

Total No. of Printed Pages—15

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MATHEMATICS

(FOR CANDIDATES WITH INTERNAL ASSESSMENT)

Full Marks : 80
Pass Marks : 24

(FOR CANDIDATES WITHOUT INTERNAL ASSESSMENT)

Full Marks : 100
Pass Marks : 30

Time : 3 hours

(FOR ALL CATEGORIES OF CANDIDATES)

General Instructions :

- (i) This Question Paper comprises of 32 questions divided into six Sections A, B, C, D, E and F.
- (ii) Marks allocated to every question are indicated against each.
- (iii) Question Nos. **1** to **30** (Section—A to Section—E) are to be answered by all candidates.
- (iv) Question Nos. **31** and **32** of Section—F are to be answered by **Candidates without Internal Assessment.**

(2)

- (v) In question on construction, the drawing should be neat and exactly as per the given measurements.
- (vi) Questions, which are meant for Visually Handicapped (Blind) Students, should be answered by them only.
- (vii) Use of Calculator/Mobile Phone is not permitted.

SECTION—A

(Marks : 8)

(Question Nos. 1 to 8 carry 1 mark each)

1. Write the exponent of 2 in the prime factorization of 288. 1
2. What is the 2nd term of the sequence $a_n = \frac{n}{n+1}$? 1
3. Check whether the equation $x + \frac{1}{x} = 2$, $x \neq 0$ is a quadratic equation or not. 1
4. Express $\cos 65^\circ + \tan 65^\circ$ in terms of angles between 0° and 30° . 1
5. State AA-similarity criterion. 1
6. Find the distance between two parallel tangents of a circle of radius 6 cm. 1

(3)

7. Write the formula of the total surface area of a right circular cylinder whose radius of the base is r units and its height is h units. 1
8. Define modal class. 1

SECTION—B

(Marks : 14)

(Question Nos. 9 to 15 carry 2 marks each)

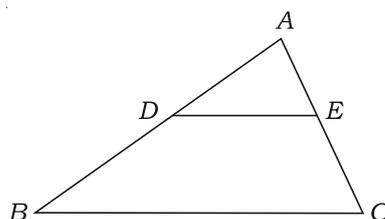
9. Find the value of k so that $8k+4$, $6k-2$ and $2k-7$ are the three consecutive terms of an A.P. 2
10. Prove that
 $\cos 48^\circ \operatorname{cosec} 42^\circ + \sec 48^\circ \sin 42^\circ = 2$ 2
11. Find the value of x ($0^\circ < x < 90^\circ$) in
 $\cos x = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$ 2
Or
If $x = a \cos^3 \theta$ and $y = b \sin^3 \theta$, then prove that
$$\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$$
 2
12. Find the distance between the pair of points $(a, 0)$ and $(0, b)$. 2
Or
Find the coordinates of the midpoint of the line segment joining the points $P(7, 0)$ and $Q(-5, 4)$. 2

(4)

13. Find the third vertex of a triangle ABC , if two of its vertices are $B(-3, 1)$ and $C(0, -2)$, and its centroid is at the origin.

2

14.



In the above figure, D and E are the points on the sides AB and AC respectively of $\triangle ABC$. Determine whether $DE \parallel BC$ or not if $AD = 5.7$ cm, $DB = 9.5$ cm, $AE = 4.8$ cm and $EC = 8$ cm.

2

[For Visually Handicapped (Blind) Students only, instead of Question No. 14 given above]

14. (a) Two geometric figures having the same shape but different sizes are said to be _____ figures.

(Fill in the blank)

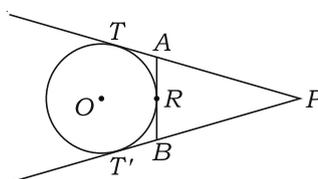
1

- (b) If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

(State whether True or False)

1

15.



In the above figure, PT and PT' are tangents from P to the circle with centre O . R is a point on the circle. Prove that $PA + AR = PB + BR$.

2

(5)

**[For Visually Handicapped (Blind) Students only,
instead of Question No. 15 given in Page No. 4]**

15. (a) Define a tangent. 1
- (b) How many tangents can be drawn through a point lying on the circle? 1

SECTION—C

(Marks : 24)

(Question Nos. 16 to 23 carry 3 marks each)

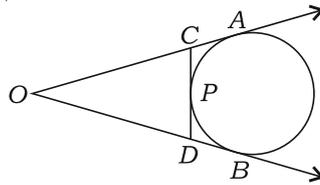
16. Using ruler and compass only, construct a triangle ABC such that each of its sides is $\frac{2}{3}$ rd of the corresponding side of $\triangle ABC$. It is given that $AB = 4$ cm, $BC = 5$ cm and $AC = 6$ cm. (Only traces of constructions are required.) 3

**[For Visually Handicapped (Blind) Students only,
instead of Question No. 16 given above]**

16. (a) Define an equilateral triangle. 1
- (b) When are two triangles said to be equiangular? 1
- (c) The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.
(State whether True or False) 1

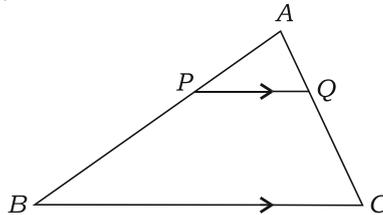
(6)

17. In the figure below, OA and OB are tangents to the circle drawn from an external point O . CD is a third tangent touching the circle at P . If $OB = 10$ cm and $CP = 2$ cm, then find the length of OC . 3



Or

- In the figure below, $PQ \parallel BC$ such that $AQ = \frac{1}{4}AC$. If $AB = 6$ cm, find AP . 3



[For Visually Handicapped (Blind) Students only,
instead of Question No. 17 given above]

17. (a) Define a circle. 1
- (b) Can a triangle have two right angles? 1
- (c) If two triangles are congruent, then their areas are ____.

(Fill in the blank) 1

(7)

18. A race track is in the form of a ring whose inner circumference is 352 m and the outer circumference is 396 m. Find the width of the track. (Use $\pi = \frac{22}{7}$) 3

Or

Find the area of the minor sector of a circle of radius 4 cm and of angle 30° . (Take $\pi = 3.14$) 3

19. Find the smallest number which when divided by 35, 56 and 91 leaves remainder of 7 in each case. 3

Or

Using Euclid's division algorithm, find the HCF of 105 and 120. 3

20. Find a quadratic polynomial whose zeroes are $3 + \sqrt{5}$ and $3 - \sqrt{5}$. 3

21. The 9th term of an A.P. is equal to 6 times its second term. If its 5th term is 22, find the A.P. 3

Or

Find the sum of the first 100 natural numbers. 3

22. Prove that

$$\frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta} = 2 \sec^2 \theta \quad 3$$

Or

If $\sin(A + B) = 1$ and $\cos(A - B) = 1$, $0^\circ < A + B \leq 90^\circ$, $A > B$, find A and B. 3

(8)

23. A bag contains 5 red, 8 white and 7 black balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is—

(a) red or white;

(b) neither black nor white.

3

SECTION—D

(Marks : 16)

(Question Nos. **24** to **27** carry 4 marks each)

24. The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction.

4

Or

Find two consecutive natural numbers whose product is 20.

4

25. An angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 9.5 m away from the wall. Find the length of the ladder.

4

Or

The angle of elevation of the top of a hill from the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30° . If the tower is 50 m high, then what is the height of the hill?

4

**[For Visually Handicapped (Blind) Students only,
instead of Question No. 25 given in Page No. 8]**

25. (a) Evaluate $\frac{\cos 17^\circ}{\sin 73^\circ}$. 1

(b) If $\cos \theta = 1$, then $\theta =$ _____.
(Fill in the blank) 1

(c) If $\sec \theta + \tan \theta = m$ and $\sec \theta - \tan \theta = n$, then prove
that $mn = 1$. 2

26. Find the ratio in which the point $P(m, 6)$ divides the line
segment joining the points $A(-1, 3)$ and $B(2, 8)$. Also,
find the value of m . 4

Or

Find the value of p for which the points $A(-1, 3)$, $B(2, p)$
and $C(5, -1)$ are collinear. 4

27. Prove that in a triangle, if the square of one side is equal
to the sum of the squares of the other two sides, then the
angle opposite to the first side is a right angle. 4

**[For Visually Handicapped (Blind) Students only,
instead of Question No. 27 given above]**

27. (a) What is the area of an equilateral triangle having
side a units? 2

(b) The greatest side of a right-angled triangle is called
hypotenuse.
(State whether True or False) 1

(c) All equilateral triangles are _____.
(Fill in the blank) 1

(10)

SECTION—E

(Marks : 18)

(Question Nos. 28 to 30 carry 6 marks each)

28. Solve the following system of linear equations graphically :

$$4x - y - 4 = 0$$

$$3x + 2y - 14 = 0$$

Find the vertices of the triangle formed by the lines and the y -axis. (Plot at least three points for each graph.) 6

[For Visually Handicapped (Blind) Students only, instead of Question No. 28 given above]

28. Solve the following system of linear equations : 6

$$2x + y = 7$$

$$4x - 3y + 1 = 0$$

29. The dimensions of a metallic cuboid are $100 \text{ cm} \times 80 \text{ cm} \times 64 \text{ cm}$. It is melted and recast into a cube. Find the total surface area of the cube. 6

Or

The volume of a hemisphere is $2425\frac{1}{2} \text{ cm}^3$. Find its curved surface area. (Use $\pi = \frac{22}{7}$) 6

30. Find the mean of the following data : 6

<i>Class Interval</i>	0-100	100-200	200-300	300-400	400-500
<i>Frequency</i>	6	9	15	12	8

Or

Find the mode of the following frequency distribution : 6

<i>Age (in years)</i>	0-5	5-10	10-15	15-20	20-25	25-30	30-35
<i>Number of Patients</i>	6	11	18	24	17	13	5

(11)

SECTION—F

(Marks : 20)

[For Candidates without Internal Assessment (WIM)]

31. Answer the following as directed (any *eight*) : $1 \times 8 = 8$

(a) Every composite number can be expressed as a product of

- (A) coprimes
- (B) primes
- (C) twin primes
- (D) None of the above

(Choose the correct option)

(b) The number of zeroes of a polynomial of degree n is

- (A) equal to n
- (B) greater than n
- (C) less than n
- (D) less than or equal to n

(Choose the correct option)

(c) A quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has real roots if

- (A) discriminant = 0
- (B) discriminant > 0
- (C) discriminant ≥ 0
- (D) discriminant ≤ 0

(Choose the correct option)

(12)

(d) The total surface area of a hemisphere of radius r units is

- (A) πr^2 sq. units
- (B) $2\pi r^2$ sq. units
- (C) $3\pi r^2$ sq. units
- (D) $4\pi r^2$ sq. units

(Choose the correct option)

(e) The degree of a constant polynomial is _____.

(Fill in the blank)

(f) Abscissa of any point on the y -axis is zero.

(State whether True or False)

(g) The different numbers in a sequence are called terms.

(State whether True or False)

(h) $\tan^2 \theta - \sec^2 \theta = 1$

(State whether True or False)

(i) Two angles are said to be complementary if their sum is _____.

(Fill in the blank)

(13)

(j) The number $\frac{343}{2 \times 7 \times 5^2}$ has terminating decimal representation.

(State whether True or False)

(k) Write the first term of 2^n .

(l) Write the maximum value of $\cos \theta$.

(m) The lengths of two tangents drawn from an external point to a circle are _____.

(Fill in the blank)

(n) Ogives are cumulative frequency curves.

(State whether True or False)

32. Answer any six from the following :

2×6=12

(a) Find the prime factorization of 2025.

(b) Without performing long division method, state whether $\frac{23}{8}$ has a terminating decimal expansion or a non-terminating repeating decimal expansion.

(14)

(c) Find the zeroes of the polynomial $x^2 - 2x - 3$.

(d) Solve the quadratic equation

$$\left(x - \frac{1}{2}\right)^2 = 4$$

(e) Write the first two terms of the sequence

$$t_n = \frac{3^n}{2^n + 1}$$

(f) Determine whether the following given sides of a triangle form a right triangle or not :

$$a = 3 \text{ cm, } b = 4 \text{ cm and } c = 7 \text{ cm}$$

(g) If $A = 30^\circ$, verify that

$$\cos 2A = 1 - 2\sin^2 A$$

(h) Eliminate 'θ' for the equations :

$$x = a \sec \theta \quad \text{and} \quad y = b \tan \theta$$

(i) If $P(E) = 0.05$, what is the probability of 'not E'?

(15)

- (j) State whether the following pair of linear equations has a unique solution, no solution or infinitely many solutions :

$$3x - 5y = 7$$

$$6x - 10y = 3$$

- (k) Find the coordinates of the centroid of the triangle whose vertices are $(-2, 3)$, $(2, -1)$ and $(4, 0)$.

- (l) Write the discriminant of the quadratic equation

$$x^2 + x - 2 = 0$$

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