1400 kHz. How aero-magnetic survey is carried out for base metal exploration?

Section – B

5. (a) Prove that \( [\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z \),

where \( \hat{L}_x, \hat{L}_y \), and \( \hat{L}_z \) are the operators corresponding to the \( x \)-, \( y \)-, and \( z \)-components of the angular momentum, respectively.

(b) Find the energy levels of a particle of mass \( m \) moving in the one-dimensional potential

\[
v(x) = \begin{cases} +\infty, & \text{for } x \leq 0 \\ \frac{1}{2} m \omega^2 x^2, & \text{for } x > 0, \end{cases}
\]

where \( \omega \) is a real constant. Give reasons for your answer.
(c) Intensity $I$ of e.m. waves passing through a laser active medium is expressed by

$$I(z) = I(0)e^{-az}$$

where $a$ is absorption coefficient and is a positive quantity;

$$a = \frac{\pi^2 C^2}{\omega^2 \tau} (N_1 - N_2)$$. $	au$ is spontaneous relaxation time and the laser cavity is along the $z$-axis.

What is the basic requirement for amplification of e.m. waves? 5

(d) The spatial part of e.m. field in a cubic cavity of side $L$ is governed by

$$\nabla^2 u(x, y, z) = -K^2 u(x, y, z)$$

(Helmholtz equation). $K$ is propagation vector. Solution of the equation yields

$$k_x = \frac{l\pi}{L}, k_y = \frac{m\pi}{L}, k_z = \frac{n\pi}{L}$$

$l, m$ and $n$ are integers and represent number of modes that a standing wave has along the $x, y$ and $z$-directions respectively.

Find the length of a parallel plane cavity for the longitudinal modes under resonance condition. 5
(e) Explain, how the change between Crystalline and Amorphous structure is used to record the data in CDRW (Phase Change Device Technology).

(f) Draw the voltage – current characteristics for a solar cell with a variable load at fixed solar insolation. Specify typical value of efficiency for modern solar cells in common use nowadays.

(g) Draw a sketch for a non-inverting unity gain amplifier using an operational amplifier and minimum possible number of external components.

(h) Define the following:

(i) Geo-stationary orbit

(ii) Gain of an antenna

(iii) SFR in a microcontroller

(iv) Quantization error in A/D converter

(v) Thermister (NTC)
6. (a) State the limitations of Einstein's theory for heat capacity of solids. How were these circumvented by Debye? Obtain the expression for heat capacity based on Debye's theory.

(b) The state of a particle of mass \( m \) in an infinite square well potential in one dimension

\[
\nu(x) = \begin{cases} 
0, & 0 < x < a \\
\infty, & \text{elsewhere} 
\end{cases}
\]

is described by

\[
\psi(x) = \begin{cases} 
A \sin \left( \frac{3\pi x}{2a} \right) \cos \left( \frac{\pi x}{2a} \right), & \text{for } 0 \leq x \leq a \\
0, & \text{elsewhere} 
\end{cases}
\]

(i) Normalize the wave function and find \( A \).

(ii) If energy is measured in this state, what is the probability of finding the particle in the first excited state?

(iii) What is the average value of energy in this state?
Consider the transition from $^2P(m_l = +1, 0, -1)$ to the lower $^2S$ state of hydrogen atom. Using the following data, find the radiative life time of the $^2P$ state. Given

**Ratio of Einstein coefficients:**

$$
\frac{A_{ba}}{B_{ab}} = \frac{g_a}{g_b} \frac{\hbar \omega_{ab}^3}{\pi^2 C^3}, \text{ } g \text{ stands statistical weight}
$$

$$
\psi_{1s} = \pi^{-1/2} a_0^{-3/2} e^{-r/a_0}
$$

$$
\psi_{2p} = 2^{-5/2} \pi^{-1/2} a_0^{-5/2} x e^{-r/2a_0}
$$

$a_0$ (Bohr radius) = $5 \times 10^{-11} m$

$$
\omega_{ba} = 1.78 \times 10^{16} \text{ s}^{-1}
$$

$e = 1.6 \times 10^{-19} C \quad \hbar = 10^{-34} Js$

$\varepsilon_0 = 8.85 \times 10^{-12} \text{ Fr} m^{-1}; \quad C = 3 \times 10^8 \text{ ms}^{-1}$

$$
B_{ab} = \frac{\pi e^2}{3 \varepsilon_0 \hbar^2} |D_{ba}|^2
$$

where $D_{ba}$ is dipole moment expressed by

$$
\int \psi_{2p} \times \psi_{1s} d\tau
$$

$$
\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta; \quad \cos 2\theta = 2 \cos^2 \theta - 1
$$

and

$$
\int_0^\infty x^n e^{-ax} = \frac{n!}{a^{n+1}}
$$
(b) $N$ longitudinal modes in a resonator of length, $d$, are mode locked with constant pulse difference, $\delta$ and the modes have the same average amplitude, $A$. The intensity of such a mode locked pulse is given by

$$I = A^2 \left[ \frac{\sin\left(\frac{Nt\Delta\omega}{2}\right)}{\sin\left(t\Delta\omega\right)} \right]^2$$

where $\Delta\omega$ is frequency separation between two modes.

(i) Show that the temporal separation between two mode locked pulses is equal to one round trip transit time of the photon.

(ii) What is the length of a resonator to achieve mode locked pulses having the peak temporal separation of 10 ns?

(iii) Assuming that time between the first minimum and the first maximum is approximately equal to the time duration, $t_d$ of the total gain profile (FNHM), find approximate number of oscillating modes for $t_d = 0.1$ ns.
8. (a) Draw functional block diagram of an eight bit microprocessor and explain its working. Show timing diagram for reading an instruction from the memory and executing it. 10

(b) Draw the truth table for logic expression
\[ \overline{A} \cdot B + C \]
and realise it using basic logic gates. 10

(c) Explain the uses of satellites placed (i) in the geostationary orbit and (ii) in the low altitude orbit. What are the criteria used for selection of uplink and downlink frequencies for the satellite communication?

Specify the most commonly used uplink and downlink frequency bands for satellite communication. 10