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HS/XII/A. Sc. Com/M/NC/21

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MATHEMATICS

(New Course)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

General Instructions :

- (i) All questions are compulsory.
- (ii) This question paper contains 36 questions divided into four Sections A, B, C and D. Section—A comprises of 20 questions of 1 mark each, Section—B comprises of 6 questions of 2 marks each, Section—C comprises of 6 questions of 4 marks each and Section—D comprises of 4 questions of 6 marks each.
- (iii) There is no overall choice. However, internal choice has been provided in 9 questions of Section—A, 5 questions of Section—B, 5 questions of Section—C and 2 questions of Section—D. You have to attempt only one of the alternatives in all such questions.
- (iv) Use of calculator is not permitted.

(2)

SECTION—A

1. If $R = \{(1, -1), (2, -2), (3, -1)\}$ is a relation, then find the domain and range of R . 1

Or

Find the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$. 1

2. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is a function defined by $f(x) = x^2, \forall x \in \mathbb{R}$, then show that f is not one-one. 1

3. Construct a 2×2 matrix $A = [a_{ij}]$, whose elements are given by

$$a_{ij} = \frac{(i+j)^2}{2} \quad 1$$

Or

Find the value of AB when $A = [1\ 2\ 3\ 4]$ and

$$B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad 1$$

4. Use determinant to find the value of K for which the points $A(3, -2)$, $B(K, 2)$ and $C(8, 8)$ are collinear. 1

(3)

Or

Find the value of λ so that the matrix

$$\begin{bmatrix} 5-\lambda & \lambda+1 \\ 2 & 4 \end{bmatrix}$$

is singular.

1

5. If

$$\begin{vmatrix} 4 & m \\ -3 & 5 \end{vmatrix} = 8$$

find the value of m .

1

Or

If

$$\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix} = 3$$

find the value of x .

1

6. Show that the matrix

$$A = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$$

is skew-symmetric.

1

7. If $y = e^{3\log x}$, then find $\frac{dy}{dx}$.

1

Or

Find the value of $\frac{d}{dx}(\sin^2 x^4 + \cos^2 x^4)^4$.

1

(4)

8. Find the value of $\int_2^3 |x| dx$. 1

Or

- Find the value of $\int_0^{\pi/2} \cos 2x dx$. 1

9. Find the slope of the tangent to the curve

$$y = 2x^2 + 3 \sin x$$

- at $x = 0$. 1

10. What is the order and the degree of the following differential equation? 1

$$\left(\frac{d^2y}{dx^2}\right)^3 + 2\left(\frac{dy}{dx}\right)^5 + 9y = \sin x$$

11. If \vec{a} and \vec{b} are two vectors such that $|\vec{a}| = \sqrt{2}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{6}$, find the angle between \vec{a} and \vec{b} . 1

Or

- Find the dot product of the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{k}$. 1

12. If $P(1, 3, 4)$ and $Q(2, 5, 3)$ be two points in space, find the unit vector along \overrightarrow{PQ} . 1

(5)

13. A and B appear for an interview for two vacancies in a company. The probability of A 's selection is $\frac{1}{5}$ and that of B 's selection is $\frac{1}{6}$. What is the probability that both of them got selected? 1

14. If A and B are events such that

$$P(A) = \frac{5}{11}, P(B) = \frac{6}{11} \text{ and } P(A \cup B) = \frac{7}{11}$$

- find $P\left(\frac{B}{A}\right)$. 1

15. Find $\bar{a} \times \bar{b}$ where $\bar{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\bar{b} = \hat{i} + 2\hat{j} - \hat{k}$. 1

Or

- Find the vector equation of the straight line joining the points $(1, 2, 3)$ and $(2, 1, 4)$. 1

Choose the correct answer :

16. The value of $\int 2^x dx$ is

(a) $\frac{2^{x+1}}{x+1} + C$

(b) $2^x \log 2 + C$

(c) $\frac{2^x}{\log 2} + C$

- (d) None of the above 1

(6)

17. The derivative of a constant function is

(a) a non-zero constant

(b) zero

(c) the function itself

(d) None of the above

1

Or

The second-order derivative of $\log x$ with respect to x is

(a) $\frac{1}{x}$

(b) $\frac{1}{x^2}$

(c) $-\frac{1}{x^2}$

(d) 1

1

18. If $f(x) = 2$, then the value of $f(2)$ is

(a) 2

(b) x

(c) x^2

(d) 2^x

1

(7)

19. If $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 9\hat{j} - 3\hat{k}$, then \vec{a} and \vec{b} are

(a) perpendicular vectors

(b) parallel vectors

(c) equal vectors

(d) None of the above

1

20. The maximum value of $Z = 4x + 3y$, subject to the constraints $x + y \leq 4$, $x \geq 0$, $y \geq 0$ is

(a) 8

(b) 10

(c) 12

(d) 16

1

SECTION—B

21. Show that the relation R in the set of real numbers \mathbb{R} defined by $R = \{(a, b) : a \leq b\}$ is transitive but not symmetric.

2

22. Show that

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) = \sin^{-1}\left(\frac{x}{a}\right)$$

2

(8)

Or

Evaluate $\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right\}$. 2

23. If

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$$

find the matrix X such that $2A + X = B$. 2

Or

Find the cofactors of the elements of the second column of the determinant

$$\begin{vmatrix} 8 & 4 & 2 \\ 2 & 9 & 4 \\ 1 & 2 & 8 \end{vmatrix} \quad \text{2}$$

24. Is the function defined by

$$f(x) = \begin{cases} 2x + 3 & , \text{ if } x \leq 2 \\ 2x - 3 & , \text{ if } x > 2 \end{cases}$$

continuous at $x = 2$? Justify. 2

Or

Find $\frac{dy}{dx}$, when $x = \sin t$ and $y = \cos 2t$. 2

25. By using the properties of definite integral, show that

$$\int_0^1 x(1-x)^5 dx = \frac{1}{42} \quad 2$$

Or

Evaluate $\int \frac{(\tan^{-1} x)^2}{4+4x^2} dx.$ 2

26. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \dots}}}}$ to ∞ , then prove that

$$\frac{dy}{dx} = \frac{1}{2y-1} \quad 2$$

Or

Solve the following equation : 2

$$(x^2 + 1) \frac{dy}{dx} = xy$$

SECTION—C

27. (a) Find the value of k , if the function defined by

$$f(x) = \begin{cases} kx + 1 & , \text{ if } x \leq 5 \\ 3x - 5 & , \text{ if } x > 5 \end{cases}$$

is continuous at $x = 5$. 2

(b) Use definition to find the derivative of x^2 . 2

28. If $y = \sin^{-1} x$, then prove that

$$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 0 \quad 4$$

(10)

Or

Find the interval in which the function

$$f(x) = 2x^3 - 3x^2 - 36x + 7$$

is

(a) strictly increasing;

(b) strictly decreasing.

4

29. Prove that

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx = \frac{\pi}{4}$$

4

Or

Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is parallel to the line $2x - y + 9 = 0$.

4

30. Find the Cartesian and vector equation of the line which passes through the point $(-2, 4, -5)$ and parallel to the line given by

$$\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$$

4

Or

Find the vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 6 = 0$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) + 5 = 0$ and the point $(1, 1, 1)$.

4

31. Find two positive numbers whose product is 49 and the sum is minimum.

4

Or

If $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express \vec{b} in the form $\vec{b} = \vec{c} + \vec{d}$ where \vec{c} is parallel to \vec{a} and \vec{d} is perpendicular to \vec{a} .

4

(11)

32. If A and B are two events such that

$$2P(A) = P(B) = \frac{5}{13} \text{ and } P\left(\frac{A}{B}\right) = \frac{2}{5}$$

find $P(\text{not } A \text{ and not } B)$.

4

Or

Solve the following LPP graphically :

4

$$\text{Maximize } Z = 4x + y$$

subject to the constraints

$$x + y \leq 50$$

$$3x + y \leq 90$$

$$x \geq 0, y \geq 0$$

SECTION—D

33. If

$$A = \begin{bmatrix} 4 & 5 & 3 \\ 1 & 0 & 6 \\ 2 & 7 & 9 \end{bmatrix}$$

verify that $A \cdot (\text{adj } A) = (\text{adj } A) \cdot A = |A| I_3$.

6

Or

Solve the system equations by matrix method :

6

$$5x - y + z = 4$$

$$3x + 2y - 5z = 2$$

$$x + 3y - 2z = 5$$

(12)

34. Integrate the following : 3×2=6

(i) $\int \frac{(x+1)e^x}{\cos^2(xe^x)} dx$

(ii) $\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$

35. Find the shortest distance between the lines

$$\bar{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\bar{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

6

36. State Bayes' theorem on probability. Use this theorem to solve the following [(a) or (b)] : 2+4=6

(a) An insurance company insured 2000 scooty drivers, 4000 taxi drivers and 6000 bus drivers in a particular year. The probability of their accidents are 0.01, 0.03 and 0.15 respectively. One of the insured drivers meets with an accident. What is the probability that the person drives a scooty?

Or

(b) First bag contains 3 red and 4 black balls, and second bag contains 5 red and 6 black balls. A ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from the second bag.

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