2024
MATHEMATICS

Full marks: 80   Time: 3 hours
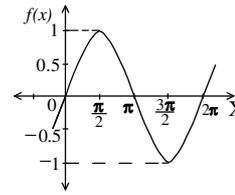
General instructions:
 i) *Approximately 15 minutes is allotted to read the question paper and revise the answers.*
 ii) *The question paper consists of 22 questions. All questions are compulsory.*
 iii) *Marks are indicated against each question.*
 iv) *Internal choice has been provided in some questions.*
 v) *Use of simple calculators (non-scientific and non-programmable) only is permitted.*
N.B: Check that all pages of the question paper is complete as indicated on the top left side.

Section – A

1. **Choose the correct answer from the given alternatives:**

   (a) Let R be the relation in the set Z of all integers defined as, R = {$(x, y) : x - y$ is an integer}, then R is **1**
   (i) Reflexive   (ii) Symmetric
   (iii) Transitive   (iv) Equivalence relation

   (b) The principal value of $\cot^{-1}\left(\dfrac{-1}{\sqrt{3}}\right)$ is **1**

   (i) $\dfrac{-\pi}{3}$ (ii) $\dfrac{\pi}{3}$ (iii) $\dfrac{-2\pi}{3}$ (iv) $\dfrac{2\pi}{3}$

   (c) If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A^T$ then **1**
   (i) $x = 0, y = 5$ (ii) $x + y = 5$
   (iii) $x = y$ (iv) $x = 5, y = 0$

   (d) If $y = e^{x^3}$ then the value of $\dfrac{dy}{dx}$ is **1**
   (i) $3x^3 e^{x^3}$ (ii) $3x^2 e^{x^3}$ (iii) $3ex^3$ (iv) e^{x^3}

(e) The graph of $f(x) = \sin x$ is given below.   **1**
    Then in the interval $(0, \pi)$
    (i) $f(x)$ is a constant function
    (ii) $f(x)$ is an increasing function
    (iii) $f(x)$ is a decreasing function
    (iv) $f(x)$ is neither increasing nor decreasing function

(f) $\int\limits_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^5 x\, dx$ is equal to   **1**

(i) 0   (ii) 1   (iii) $\dfrac{\pi}{2}$ (iv) $\dfrac{-\pi}{2}$

(g) The value of $\int \dfrac{\sec^2 x}{\cosec^2 x}\, dx$ is   **1**
    (i) $\sec x + x + C$ (ii) $\tan x - x + C$
    (iii) $\sec x - x + C$ (iv) $\tan x + x + C$

(h) The degree of the differential equation
$$\left(\frac{d^2 y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0 \text{ is}$$ **1**
    (i) 3   (ii) 1   (iii) 0   (iv) not defined

(i) If $\vec{a}$ is a nonzero vector of magnitude 'a' and λ a nonzero scalar, then $\lambda\,\vec{a}$ is
    unit vector if   **1**

(i) $\lambda = 1$ (ii) $\lambda = -1$ (iii) $a = |\lambda|$ (iv) $a = \dfrac{1}{|\lambda|}$

(j) If P(A) = 0.8, P(B) = 0.5 and P(B/A) = 0.4 then the value of $P(A \cap B)$ is   **1**
    (i) 0.32   (ii) 0.64   (iii) 0.98   (iv) 0.40

Section – B

2. Find the value of $\sin^{-1}\left(\sin \dfrac{2\pi}{3}\right)$.   **2**

3. Find x and y, if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$.   **2**

4. Find the value of k so that the function defined by
$$f(x) = \begin{cases} kx^2, & if\ x \le 2 \\ 3, & if\ x > 2 \end{cases} \text{ is continuous at } x = 2.$$ **2**

5. The side of a square sheet of metal is increasing at the rate of 3 cm/minute. At what rate is the area of the sheet increasing when its side is 10 cm long? **2**

6. Evaluate $\int \cot x \log(\sin x) dx$. **2**

7. Integrate $\int \sin^2 x \, dx$. **2**

8. Verify, if $y = Ax$ is a solution of the differential equation $xy' = y \, (x \neq 0)$. **2**

9. Find a vector in the direction of the vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units. **2**

10. If A and B are two events such that $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find P(not A and not B). **2**

Section – C

11. **a.** Let L be the set of all lines in XY plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$. Show that R is equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

   Or **4**

   b. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3} \right)$. Is f one-one and onto? Justify your answer.

12. Three schools A, B and C organized a fair for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats and plates from recycled material at a cost of `25, `100 and `50 each. The number of articles sold are given below: **2+2=4**

School / Article	A	B	C
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	30	40

   Based on the above information, answer the following questions.
   a) If P be a 3×3 matrix representing the sale of handmade fans, mats and plates by three schools A, B and C and if Q be a 3×1 matrix representing the sale prices in ` of given products per unit, then, find the matrices P and Q.
   b) Find the funds collected by the schools A and B.

13. If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$, show that $\dfrac{dy}{dx} = -\dfrac{y}{x}$ **4**

14. **a.** Evaluate $\displaystyle\int \dfrac{5x+3}{\sqrt{x^2+4x+10}}\,dx$

    Or   **4**

    b. Evaluate $\displaystyle\int \dfrac{2}{(1-x)(1+x^2)}\,dx$

15. **a.** Solve the differential equation $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \dfrac{x}{y}\right)dy = 0$

    Or   **4**

    b. Find the general solution of the differential equation
    $(1 + x^2)dy + 2xy\,dx = \cot x\,dx\,(x \neq 0)$

16. Find the values of p so that the lines $\dfrac{1-x}{3} = \dfrac{7y-14}{2p} = \dfrac{z-3}{2}$ and

    $\dfrac{7-7x}{3p} = \dfrac{y-5}{1} = \dfrac{6-z}{5}$ are at right angles.   **4**

17. Solve the Linear Programming Problem graphically: Minimise $Z = x + 2y$ subject to the constraints $x + 2y \geq 6, 2x + y \geq 3;\ x, y \geq 0$.   **4**

18. **a.** In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B?

    Or   **4**

    b. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Section – D

19. **a.** Solve the system of linear equations by using matrix method.

    $3x - 2y + 3z = 8$

    $2x + y - z = 1$

    $4x - 3y + 2z = 4$

    Or   **5**

b. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$. Prove that $A^3 - 6A^2 + 7A + 2I = 0$

20 **a.** Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic centimeters, find the dimensions of the can which has the minimum surface area?

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Or

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b. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

21. **a.** Find the area of the region bounded by the ellipse $9x^2 + 4y^2 = 36$ using integration.

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Or

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b. Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.

22. **a.** Find the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

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Or

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b. Find the vector equation of the line passing through the point $(1, 2, -4)$ and perpendicular to the two lines:
$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

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