Total No. of Printed Pages—12

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MATHEMATICS

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

General Instructions :

(i) All questions are compulsory.

(ii) This question paper contains 36 questions divided into four Sections—A, B, C and D. Section—A comprises of 20 questions of *1* mark each, Section—B comprises of 6 questions of *2* marks each, Section—C comprises of 6 questions of *4* marks each and Section—D comprises of 4 questions of *6* marks each.

(iii) There is no overall choice. However, internal choice has been provided in 9 questions of Section—A, 5 questions of Section—B, 5 questions of Section—C and 3 questions of Section—D. You have to attempt only one of the alternatives in all such questions.

(iv) Use of calculator is not permitted.

[P.T.O.

(**2**)

SECTION—A

1. Find the principal value of $\cot^{-1}(-\sqrt{3})$.   1

Or

Evaluate : $\sin\left\{\dfrac{\pi}{3} - \sin^{-1}\left(-\dfrac{1}{2}\right)\right\}$ 1

2. Write the elements a_{23} and a_{32} of a 3×3 matrix $A = [a_{ij}]$, whose elements a_{ij} are given by $a_{ij} = \dfrac{|i - j|}{2}$.   1

3. Check the continuity of the function f given by $f(x) = 2x + 3$ at $x = 1$.   1

Or

Find the value of k, so that the function

$$f(x) = \begin{cases} kx^2, & \text{if } x \le 2 \\ 3, & \text{if } x > 2 \end{cases}$$

is continuous at $x = 2$.   1

4. Define an equivalence relation.   1

Or

If $R = \{(1, -1), (2, -2), (3, -1)\}$ is a relation, then find the domain and the range of R.   1

(3)

5. Evaluate : $\int x \log 2x \, dx$

1

6. Find the equation of the line joining (1, 2) and (3, 6) using determinants.

1

Or

For what values of λ the matrix $\begin{bmatrix} 5 - \lambda & \lambda + 1 \\ 2 & 4 \end{bmatrix}$ is invertible?

1

7. The random variable X has a probability distribution $P(X)$ of the following form, where k is some real number :

$$P(X) = \begin{cases} k, & \text{if } X = 0 \\ 2k, & \text{if } X = 1 \\ 3k, & \text{if } X = 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of k.

1

8. Find an anti-derivative of $(ax + b)^2$ by the method of inspection.

1

Or

Find the order and degree of the differential equation

$$\frac{d^2 y}{dx^2} + 5x \left(\frac{dy}{dx} \right)^2 - 6y = \log x$$

1

9. If θ is the angle between any two non-zero vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$, then find θ.   1

Or

If a line makes angles 90°, 135° and 45° with the x-, y- and z-axes respectively, then find its direction cosines.   1

10. Given two independent events A and B such that $P(A) = 0 \cdot 3$ and $P(B) = 0 \cdot 6$. Find P (neither A nor B).   1

11. Find the second-order derivative of $x \cos x$ w.r.t. x.   1

12. If $f : \mathbb{R} \to \mathbb{R}$ is a function defined by $f(x) = x^2$, $\forall \ x \in \mathbb{R}$, then show that f is not one-one.   1

13. If $\int_0^a 3x^2 dx = 8$, then write the value of a.   1

Or

Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$.   1

14. Show that the function $f(x) = 3x + 17$ is strictly increasing on $\mathbb{R}$.   1

15. Show that the relation R in $\mathbb{R}$ defined as $R = \{(a, b) : a \le b\}$ is transitive.   1

16. Find $\dfrac{dy}{dx}$, if $ax + by^2 = \cos y$.   1

Or

Find the integral $\displaystyle\int \dfrac{\log x}{x}\, dx$.   1

Choose the correct answer :

17. The integrating factor of the differential equation $x\dfrac{dy}{dx} - y = 2x^2$ is

    (a) e^{-x}

    (b) e^{-y}

    (c) $\dfrac{1}{x}$

    (d) x 1

18. The maximum value of the objective function $Z = 3x + 4y$ subject to the constraints $x \ge 0$, $y \ge 0$ and $x + y \le 1$ is

    (a) 7

    (b) 4

    (c) 3

    (d) 10   1

19. If $\vec{a}$ is a non-zero vector of magnitude 'a' and 'λ' is a non-zero scalar, then $\lambda\vec{a}$ is a unit vector if

(a) $\lambda = 1$

(b) $\lambda = -1$

(c) $a = |\lambda|$

(d) $a = \dfrac{1}{|\lambda|}$ 1

Or

The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$ is

(a) 0

(b) −1

(c) 1

(d) 3   1

20. $\int e^x \sec x\,(1 + \tan x)\,dx$ is equal to

(a) $e^x \cos x + c$

(b) $e^x \sec x + c$

(c) $e^x \sin x + c$

(d) $e^x \tan x + c$ 1

(7)

SECTION—B

21. Find the vector and the Cartesian equations of the line through the point $(5, 2, -4)$ and which is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$. 2

22. Evaluate the integral : 2

$$\int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x}\, dx$$

Or

Find $\dfrac{dy}{dx}$, if $x = a\,(\cos\theta + \theta\sin\theta)$ and $y = a\,(\sin\theta - \theta\cos\theta)$. 2

23. Find the unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points $(1, 2, 3)$ and $(4, 5, 6)$ respectively. 2

Or

Find $|\vec{a} - \vec{b}|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$. 2

24. Find the values of x, y and z from the following equation :   2

$$\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

Or

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 5A + 7I = 0$.   2

25. Find the interval in which the function given by $f(x) = x^2 + 2x - 5$ is strictly increasing.   2

Or

Evaluate : $\int e^x (\sin x + \cos x)\, dx$ 2

26. Find the values of p so that the lines

$$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2} \quad \text{and} \quad \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

are at right angles.   2

Or

If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.   2

SECTION—C

27. By using the properties of definite integrals, evaluate the integral

$$\int_0^{\pi/4} \log (1 + \tan x)\, dx \qquad\qquad 4$$

Or

Find $\int \dfrac{3x - 2}{(x + 1)^2 (x + 3)}\, dx.$ 4

28. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.   4

Or

Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is *(a)* increasing and *(b)* decreasing.   2+2

29. Let $f : X \to Y$ be a function. Define a relation R in X given by $R = \{ (a, b) : f(a) = f(b) \}$. Show that R is an equivalence relation.   4

Or

Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3 - 4x$ is bijective.   4

30. Find $\dfrac{dy}{dx}$, if $y = x^{x\cos x} + \dfrac{x^2 + 1}{x^2 - 1}$.   4

Or

If $y = 3\cos(\log x) + 4\sin(\log x)$, then show that $x^2 y_2 + x y_1 + y = 0$.   4

31. Solve the following linear programming problem graphically :   4

Maximize $Z = 4x + y$

subject to the constraints

$$x + y \le 50$$
$$3x + y \le 90$$
$$x \ge 0, \; y \ge 0$$

32. Find the equation of the curve passing through the point $\left(0, \dfrac{\pi}{4}\right)$ whose differential equation is

$$\sin x \cos y \, dx + \cos x \sin y \, dy = 0 \qquad 4$$

Or

Solve the homogeneous differential equation

$$(x^2 + xy)\, dy = (x^2 + y^2)\, dx \qquad 4$$

SECTION—D

33. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\dfrac{2R}{\sqrt{3}}$. Also find the maximum volume. 6

Or

Find the area of the region bounded by the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ above x-axis. 6

34. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Using A^{-1}, solve the following system of equations : 6

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

35. Find the Cartesian equation of the line passing through the point $(1, 2, -4)$ and perpendicular to the two lines

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \quad \text{and} \quad \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

Also write the vector form of the line so obtained. 5+1

Or

Find the shortest distance between the lines whose vector equations are

$$\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k} \quad \text{and}$$
$$\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$$

6

36. State Bayes' theorem on probability. Use this theorem to solve any *one* of the following : 1+5

(a) Bag *I* contains 3 red and 4 black balls and Bag *II* contains 4 red and 5 black balls. One ball is transferred from Bag *I* to Bag *II* and then one ball is drawn from Bag *II*. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Or

(b) A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

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