GEO-PHYSICS
Paper – II

Time Allowed: Three Hours

Maximum Marks: 200

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions:

There are TEN questions divided under TWO sections.
Candidate has to attempt SIX questions in all.
Questions no. 1 and 6 are compulsory.
Out of the remaining EIGHT questions, FOUR questions are to be attempted choosing TWO from each section.

The number of marks carried by a question/part is indicated against it.

Neat sketches may be drawn to illustrate answers, wherever required. These shall be drawn in the space provided for answering the question itself.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

Answers must be written in ENGLISH only.

Physical Constants:

Electron rest mass, \( m_e \) = \( 9.109 \times 10^{-31} \) kg
Proton rest mass, \( m_p \) = \( 1.672 \times 10^{-27} \) kg
Neutron rest mass, \( m_n \) = \( 1.675 \times 10^{-27} \) kg
Atomic mass unit (C\(^{12}\) = 12), a.m.u. = \( 1.661 \times 10^{-27} \) kg
Bohr magneton, \( \mu_B \) = \( 9.27 \times 10^{-24} \) J/tesla
Nuclear magneton, \( \mu_N \) = \( 5.05 \times 10^{-27} \) J/tesla
Boltzmann constant, \( k_B \) = \( 1.381 \times 10^{-23} \) J/K
SECTION A

Q1. (a) Draw a neat sketch explaining the elements of the Earth’s magnetic field. Write down the Poisson’s relation between gravity potential and magnetic potential.

(b) The apparent resistivity measured with Wenner and Schlumberger electrodes configurations is the same, when equal amount of current is passed through the current electrodes and equal potential difference is recorded between pair of potential electrodes. If the Wenner configuration electrodes spacing is 10 m and in Schlumberger configuration potential electrodes spacing is 20 m, estimate the current electrodes spacing for the Schlumberger configuration.

(c) Define multiples. Distinguish between short-path and long-path multiples with the help of neat sketch for the case of horizontal beds.

(d) Find coefficient of anisotropy of a geoelectrical section consisting of an alternating series of beds with a total thickness of 10 m, the individual bed being homogeneous and isotropic, one meter thick and with resistivities alternating between 10 Ωm and 100 Ωm.

(e) Define the term ‘Hydrogen Index’ used in neutron porosity logging. How can the combination of neutron and density porosity logs be used as gas log?

Q2. (a) In frequency domain electromagnetic prospecting survey, a Transmitter (T) and a Receiver (R) loops are placed at a test site in free space. If harmonically time varying current of 1000 Hz and magnitude, I₀ of 2 A in the transmitter loop produces induced emf in receiver loop of magnitude 1 volt, calculate mutual inductance between transmitter and receiver loop along with the phase difference between transmitter current and induced emf in the receiver loop for harmonic transmitter current represented by \(I_T = I_0 \cos \omega t\).
(b) In a circuit theory approach, frequency domain electromagnetic prospecting is represented by three loops defined as a Transmitter (T), a Receiver (R) and a buried Conductor (C) in the ground. Buried conductor is represented by a Resistor (R) and an inductor (L) connected in series. Assuming mutual inductance between each pair of loops as $M_{TC}$ (Transmitter-Conductor), $M_{CR}$ (Conductor-Receiver) and $M_{TR}$ (Transmitter-Receiver); obtain secondary emf normalized by primary emf induced in the receiver loop for a harmonic transmitter current ($I_T$) with magnitude $I_0$ and angular frequency, $\omega$, as $I_T = I_0 e^{i\omega t}$. Draw neat response curve in terms of in-phase, quadrature and phasor diagram.

(c) What do you understand by first approximation in the interpretation of three-dimensional anomalies of gravity data? Derive expression for gravity anomaly due to buried spherical body. Explain the methodology to obtain the depth of centre of spherical body.

Q3. (a) What do you understand by resonance magnetometers? Write down the names of two resonance magnetometers. Explain the working principle of the Alkali Vapour Magnetometer.

(b) What is meant by stable and unstable gravimeters? Write down at least two names from each category. Write down the names of at least five corrections to gravity observations. Briefly explain the drift correction.

(c) Applying appropriate electromagnetic boundary conditions, show that TM-mode magnetotelluric apparent resistivity curve, plotted along a profile passing through a vertical resistivity interface boundary, is discontinuous. Draw TM-mode magnetotelluric apparent resistivity curve along a profile passing through vertical resistivity interface boundary separating two mediums of resistivities $10 \, \Omega m$ and $100 \, \Omega m$. Explain characteristics of this apparent resistivity curve.

(d) Starting from the expression of magnetotelluric impedance for a two-layer medium, find the phase of impedance at low frequency part for perfectly conducting and insulating second basement layer.
Q4. (a) Draw ray path geometry for a dipping interface and its travel
time – distance graph along with suitable notations for different
parameters.
Explain the procedure to compute travel time for dipping interface.
(b) Compute depth of a horizontal refractor/interface using intercept time
method for
\[ V_1 = 450 \text{ m/sec.} \]
\[ V_2 = 1200 \text{ m/sec.} \]
and intercept time, \( T_i = 75 \) milliseconds.
(c) Using an expression for potential due to a point current source, \( I \), placed
at a distance, \( r \), in uniform medium of constant resistivity, \( \rho \), derive an
expression for apparent resistivity response for lateral log device. Draw
appropriate response curve of lateral resistivity logging device passing
through a thick resistive bed. Also explain various features of the
response curve.

Q5. (a) Explain different layouts for a common-shot gather, a common
mid-point gather and a common receiver gather using suitable sketches.
What do you understand by constant (optimum) offset method of
shallow seismic reflection survey?
Write down the sequential steps of the basic seismic data processing
flow chart.
(b) Resistivity log response is recorded using lateral device consisting of
three electrodes defined by characters A, M and N respectively. The
distance between electrodes : \( AM = 5 \) m and \( AN = 5.5 \) m. When a 2 A
current is circulated between current electrode, A, and a distant
electrode, B, situated at the surface, a signal of 10 mV is measured
between electrodes M and N. Calculate the apparent resistivity of the
medium that surrounds the logging device.
(c) Define the condition in which spontaneous potential (SP) log response
cannot be used to differentiate between clean sand and shale formations.
How can natural gamma ray log response be used as substitute of SP log
to identify clean sand and shale formations? Also define the term ‘shale
index’ used in natural gamma ray log.
SECTION B

Q6. (a) Derive an expression for Hamiltonian for a particle having charge q, moving in an electromagnetic field. The potential energy of the particle is \( V = q (\theta - \vec{\theta} \cdot \vec{A}) \), where \( \theta \) and \( \vec{A} \) are scalar and vector potentials respectively.

(b) With what velocity should a rod be moved in the direction inclined at 45° to its length for getting contraction of 20% in the length of rod?

(c) Show that the entropy of a system in canonical ensemble can be expressed as

\[ \sigma = - \sum_i \rho_i \log \rho_i \]

where \( \rho_i \) is the probability of the system to be found in \( i \)th state.

(d) The quantum numbers of two electrons in a two valence electron atom are

\[ n_1 = 6, \quad l_1 = 3, \quad s_1 = \frac{1}{2} \]
\[ n_2 = 5, \quad l_2 = 1, \quad s_2 = \frac{1}{2} \]

(i) Assuming \( L - S \) coupling, find the possible values of \( L \) and hence of \( J \).
(ii) Assuming \( j - j \) coupling, find the possible value of \( J \).

(e) If \( 10^{-3} \) kg of radioactive isotope (atomic mass 226) emits \( 3.72 \times 10^{10} \) alpha particles in a second, (i) calculate the half-life of the isotope. (ii) Calculate the average energy of the \( \alpha \)-particles if \( 4.2 \times 10^2 \) joules energy is released in an hour.

Q7. (a) Using Hamilton's equation of motion find the differential equation for planetary motion. Let us assume force \( f(r) = - \frac{K}{r^2} \).
Show that areal velocity is constant.

(b) At normal pressure \( 1.031 \times 10^5 \) N/m\(^2\) and temperature 0°C the volume of 2 grams of water increases from 1.0 cc to 1.81 cc on freezing. What will be the change in its internal energy? The latent heat of melting of ice = 80 cal/gram.
Q8. 
(a) Using first law of thermodynamics, establish the relation between two principal specific heats of a gas.

(b) Define canonical transformation. For a certain canonical transformation it is known that 
\[ Q = \sqrt{(q^2 + p^2)} \]
\[ F = \frac{1}{2} (q^2 + p^2) \tan^{-1} \left( \frac{q}{p} + \frac{1}{2} qp \right) \]
Find \( P(q, p) \) and \( P(q, Q) \).

(c) (i) Why is the reaction \( e^+ + e^- \rightarrow \mu^+ + \pi^- \) forbidden?

(ii) A pion at rest decays into a muon and a neutrino. Find the energy and velocity of the outgoing muon, in terms of two masses \( m_\pi \) and \( m_\mu \). (Assume \( m_\nu = 0 \))

Q9. 
(a) (i) Explain 'Normal Zeeman Effect' and 'Anomalous Zeeman Effect'.

(ii) In a Normal Zeeman Effect experiment, spectral splitting of the line at the wavelength 643.8 nm corresponding to the transition \( 5^1D_2 \rightarrow 5^1P_1 \) of cadmium atoms is to be observed. The spectrometer has a resolution of 0.01 nm. Calculate the minimum magnetic field needed to observe this.

\[ m_e = 9.1 \times 10^{-31} \text{ kg}, \ e = 1.6 \times 10^{-19} \text{ C}, \ c = 3 \times 10^8 \text{ m/s} \]

(b) (i) Why does fusion reaction take place at high temperature?

(ii) A nuclear reactor using \( ^{235}_{92}U \) is operated at a power level of 250 mega watts. If the energy released per fission of \( ^{235}_{92}U \) is 200 MeV, calculate the rate of consumption of \( ^{235}_{92}U \) per year. Assuming that there are no losses and the mass of atom is equal to the sum of mass of nucleus where each nucleon has a mass equal to 1 amu.

(c) A particle has kinetic energy of 72 MeV and momentum 375 MeV/c. Find its mass (in MeV/c^2) and speed (as a fraction of c).
Q10. (a) (i) Explain the terms mass defect, missing energy and binding energy of a nucleus.

(ii) The binding energy per nucleon of deuteron ($^2_1\text{H}$) and helium nucleus ($^4_2\text{He}$) is 1.1 MeV and 7 MeV respectively. If two deuteron nuclei react to form a single helium nucleus, how much energy will be released?

(b) (i) The spectroscopic symbol for the ground state of $^{19}\text{Al}$ is $^2\text{P}_{1/2}$. Under the action of a strong magnetic field (when $L - S$ coupling can be neglected) the ground state energy level will split. What will be the number of splitting levels?

(ii) A diatomic molecule has vibrational states with energy $E_v = \hbar \omega \left(v + \frac{1}{2}\right)$ and rotational states with energies $E_J = B_J (J + 1)$, where $v$ and $J$ are non-negative integers. Consider the transitions in which both the initial and final states are restricted to $v \leq 1$ and $J \leq 2$ and subject to the selection rule $\Delta v = \pm 1$ and $\Delta J = \pm 1$. Calculate the largest allowed energy of transition.

(c) Calculate the value of partition function for the four energy levels shown in the figure assuming that the particles obey Maxwell-Boltzmann statistics and $\beta = 2.0$ per energy unit. Also compute the fractional populations of the four energy levels.

| 1 energy unit (eu) = minimum quantum increment |
|---|---|---|---|---|
| $\varepsilon_i = 0$ | $\varepsilon_i = 1$ eu | $\varepsilon_i = 2$ eu | $\varepsilon_i = 3$ eu |
| KE_x KE_y KE_z | KE_x KE_y KE_z | KE_x KE_y KE_z | KE_x KE_y KE_z |
| 0 0 0 | 1 0 0 | 1 1 0 | 1 1 1 |
| 0 1 0 | 0 1 1 | 1 0 1 | 1 2 0 |
| 0 0 1 | 2 0 0 | 2 1 0 | 0 1 2 |
|  | 0 2 0 | 2 0 1 |  |
|  | 0 0 2 |  |  |
| $g_i = 1$ | $g_i = 3$ | $g_i = 6$ | $g_i = 10$ |