2025
MATHEMATICS

Full marks: 80 Time: 3 hours

General instructions:

i) *Approximately 15 minutes is allotted to read the question paper and revise the answers.*
ii) *The question paper consists of 22 questions. All questions are compulsory.*
iii) *Marks are indicated against each question.*
iv) *Internal choice has been provided in some questions.*
v) *Use of simple calculators (non-scientific and non-programmable) only is permitted.*

N.B: Check that all pages of the question paper are complete as indicated on the top left side.

Section – A

1. **Choose the correct answer from the given alternatives:**

(a) Let $f: \boldsymbol{Z} \rightarrow \boldsymbol{Z}$ define as $f(x) = x^2$. Then *f* is **1**

(i) injective (ii) surjective
(iii) bijective (iv) neither injective nor surjective

(b) The principal value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ is **1**

(i) $\frac{\pi}{4}$ (ii) $\frac{-\pi}{4}$ (iii) $\frac{3\pi}{4}$ (iv) $\frac{-3\pi}{4}$

(c) If $\mathrm{A} = \begin{bmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{bmatrix}$ is a symmetric matrix, then **1**

(i) $x = -4$ (ii) $x = 4$
(iii) $x = -3$ (iv) $x = 3$

(d) If $y = \log(\cos x)$, then the value of $\frac{dy}{dx}$ is **1**

(i) $\frac{1}{\cos x}$ (ii) $-\tan x$ (iii) $-\cot x$ (iv) $\frac{-\sin x}{x}$

(e) The function $f(x) = \sin x$ is increasing in the interval **1**

(i) $\left(0, \frac{\pi}{2}\right)$ (ii) $\left(\frac{\pi}{2}, \pi\right)$ (iii) $(0, \pi)$ (iv) $(0, 2\pi)$

(f) The value of $\int \frac{e^{2x}-1}{e^{2x}+1} dx$ is **1**

(i) $log(e^{2x} + 1) + c$ (ii) $log(e^{2x} - 1) + c$
(iii) $log(e^{x} + e^{-x}) + c$ (iv) $log(e^{x} - e^{-x}) + c$

(g) $\int_0^{\pi/4} \sin 2x \, dx$ is equal to **1**

(i) 0 (ii) 1 (iii) 2 (iv) $\frac{1}{2}$

(h) The order of the differential equation $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$ is **1**

(i) 4 (ii) 2 (iii) 1 (iv) not defined

(i) The unit vector in the direction of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ is **1**

(i) $\frac{1}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath} + \hat{k}$ (ii) $\frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{2}{\sqrt{3}}\hat{k}$

(iii) $\frac{1}{\sqrt{5}}\hat{\imath} + \frac{1}{\sqrt{5}}\hat{\jmath} + \frac{2}{\sqrt{5}}\hat{k}$ (iv) $\frac{1}{\sqrt{6}}\hat{\imath} + \frac{1}{\sqrt{6}}\hat{\jmath} + \frac{2}{\sqrt{6}}\hat{k}$

(j) If $P(\mathrm{A}) = \frac{1}{2}$, $P(\mathrm{B}) = 0$, then $P(\mathrm{A/B})$ is **1**

(i) 0 (ii) $\frac{1}{2}$ (iii) 1 (iv) not defined

Section – B

2. Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$. **2**
3. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling price are ₹ 80, ₹ 60 and ₹ 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra. **2**
4. Examine the continuity of the function *f*, defined by

   $f(x) = \begin{cases} x + 1, if\ x \geq 1 \\ x^2 + 1, if\ x < 1 \end{cases}$ at $x = 1$. **2**
5. A balloon, which always remains spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm. **2**
6. Evaluate $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$. **2**
7. Find the general solution of the differential equation $\frac{dy}{dx} = \sqrt{4 - y^2}, (-2 < y < 2)$. **2**
8. Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$. **2**
9. If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ. **2**
10. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that

    i) first ball is black and second ball is red.

    ii) one of them is black and other is red. **2**

Section – C

11. A confectionery shop is a place where sweets and chocolates are sold. The table below gives information on four varieties of chocolates sold there. **4**

Chocolate name	Cost price (₹)
Dairy milk (D)	5
5-Star (S)	10
Crunch (C)	20
Kit-kat (K)	50

Let A= {D, S, C, K} be the set containing the chocolates and B = {5, 10, 20, 50} be the set containing their costs. A relation R is defined on set A as, R = {(x, y): cost of $x \leq$ cost of y}.
Read the information given above and answer the following:
(i) Express the relation R in roster form.
(ii) Is R reflexive relation? Justify.
(iii) Is R symmetric relation? Justify.
(iv) Is R transitive relation? Justify.

12. **a.** If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. Using this equation find A^{-1}.

Or **4**

b. Express the matrix $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

13. Differentiate $x^{\sin x} + (\sin\ )^{\cos x}$ with respect to x. **4**

14. **a.** Integrate $\frac{3x+5}{x^3-x^2-x+1}$

Or **4**

b. Evaluate $\int_0^{\pi/4} \log(1 + \tan x)\, dx$.

15. **a.** Find the general solution of the differential equation $(x + y)\frac{dy}{dx} = 1$.

Or **4**

b. Show that the given differential equation is homogeneous and solve it.
$(x^2 - y^2)dx + 2xydy = 0$

16. Find the angle between the pair of lines $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$. **4**

17. Solve the Linear Programming problem graphically: Minimise Z = $3x + 5y$ subject to constraints $x + 3y \geq 3, x + y \geq 2;\ \ x, y \geq 0$. **4**

18. **a.** Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholar attain A grade in their annual Examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hosteller?

Or **4**

b. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

Section – D

19. **a.** The sum of three number is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third number, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

Or **5**

b. If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then verify that $A\ adj\ A = |A|I$. Also find A^{-1}.

20 **a.** Prove that the volume of the largest cone that can be inscribe in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

Or **5**

b. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

21. **a.** Using integration prove that the area of circle with centre at the origin and radius 'a' is πa^2.

Or **5**

b. Find the area of the region bounded by the ellipse $9x^2 + 16y^2 = 144$.

22. **a.** Find the shortest distance between the lines $\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$ and $\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$.

Or **5**

b. Find the shortest distance between the lines:
$\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2}$ and $\frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}$

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