**2022**
**MATHEMATICS**

Total marks : 80                                              Time : 3 hours

**General Instructions:**
i) *Approximately 15 minutes is allotted to read the question paper and revise the answers.*
ii) *The question paper consists of 18 questions.*
iii) *All questions are compulsory.*
iv) *Internal & general choice have been provided in some questions.*
v) *Marks allocated to every question are indicated against it.*
**N.B: Check to ensure that all pages of the question paper is complete as indicated on the top left side.**
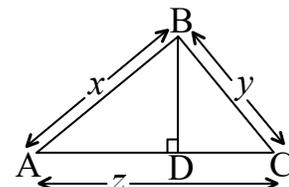
**Section – A**

1. **Choose the correct answer from the given alternatives.**

   (a) If α and β are the zeroes of the polynomial $f(x) = px^2 - 2x + 3p$ and $\alpha + \beta = \alpha\beta$ then the value of $p$ is                                                          1

   (i) $-\dfrac{2}{3}$          (ii) $\dfrac{2}{3}$          (iii) $\dfrac{1}{3}$          (iv) $-\dfrac{1}{3}$

   (b) The pair of equations $x + y - 40 = 0$ and $x - 2y + 14 = 0$ have          1
   (i) a unique solution.                    (ii) exactly two solutions.
   (iii) infinitely many solutions.          (iv) no solution.

   (c) If $x = 1$ is a common root of $ax^2 + ax + 2 = 0$ and $x^2 + x + b = 0$, then $ab =$          1
   (i) 1          (ii) 2          (iii) 3          (iv) 4

   (d) If sum of the first five terms of an AP is 30 and the $5^{th}$ term is 10, then the sum of the first four terms of the AP is          1
   (i) 5          (ii) 10          (iii) 20          (iv) 30

   (e) If $\tan 2\theta = \cot(\theta + 15°)$, where 2θ and $(\theta + 15°)$ are acute, the value of θ is          1
   (i) 22°          (ii) 25°          (iii) 30°          (iv) 35°

   (f) *y*-axis divides the join of P(–4, 2) and Q(8, 3) in the ratio          1
   (i) 3 : 1          (ii) 1 : 3          (iii) 2 : 1          (iv) 1 : 2

   (g) If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then ∠POA is equal to          1
   (i) 50°          (ii) 60°          (iii) 70°          (iv) 80°

(h)   An arc of a circle is of length $5\pi$ cm and the sector it bounds has an area of $20\pi$ cm². The radius of the circle is **1**
    (i) 16 cm        (ii) 12 cm      (iii) 8 cm       (iv) 4 cm

(i)   The ratio of the total surface area to the lateral surface area of a cylinder with base radius 80 cm and height 20 cm is **1**
    (i) 2 : 1        (ii) 3 : 1      (iii) 4 : 1       (iv) 5 : 1

(j)   Two dice are rolled once. The probability of getting such numbers on two dice, whose product is a perfect square, is **1**
    (i) 8        (ii) $\dfrac{1}{8}$      (iii) $\dfrac{9}{2}$       (iv) $\dfrac{2}{9}$

## Section – B

2.   An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march? **2**

3.   Find the value of $k$ for the quadratic equation $kx(x-2)+6=0$ has equal roots. **2**

4.   Find the values of $y$ for which the distance between the points P(2, −3) and Q(10, $y$) is 10 units. **2**

5.   In the adjoining figure, $\triangle$ABC is a right triangle in which $\angle$B = 90° and BD $\perp$ AC. If AB = $x$ units, BC = $y$ units and AC = $z$ units, then find BD. **2**

6.   The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes. **2**

## Section – C

7.   **Answer any three from the following questions (a) to (e).**     **3 × 3 = 9**

(a)   On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g($x$), the quotient and remainder were $(x - 2)$ and $(-2x + 4)$ respectively. Find g($x$).

(b)   Meena went to a bank to withdraw ₹2000. She asked the cashier to give her ₹50 and ₹100 notes only. Meena got 25 notes in all. Find how many notes of ₹50 and ₹100 she received.

(c)   Solve the following pair of linear equations by cross-multiplication method:

$$x - 3y - 7 = 0$$
$$3x - 3y - 15 = 0$$

**(d)** A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

**(e)** A sum of ₹700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each of the prizes.

8. **Answer any two from the following questions (a) to (d).** **2 × 3 = 6**

**(a)** In triangle ABC, right angled at B, if $\tan A = \dfrac{1}{\sqrt{3}}$, find the value of $\cos A \cos C - \sin A \sin C$, with the help of a right triangle.

**(b)** If $\sin(A - B) = \dfrac{1}{2}$, $\cos(A + B) = \dfrac{1}{2}$, $0° < A + B \le 90°$, $A > B$, find A and B.

**(c)** Prove that: $\dfrac{1 + \sec A}{\sec A} = \dfrac{\sin^2 A}{1 - \cos A}$, where angle A is an acute angle.

**(d)** A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

9. **Case Study based question:** **3**
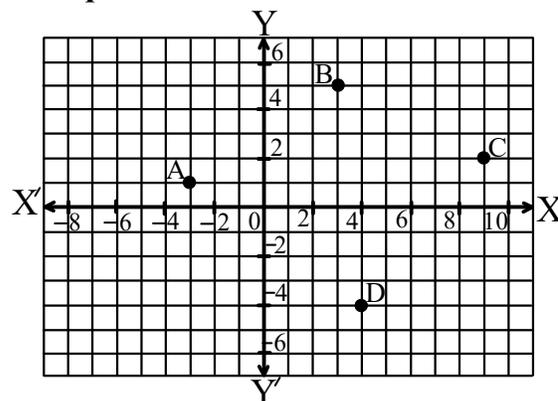


Four friends are seated at the points A, B, C and D on a lawn keeping social distancing, as shown in the figure above. One more friend wants to join them and sit exactly at the middle position E on a straight line between A and C. Based on the above information, answer the following questions (i) to (iii).

(i)　The distance between A and D is

    (a) $\sqrt{72}$ units　　　　(b) $\sqrt{73}$ units　　(c) $\sqrt{74}$ units　　　　(d) $\sqrt{75}$ units

(ii)　The coordinates of the position of E are

    (a) $\left(\dfrac{3}{2}, 3\right)$　　　　　(b) $\left(3, \dfrac{3}{2}\right)$　　　　(c) $(6, 3)$　　　　　(d) $(3, 6)$

(iii)　$x$-axis divides the distance/length between B and D in the ratio

    (a) $3 : 4$　　　　　(b) $4 : 3$　　　　　(c) $5 : 4$　　　　　(d) $4 : 5$

10.　**a.**　Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $1\dfrac{1}{2}$ times the corresponding sides of the isosceles triangle. (Traces of construction only is required.)

                             **Or**　　　　　　　　　　　　　　**3**

   **b.**　Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle. (Traces of construction only is required.)

11.　**a.**　A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. [Use $\pi = 3.14$ and $\sqrt{3} = 1.73$]

                             **Or**　　　　　　　　　　　　　　**3**

   **b.**　How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

12.　**Answer any two from the following questions (a) to (c).**　　　　　**2 × 3 = 6**

**(a)**　A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data:

| Number of cars | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
|---|---|---|---|---|---|---|---|---|
| Frequency | 7 | 14 | 13 | 12 | 20 | 11 | 15 | 8 |

**(b)**　The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table:

| Length (in mm) | 118-126 | 127-135 | 136-144 | 145-153 | 154-162 | 163-171 | 172-180 |
|---|---|---|---|---|---|---|---|
| Number of leaves | 3 | 5 | 9 | 12 | 5 | 4 | 2 |

Find the median length of the leaves.

**(c)** A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears: (i) a two-digit number, (ii) a perfect square number, (iii) a number divisible by 5.

## Section – D

13. **a.** Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the *x*-axis and shade the triangular region.

<div align="center">Or</div> 5

**b.** Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

14. **a.** A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

<div align="center">Or</div> 5

**b.** A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After sometime, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.
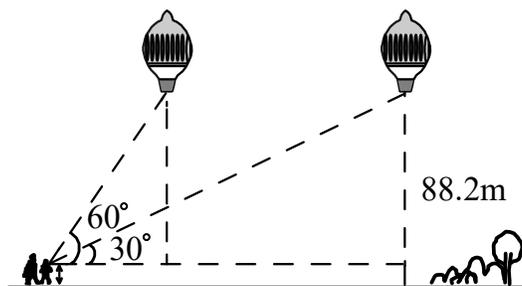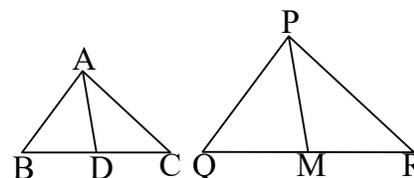
15. **a.** State and Prove Pythagoras Theorem.

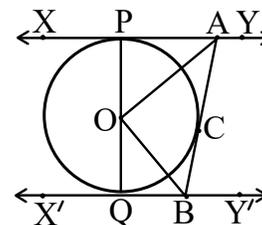<div align="center">Or</div> 5

**b.** Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle$PQR. Show that $\triangle$ABC ~ $\triangle$PQR.
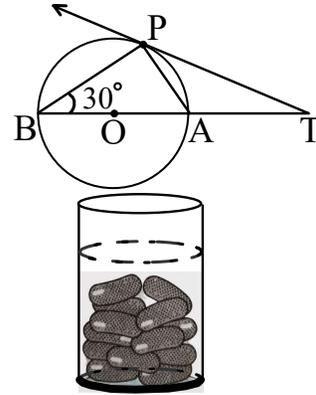
16. **a.** In the adjoining figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that $\angle$AOB = 90°

<div align="center">Or</div> 5

**b.** In the adjoining figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If ∠PBT = 30°, prove that BA : AT = 2 : 1

**17. a.** A *gulab jamun*, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 *gulab jamuns*, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm.

**Or**                                                                5

**b.** A container, opened from the top and made up of a metal sheet, is in the form of a frustrum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container @ of ₹20 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹8 per 100 cm². [Take π = 3.14]

**18. a.** The following distribution shows the daily pocket allowance of children of a locality. Find the mean pocket allowance by step-deviation method.

| Daily pocket allowance (in ₹) | More than 11 | More than 13 | More than 15 | More than 17 | More than 19 | More than 21 | More than 23 |
|---|---|---|---|---|---|---|---|
| Number of children | 64 | 57 | 51 | 42 | 29 | 9 | 4 |

**Or**                                                                5

**b.** The distribution of heights (in cm) of 96 children is given below:

| Height (in cm) | Number of children |
|---|---|
| 124-128 | 5 |
| 128-132 | 8 |
| 132-136 | 17 |
| 136-140 | 24 |
| 140-144 | 16 |
| 144-148 | 12 |
| 148-152 | 6 |
| 152-156 | 4 |
| 156-160 | 3 |
| 160-164 | 1 |

Draw a less than type cumulative frequency curve for this data and use it to compute median height of the children.

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