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**HS/XII/A.Sc.Com/M/OC/20**

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**MATHEMATICS**

( Old Course )

*Full Marks : 100*

*Time : 3 hours*

*General Instructions :*

- (i) Write all the answers in the Answer Script.
- (ii) The question paper consists of three Sections—A, B and C.
- (iii) Section—A consists of 15 questions, carrying 2 marks each.
- (iv) Section—B consists of 10 questions, carrying 4 marks each, out of which 3 questions have internal choices.
- (v) Section—C has 5 questions, carrying 6 marks each, out of which 2 questions have internal choices.

**SECTION—A**

- 1.** Let  $S$  be the set of all real numbers and let  $R$  be a relation in  $S$  defined by

$$R \equiv \{(a, b) : (1 + ab) > 0\}$$

Show that  $R$  is reflexive and symmetric.

2

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2. Let

$$R = \left\{ \left( a, \frac{1}{a} \right) : a \in N \text{ and } 1 \leq a \leq 5 \right\}$$

List the elements of the above relation. Find the domain and range.

2

3. Show that the function  $f : \mathbb{R} \rightarrow \mathbb{R} : f(x) = |x|$  is neither one-one nor onto.

2

4. Show that the operation  $*$  on  $\mathbb{Z}$ , defined by

$$a * b = a + b + 1 \quad \forall a, b \in \mathbb{Z}$$

satisfies (a) closure property and (b) commutative law.

2

5. Construct a  $3 \times 2$  matrix whose elements are given by  $a_{ij} = i + 2j$ .

2

6. Let

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & -5 & 7 \end{bmatrix}$$

Verify that  $(A')' = A$ .

2

7. Show that  $f(x) = x^3$  is continuous at the point  $x = 2$ .

2

8. Differentiate  $y = \tan \sqrt{x}$  with respect to  $x$ .

2

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9. If  $x^3 + y^3 = 3axy$ , find  $\frac{dy}{dx}$ . 2

10. Find  $\frac{dy}{dx}$ , when  $x = at^2$ ,  $y = 2at$ . 2

11. A stone is dropped into a quiet lake and the waves move in circles. If the radius of a circular wave increases at the rate of 4 cm/s, find the rate of increase in its area at the instant when its radius is 10 cm. 2

12. Evaluate : 2

$$\int x^2 e^x dx$$

13. Let

$$\bar{a} = 3\hat{i} + 2\hat{j} \quad \text{and} \quad \bar{b} = 2\hat{i} + 3\hat{j}$$

Is  $|\bar{a}| = |\bar{b}|$ ? Is  $\bar{a} = \bar{b}$ ? 2

14. If

$$\bar{a} = 3\hat{i} + \hat{j} - 4\hat{k} \quad \text{and} \quad \bar{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$$

find  $\bar{a} \times \bar{b}$  and  $|\bar{a} \times \bar{b}|$ . 2

15. If  $E_1$  and  $E_2$  are two independent events such that  $P(E_1) = 0.35$  and  $P(E_1 \cup E_2) = 0.60$ , find  $P(E_2)$ . 2

SECTION—B

16. Express the matrix

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

as the sum of a symmetric and a skew-symmetric matrix. 4

Or

If

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

verify that  $A^2 - 4A - I = 0$  and hence find  $A^{-1}$ . 4

17. Show that the semivertical angle of a right circular cone of given surface area and maximum volume is

$$\sin^{-1}\left(\frac{1}{3}\right) 4$$

18. Verify Rolle's theorem for the function  $f(x) = e^x \cos x$  in

$$\left[ \frac{-\pi}{2}, \frac{\pi}{2} \right] 4$$

Or

Using Lagrange's mean value theorem, find a point on the curve  $y = \sqrt{x-2}$ , defined in the interval  $[2, 3]$ , where the tangent is parallel to the chord joining the end points of the curve. 4

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19. (a) The side of a square is increasing at the rate of 0.2 cm/s. Find the rate of increase of the perimeter of the square. 2

(b) Find the approximate value of the cube root of 127. 2

20. Water is leaking from a conical funnel at the rate of 5 cm<sup>3</sup>/s. If the radius of the base of the funnel is 5 cm and its altitude is 10 cm, find the rate at which the water level is dropping when it is 2.5 cm from the top. 4

21. (a) If  $y = e^x(\sin x + \cos x)$ , prove that

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0 \quad 2$$

(b) Solve the differential equation

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0 \quad 2$$

22. (a) Evaluate : 2

$$\int_0^a \frac{dx}{\sqrt{ax-x^2}}$$

(b) Evaluate : 2

$$\int_0^{\sqrt{2}} \sqrt{2-x^2} dx$$

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Or

Evaluate : 4

$$\int_0^{\pi} \frac{x}{(a^2 \cos^2 x + b^2 \sin^2 x)} dx$$

**23.** Find the intervals in which the function

$$f(x) = -2x^3 - 9x^2 - 12x + 1$$

is (a) strictly increasing and (b) strictly decreasing. 4

**24.** A line passes through the point (3, 4, 5) and is parallel to the vector  $(2\hat{i} + 2\hat{j} - 3\hat{k})$ . Find the equations of the line in the vector as well as Cartesian forms. 4

**25.** Find the equations of the normals to the curve  $3x^2 - y^2 = 8$  parallel to the line  $x + 3y = 4$ . 4

SECTION—C

**26.** Using matrices, solve the following system of equations : 6

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

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Or

(a) Without expanding the determinants, prove that

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} \quad 3$$

(b) Solve for  $x$  : 3

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

27. Evaluate

$$\int_a^b \sin x \, dx$$

from the first principle. 6

Or

Find the area of the smaller region bounded by the

ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and the straight line  $\frac{x}{a} + \frac{y}{b} = 1$ . 6

28. (a) Find the angle between the lines whose direction ratios are 2, 3, 6 and 1, 2, 2. 2

(b) Show that the lines

$$\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5} \quad \text{and} \quad \frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$$

intersect each other. Also, find the point of their intersection. 4

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- 29.** A factory has three machines  $X$ ,  $Y$  and  $Z$ , producing 1000 bolts, 2000 bolts and 3000 bolts per day respectively. The machine  $X$  produces 1% defective bolts,  $Y$  produces 1.5% defective bolts and  $Z$  produces 2% defective bolts. At the end of the day, a bolt is drawn at random and it is found to be defective. What is the probability that this defective bolt has been produced by machine  $X$ ? 6
- 30.** A firm manufactures two types of products  $A$  and  $B$  and sells them at a profit of ₹ 5 per unit of type  $A$  and ₹ 3 per unit of type  $B$ . Each product is processed on two machines  $M_1$  and  $M_2$ . One unit of type  $A$  requires one minute of processing time on  $M_1$  and two minutes of processing time on  $M_2$ , whereas one unit of type  $B$  requires one minute of processing time on  $M_1$  and one minute of processing time on  $M_2$ . Machines  $M_1$  and  $M_2$  are respectively available for at most 5 hours and 6 hours in a day. Find out how many units of each type of products should the firm produce a day in order to maximize the profit. Solve the problem graphically. 6

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