Total No. of Printed Pages—11

HS/XII/A. Sc. Com/M/24

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MATHEMATICS

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

General Instructions :

(i) All questions are compulsory.

(ii) This question paper contains 36 questions divided into four Sections—A, B, C and D. Section—A comprises of 20 questions of *1* mark each, Section—B comprises of 6 questions of *2* marks each, Section—C comprises of 6 questions of *4* marks each and Section—D comprises of 4 questions of *6* marks each.

(iii) There is no overall choice. However, internal choice has been provided in 7 questions of Section—A, 3 questions of Section—B, 5 questions of Section—C and 2 questions of Section—D. You have to attempt only one of the alternatives in all such questions.

(iv) Use of calculator is not permitted.

SECTION—A

1. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$. 1

2. Differentiate w.r.t. x : 1

$$y = \log(\log x), \quad x > 1$$

3. Construct a 2×2 matrix $A = [a_{ij}]$, where $a_{ij} = \frac{(i+2j)^2}{2}$. 1

Or

Find the values of x and y, if

$$\begin{bmatrix} 3x+y & -y \\ 2x-y & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$ 1

4. Find the unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$. 1

5. Find the Cartesian equation of the line which passes through the point (–2, 4, – 5) and parallel to the line given by

$$\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$$ 1

6. Integrate the function $\sin x \cdot \sin(\cos x)$. 1

Or

Evaluate the definite integral $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$. 1

7. Find $\frac{dy}{dx}$ for the following : 1

$$2x + 3y = \sin y$$

8. Evaluate : 1

$$\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx$$

9. Find the direction cosines of the vector $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$. 1

Or

Find the scalar and vector components of the vector with initial point $(2, 1)$ and terminal point $(-5, 7)$. 1

10. Find the value of k, if the function given by

$$f(x) = \begin{cases} kx^2, & x \geq 1 \\ 4, & x < 1 \end{cases}$$

is continuous at $x = 1$. 1

11. Differentiate $\log(\cos e^x)$ w.r.t. x. 1

Or

Find $\frac{dy}{dx}$, if $x = a\cos\theta$ and $y = a\sin\theta$. 1

12. The radius of a circle is increasing at the rate of 0·7 cm/s. What is the rate of increase in its circumference? 1

13. Find the second-order derivative of $\log x$. 1

14. Examine the continuity of the function $f(x) = 5x - 3$ at $x = 5$. 1

15. Evaluate : 1

$$\int \frac{dx}{\sqrt{9 - 25x^2}}$$

Or

Integrate w.r.t. x : 1

$$\frac{1}{x + x\log x}$$

16. Verify that $y = e^x + 1$ is a solution of the differential equation $y'' - y' = 0$. 1

Or

Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}$. 1

Choose the correct answer :

17. Let R be the relation in the set $\mathbb{N}$ given by $R = \{(a, b) : a = b - 2, \ b > 6\}$. Then

(a) $(2, 4) \in R$

(b) $(3, 8) \in R$

(c) $(6, 8) \in R$

(d) $(8, 7) \in R$ 1

18. The corner points of the feasible region determined by a system of linear constraints are (0, 0), (0, 40), (20, 40), (60, 20) and (60, 0). The maximum value of $Z = 4x + 3y$ is

(a) 120

(b) 240

(c) 300

(d) 200 1

19. The matrix $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ is a

(a) diagonal matrix

(b) row matrix

(c) column matrix

(d) unit matrix 1

Or

The square matrices A and B will be inverse of each other, if

(a) $AB = O, \ BA = I$

(b) $AB = BA = O$

(c) $AB = BA = I$

(d) $AB = I, BA = O$ 1

20. The value of $(\hat{i} \times \hat{j}) \cdot \hat{k} + (\hat{j} \times \hat{k}) \cdot \hat{i}$ is

(a) 1

(b) 2

(c) 0

(d) −1 1

SECTION—B

21. Show that the function $f : \mathbb{R}^* \to \mathbb{R}^*$ defined by $f(x) = \frac{1}{2x}$ is onto. Here, $\mathbb{R}^*$ is the set of non-zero real numbers. 2

Or

Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f : A \to B$ defined by $f(x) = \frac{x-2}{x-3}$. Show that f is one-one. 2

22. Prove that the function $f(x) = \log x$ is strictly increasing on the interval $(0, \infty)$. 2

23. Evaluate : 2

$$\int \frac{dx}{(x+1)(x+2)}$$

Or

Find $\frac{dy}{dx}$, if $xy + y^2 = \tan x + y$. 2

24. If $y = 5\cos x - 3\sin x$, then prove that

$$\frac{d^2y}{dx^2} + y = 0$$ 2

Or

Find $\frac{dy}{dx}$, if $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$. 2

25. Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = p$ and $P(A \cup B) = \frac{3}{5}$. Find p, if A and B are independent. 2

26. Using determinants, find the value of k so that the points $(3, -2)$, $(k, 2)$ and $(8, 8)$ lie on the same line. 2

SECTION—C

27. Using integrals, find the area enclosed by the circle $x^2 + y^2 = 4$. 4

Or

Evaluate : 4

$$\int \frac{2x+1}{\sqrt{x^2 + 4x + 3}}\, dx$$

28. Differentiate $(\log x)^{\cos x}$ w.r.t. x. 4

Or

Using the properties of definite integral, evaluate

$$\int_0^{\pi/2} (2\log \sin x - \log \sin 2x)\, dx$$ 4

29. Show that the relation R on $\mathbb{Z}$ given by $R = \{(x, y) : x - y \text{ is divisible by } 2\}$ is an equivalence relation. 4

Or

Let A and B be two sets. Show that $f : A \times B \to B \times A$ such that $f(a, b) = (b, a)$ is one-one and onto. 4

30. Solve the following LPP graphically : 4

Minimize $Z = 200x + 500y$

subject to

$$x + 2y \geq 10$$
$$3x + 4y \leq 24$$
$$x, \; y \geq 0$$

31. Find the angle between the two lines whose equations are

$$\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4} \quad \text{and} \quad \frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$$ 4

Or

Find λ and μ, if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$. 4

32. Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is—

(a) strictly increasing;

(b) strictly decreasing. 4

Or

If $y = e^{a\cos^{-1}x}$, $-1 \le x \le 1$, then show that

$$(1 - x^2)y_2 - xy_1 - a^2y = 0$$ 4

SECTION—D

33. Solve the following system of linear equations : 6

$$\begin{aligned} x + y + z &= 6 \\ x - y + z &= 2 \\ 2x + y - z &= 1 \end{aligned}$$

34. Find the shortest distance between the lines l_1 and l_2, whose vector equations are

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$$ 6

35. Show that a closed right circular cylinder of given total surface area and maximum volume is such that its height is equal to its diameter. 6

Or

Solve the linear differential equation

$$\cos^2 x \frac{dy}{dx} + y = \tan x \quad \left(0 \le x \le \frac{\pi}{4}\right)$$

given that $y = 1$, when $x = 0$. 6

36. State Bayes' theorem and use it to solve the following : 2+4=6

In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3 / 4 be the probability that he knows the answer and 1 / 4 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability 1 / 4, what is the probability that the student knows the answer given that he answered it correctly?

Or

A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Determine the following : 6

(i) k

(ii) $P(X < 3)$

(iii) $P(X > 6)$

(iv) $P(0 < X < 3)$

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