Combined Geo-Scientist
(Main) Exam,2021

SDT-F-GPH

GEO-PHYSICS

Paper – I

Time Allowed : ***Three*** *Hours*

Maximum Marks : **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

There are ***TEN*** *questions divided under* ***TWO*** *sections.*

Candidate has to attempt ***SIX*** *questions in all.*

Questions No. ***1*** *and* ***6*** *are* ***compulsory****.*

Out of the remaining ***EIGHT*** *questions,* ***FOUR*** *questions are to be attempted choosing* ***TWO*** *from each section.*

The number of marks carried by a question / part is indicated against it.

Neat sketches may be drawn to illustrate answers, wherever required. These shall be drawn in the space provided for answering the question itself.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

Answers must be written in ***ENGLISH*** *only.*

SECTION A

Q1. (a) What is the difference in the length, in kilometers, at the Earth's surface, between a degree of latitude at equator and at the pole ? *10*

(b) The angle of inclination and declination of a remanent magnetization of basalt are observed as 45° and 60°, respectively, at present location of 30°N, 25°E. Calculate the magnetic latitude of this site at the time of magnetization of basalt. Find out the latitude and longitude of the palaeomagnetic pole. *10*

(c) Calculate the amount of increase in seismic energy released from the earthquake for the following : *10*

(i) When surface-wave magnitude increases from 5 to 6; 6 to 7 and 7 to 8.

(ii) When body-wave magnitude increases from 5 to 6; 6 to 7 and 7 to 8.

(d) Explain the concept of Tikhonov regularization in context to the inverse problem, and explain what is an L-curve solution. *10*

Q2. (a) Calculate the depth and density beneath a 5-km-high Himalayan mountain chain in isostatic equilibrium with a 35-km-thick continental crust of density 2·8 g/cc and a mantle of density 3·3 g/cc by using the hypotheses of (i) Pratt, and (ii) Airy. *5+5*

(b) Derive heat-conduction equations for an Earth model having low temperature at upper surface as compared to high temperature at lower surface. *10*

(c) How do density, gravity and pressure vary radially in the interior of the Earth ? Derive equations and discuss them with suitable figures related to variations of these physical properties. *10*

Q3. (a) How are seismic waves attenuated during propagation from the source ? *10*

(b) What is earthquake magnitude ? Describe body-wave, surface-wave and moment magnitude with the help of suitable equations. *10*

(c) The first-arrival time-distance data from a seismic refraction survey were plotted and the best line fits have the slope of 0·22222 s km^{-1}, 0·164474 s km^{-1}, 0·153846 s km^{-1} and 0·119760 s km^{-1}, respectively. Intercept times were observed to be 1·00 s, 1·90 s and 8·70 s, respectively, from the plot. Compute the velocity-depth structure and thickness of the crustal layer. *10*

Q4. (a) Show that the least square solution is same as the maximum likelihood solution when errors are normally distributed. *10*

(b) Consider you have a constitutive equation that relates $\bar{x}$ and $\bar{m}$ as follows :

$$F(\bar{x}, \bar{m}) = F(x_1, x_2, x_3, m_1, m_2)$$
$$= 3m_1x_1x_2x_3 + 2m_1m_2x_1^2x_2 - 4x_1 + 2m_1 + 6$$

Find the gradient with respect to $\bar{x}$ and $\bar{m}$. (Find $\nabla_x F$ and $\nabla_m F$). *10*

(c) Consider an earthquake is recorded at N stations. Using the P-wave arrival time at all the stations and the triangulation approach, we wish to find the origin location and time of event. Consider the earthquake occurred in a homogeneous space with a known wave velocity V.

(i) Define the data space and model space, and the constitutive relation between them.

(ii) What does the partial derivative matrix G look like ? (Show derivation). What is the dimension of G ?

(iii) Demonstrate how you can find the solution to this problem. *2+4+4*

Q5. (a) Solve the following system of equations using both — Jacobi method and Gauss-Seidel method, and show results after 3 iterations :

$$\begin{bmatrix} 2 & 1 & 1 \\ -1 & 3 & 1 \\ 1 & 2 & -4 \end{bmatrix} X = \begin{bmatrix} 4 \\ -5 \\ 6 \end{bmatrix}$$

Let the starting vector be $\bar{X}^0 = (0, 0, 0)$. *10*

(b) Derive the expression for gravitational field due to a uniform ring of mass M along its axial direction. Find the distance from the centre of the ring at which the gravitational field will be maximum. *10*

(c) Describe Gauss divergence theorem. Compute the flux of the vector field $\vec{F} = \langle x - y, x + z, z - y \rangle$ through the surface of the cone $x^2 + y^2 = z^2$, $0 \le z \le 1$ along with circular top. *10*

SECTION B

Q6. (a) Write matrix eigenvalue equation showing eigenvalue and eigenvector. Find eigenvalues of the matrix A. *10*

$$A = \begin{bmatrix} 3 & -5 & -4 \\ -5 & -6 & -5 \\ -4 & -5 & 3 \end{bmatrix}$$

(b) (i) State Gauss's law and express this law in both integral and differential forms (no derivation). Show that Gauss's law and Laplace-Poisson's equations are just the mathematical expressions for the same.

(ii) A charge of 1 μC is placed at the centre of a hollow cube. Calculate the electric flux diverging through each face. *7+3*

(c) A plane electromagnetic wave propagating in x-direction has a wavelength of 5·0 mm. The electric field is in y-direction and its maximum magnitude is 30 V/m. Write suitable equations for electric and magnetic fields as a function of x and t. *10*

(d) (i) Qualitatively discuss ground wave propagation with reference to :

I. Electrical properties of surfaces

II. Flat earth

III. Curved earth

(ii) What are the errors in Global Positioning Systems (GPS) ? *7+3*

Q7. (a) Find the Laurent series for the function $e^{1/2}$ in the range $0 \leq |z| \leq \infty$.

Using this expansion, show that for n = 0, 1, 2, 3, ...

$$\frac{1}{\pi} \int_0^{\pi} \exp(\cos\theta) \cos(\sin\theta - n\theta)\, d\theta = \frac{1}{n!}$$ *10*

(b) State Biot-Savart law and Ampere's law. Calculate the magnetic induction at a distance 'd' from an infinitely long straight wire in which a current 'I' flows using (i) Biot-Savart law, and (ii) Ampere's law. *10*

(c) An observer is 1·8 m away from a light source (of dimension much smaller than 1·8 m) whose power output P is 250 W. Calculate RMS value of electric and magnetic fields at the position of observer. Assume that the source radiates uniformly in all directions. *10*

Q8. (a) (i) Taking generating function of the Bessel differential equation as

$$\sum_{n=-\infty}^{\infty} h^n J_n(x) = \exp\left[\frac{x}{2}(h - h^{-1})\right].$$

Show that :

I. $\cos x = J_0(x) + 2\sum_{n=1}^{\infty} (-1)^n J_{2n}(x)$

II. $\sin x = 2\sum_{n=0}^{\infty} (-1)^n J_{2n+1}(x)$

(ii) Plot the Bessel polynomials $J_0(x)$ and $J_1(x)$ as the function of x. *8+2*

(b) (i) Within mathematical framework, discuss magnetic vector potential and magnetic scalar potential. Why are they called so ?

(ii) Show that magnetic induction is given by the curl of magnetic vector potential.

(iii) What are Lienard-Wiechert potentials of a moving charge (no derivations) ? *3+4+3*

(c) (i) Discuss the relative merits of Low Frequency (LF) and Very Low Frequency (VLF) wave propagation.

(ii) Discuss the working principle of satellite navigation system and mention its applications. *5+5*

Q9. (a) Solve the initial value problem

$u' = -2tu^2 \qquad u(0) = 1$

with h = 0·2 on the interval [0, 0·4] by using Runge-Kutta method. *10*

(b) (i) Prove that three-dimensional volume element 'dx dy dz' is not invariant but four-dimensional volume element is invariant under Lorentz transformation.

(ii) Establish relativistic invariance of $\vec{E} \cdot \vec{B}$. *4+6*

(c) Find the energy stored in a 60 cm length of laser beam operating at 4 mW. *10*

Q10. (a) Determine Laplace transform of $f(t) = e^{2t} + \sin 4t$. *10*

(b) What is a Poynting vector ? Derive expression for energy density, and intensity of electromagnetic field in terms of parameters of electric and magnetic field components of EM wave. Express the Poynting vectors in terms of power. *10*

(c) (i) Justify the importance of ionosphere in atmospheric science. Discuss different layers and ionospheric parameters.

(ii) What are the characteristic features of troposphere ? *7+3*