A-IGQ-O-IRA

GEO-PHYSICS

Paper I

Time Allowed : Three Hours *Maximum Marks . 200*

INSTRUCTIONS

Please read each of the following instructions carefully before attempting the questions :

There are NINE questions divided under TWO sections.

Candidate has to attempt ALL the NINE questions.

***ALL** the parts in the ONLY question in Section A are compulsory.*

In Section B, TWO parts out of THREE are to be attempted in each of the EIGHT questions.

The number of marks carried by a question/part is indicated against it.

All parts and sub-parts of a question are to be attempted together in the answer book.

Attempts of questions shall be counted in chronological order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the answer book must be clearly struck off.

Answers must be written in ENGLISH only.

Neat sketches are to be drawn to illustrate answers, wherever required.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

SECTION A

1. Attempt ***all*** of the following : $8 \times 5 = 40$

(a) Explain the concept of terrestrial planets in the solar system, and state the Kepler's laws of planetary motion. 5

(b) Earthquake refers to what state of crustal plate. Determine the Richter Magnitude (M) of an earthquake whose largest wave amplitude recorded by standard seismograph is 10 mm at a distance of 100 km from the epicentre. 5

(c) Draw the vectors $\vec{a}$ and $\vec{b}$ satisfying the following conditions : 5

(i) $\vec{a} \times \vec{b} = 0$

(ii) $\left|\vec{a} \times \vec{b}\right| = -\vec{a} \cdot \vec{b}$

(iii) $\left|\vec{a} \times \vec{b}\right| = \vec{a} \cdot \vec{b}$

(d) Explain the physical meanings of 'Eigenvalues and 'Eigenvectors' of a matrix, and find out the eigenvalues and eigenvectors of the given matrix 5

$$A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}.$$

(e) If $\phi = 2z - x^3y$, calculate a unit normal to the surface $2z - x^3y = 3$ at the point $(1, -1, 1)$. 5

(f) Distinguish clearly between microcanonical, canonical and grand canonical ensembles. Which ensemble is most commonly used and why ? 5

(g) Find out the magnetic field $\vec{B}$ of a plane electromagnetic wave travelling along the z-axis, whose electric field is given by

$$\vec{E} = (E_{0x} \hat{i} + E_{0y} \hat{j}) \sin (\omega t - kz + \varphi).$$ 5

(h) What is half wave dipole antenna ? Discuss its advantage over Hertzian antenna in terms of radiation resistance. 5

SECTION B

2. Attempt any ***two*** parts of the following : *2×10=20*

(a) Write International Gravity Formula (IGF) as adopted by IUGG, and explain how the value of 'g' varies from equator to pole over the Earth. *10*

(b) (i) In what fashion are Bouguer anomalies observed over Abyssal plains, recently uplifted younger mountains and ancient continental shields ? *5*

(ii) Which features of Plate Tectonics interaction mechanisms support that the Earth is neither expanding nor contracting ? *5*

(c) How does the seismic velocity of body waves (P, S-wave) vary with depth inside the Earth ? Based on this, give broad subdivisions of the Earth's interior. *10*

3. Attempt any ***two*** parts of the following : *2×10=20*

(a) (i) Explain the presence of 'seismic shadow zone' for P-wave with suitable ray path diagram. *6*

(ii) Why is the Earth's mantle considered to be solid ? State. *4*

(b) (i) Rate of 'Heat flow' inside the Earth depends on which two basic factors ? Define its standard unit of measurement. *4*

(ii) Write the characteristic average heat flow values in continental shield, ocean basin and mid-oceanic ridge region. *6*

(c) (i) Explain the terms 'Seismicity', 'Seismic Risk' and 'Seismic Coefficients'. *5*

(ii) How are the earthquakes geographically distributed over the Indian subcontinent ? Illustrate with figures and state the associated seismic hazard as per standard seismic zoning map of India. *5*

4. Attempt any ***two*** parts of the following : *2×10=20*

(a) Write down the 3D Laplace equation in Cartesian, cylindrical and spherical coordinate systems. Simplify the equations in cylindrical and spherical coordinates for cylindrical and spherical symmetry respectively. *10*

(b) Solve the Laplace equation for spherical symmetry, and hence derive the expression of potential over the surface of a homogeneous half-space of resistivity (ρ) at a point 'r' distant away from the point current electrode carrying current (I) at the surface using appropriate boundary conditions. *10*

(c) What do you understand by upward and downward continuation of potential field data ? State their applications as well as limitations in interpretation of gravity and magnetic data in geophysics. *10*

5. Attempt any ***two*** parts of the following : *2×10=20*

(a) In an approach to linearize an inverse problem, establish the linear relationship between perturbation in the data (Δd) and perturbation in the model (Δm). *10*

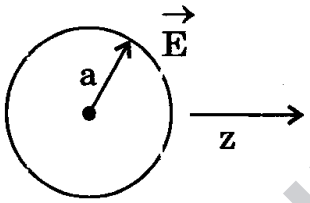
(b) Explain the method of least square solution for a linear inverse problem, and on the basis of the solution, discuss the cases of Over-determined, Under-determined, Even-determined and Mixed-determined problems. *10*

(c) Name the various steps of 'genetic algorithm' in geophysical optimization approach and discuss in brief (i) Crossover, (ii) Mutation, and (iii) Fitness function. *10*

6. Attempt any ***two*** parts of the following : *2×10=20*

(a) A grounded conducting sphere of radius a is placed in a uniform electric field $\vec{E} = E_0 \hat{z}$. Determine the electric potential $\Phi(r, \theta)$ from Laplace's equation given that

$$\Phi(r, \theta) = \left[Ar + \frac{B}{r^2} \right] \cos \theta .$$ *10*



(b) Use Ampere's law to derive the magnetic field of a toroid (N turns each carrying current I) of inner radius a and outer radius b at a distance r midway between a and b. *10*

(c) Obtain the plane wave solution of electromagnetic (EM) wave for electric field $\vec{E}$ from Maxwell's equation.

A plane wave solution of the EM wave equation is

$$\vec{E} = \hat{y}\, E_{0y} \cos(\omega t - kx + \alpha) + \hat{z}\, E_{0z} \cos(\omega t - kx + \beta).$$

Under what conditions is this light wave circularly polarised ? *10*

7. Attempt any ***two*** parts of the following : 2×10=20

(a) Considering the ionosphere as a dielectric medium of refractive index $n = n(\omega_N)$, where ω_N is the plasma angular frequency, write down the well-known expression of $n(\omega_N)$ in terms of usual symbols. Calculate the phase and group velocities of a radio wave of angular frequency, $\omega = \sqrt{2}\,\omega_N$. 10

(b) Explain with reference to suitable diagram and formula, why the polarisation of the receiving antenna must be same as that of the transmitting antenna. 10

(c) Elaborate the basic concepts of GPS systems. Explain the various GPS error sources. 10

8. Attempt any ***two*** parts of the following : 2×10=20

(a) Show that (i) all the eigenvalues of a Hermitian matrix are real and (ii) the eigenvectors corresponding to distinct eigenvalues are orthogonal. 10

(b) Solve the following linear differential equations : 10

(i) $\frac{dy}{dt} - y = e^{2x}$

(ii) $\frac{dy}{dt} + y \tan x = \sin 2x$

(c) Obtain the Fourier series for

$$f(x) = x^2 \quad \text{for} \quad -\pi \le x \le \pi.$$ 10

9. Attempt any ***two*** parts of the following : *2×10=20*

(a) Use Maxwell's relations to derive the relation

$$T\,dS = C_V\,dT + T\left(\frac{\partial P}{\partial T}\right)_V dV, \qquad \text{(i)}$$

where the symbols have their usual meaning.

Use (i) to derive the relation

$$\left(\frac{\partial U}{\partial Y}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P, \qquad \text{(ii)}$$

where U is the internal energy. *10*

(b) (i) Write down Planck's law of black-body radiation for the energy density $u_\nu\,d\nu$, between ν and $\nu + d\nu$, in a black-body enclosure at a temperature T. Obtain Rayleigh – Jeans law and Wien's law as limiting cases of this law. *5*

(ii) Show that the total energy density u is proportional to T^4. What is this law called ? *5*

(c) Derive an expression for the Fermi energy of an ideal gas, consisting of fermions of spin $\frac{1}{2}\hbar$.

What do you mean by Fermi temperature ? *10*