

2022	III	14	1030	J-762	(E)
<b>MATHEMATICS &amp; STATISTICS (40)</b> <b>(ARTS &amp; SCIENCE)</b>					
Time : 3 Hrs.		(7 Pages)		Max. Marks : 80	

**General instructions :**

The question paper is divided into **FOUR** sections.

- (1) **Section A:** Q. 1 contains **Eight** multiple choice type of questions, each carrying **Two** marks.  
Q. 2 contains **Four** very short answer type questions, each carrying **one** mark.
- (2) **Section B:** Q. 3 to Q. 14 contain **Twelve** short answer type questions, each carrying **Two** marks. (Attempt any **Eight**)
- (3) **Section C:** Q. 15 to Q. 26 contain **Twelve** short answer type questions, each carrying **Three** marks. (Attempt any **Eight**)
- (4) **Section D:** Q. 27 to Q. 34 contain **Eight** long answer type questions, each carrying **Four** marks. (Attempt any **Five**)
- (5) Use of log table is allowed. Use of calculator is not allowed.
- (6) Figures to the right indicate full marks.
- (7) Use of graph paper is not necessary. Only rough sketch of graph is expected.
- (8) For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabet, e.g., (a) ..... / (b) ..... / (c) ..... / (d)....., etc. No marks shall be given, if **ONLY** the correct answer or the alphabet of correct answer is written. Only the first attempt will be considered for evaluation.
- (9) Start answer to each section on a new page.

## SECTION - A

Q. 1. Select and write the correct answer for the following multiple choice type of questions :

[16]

(i) The negation of  $p \wedge (q \rightarrow r)$  is \_\_\_\_\_.

- (a)  $\sim p \wedge (\sim q \rightarrow \sim r)$                       (b)  $p \vee (\sim q \vee r)$   
 (c)  $\sim p \wedge (\sim q \rightarrow r)$                       (d)  $p \rightarrow (q \wedge \sim r)$                       (2)

(ii) In  $\Delta ABC$  if  $c^2 + a^2 - b^2 = ac$ , then  $\angle B =$  \_\_\_\_\_.

- (a)  $\frac{\pi}{4}$     (b)  $\frac{\pi}{3}$   
 (c)  $\frac{\pi}{2}$     (d)  $\frac{\pi}{6}$     (2)

(iii) Equation of line passing through the points  $(0, 0, 0)$  and  $(2, 1, -3)$  is \_\_\_\_\_.

- (a)  $\frac{x}{2} = \frac{y}{1} = \frac{z}{-3}$                                       (b)  $\frac{x}{2} = \frac{y}{-1} = \frac{z}{-3}$   
 (c)  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$                                       (d)  $\frac{x}{3} = \frac{y}{1} = \frac{z}{2}$                                       (2)

(iv) The value of  $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{i} \times \hat{j})$  is \_\_\_\_\_.

- (a) 0    (b) -1  
 (c) 1    (d) 3    (2)

(v) If  $f(x) = x^5 + 2x - 3$ , then  $(f^{-1})'(-3) =$  \_\_\_\_\_.

- (a) 0    (b) -3  
 (c)  $-\frac{1}{3}$     (d)  $\frac{1}{2}$     (2)

0 7 6 2

(vi) The maximum value of the function  $f(x) = \frac{\log x}{x}$  is \_\_\_\_\_.

- (a)  $e$  (b)  $\frac{1}{e}$   
(c)  $e^2$  (d)  $\frac{1}{e^2}$  (2)

(vii) If  $\int \frac{dx}{4x^2 - 1} = A \log \left( \frac{2x-1}{2x+1} \right) + c$ , then  $A =$  \_\_\_\_\_.

- (a) 1 (b)  $\frac{1}{2}$   
(c)  $\frac{1}{3}$  (d)  $\frac{1}{4}$  (2)

(viii) If the p.m.f. of a r.v.  $X$  is

$$P(x) = \frac{c}{x^3}, \text{ for } x = 1, 2, 3$$
$$= 0, \text{ otherwise,}$$

then  $E(X) =$  \_\_\_\_\_.

- (a)  $\frac{216}{251}$  (b)  $\frac{294}{251}$   
(c)  $\frac{297}{294}$  (d)  $\frac{294}{297}$  (2)

**Q. 2. Answer the following questions :** [4]

(i) Find the principal value of  $\cot^{-1} \left( \frac{-1}{\sqrt{3}} \right)$ . (1)

(ii) Write the separate equations of lines represented by the equation  $5x^2 - 9y^2 = 0$  (1)

(iii) If  $f'(x) = x^{-1}$ , then find  $f(x)$  (1)

(iv) Write the degree of the differential equation  $(y''')^2 + 3(y'') + 3xy' + 5y = 0$  (1)

## SECTION – B

Attempt any EIGHT of the following questions :

[16]

Q. 3. Using truth table verify that :

$$(p \wedge q) \vee \sim q \equiv p \vee \sim q \quad (2)$$

Q. 4. Find the cofactors of the elements of the matrix  $\begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$  (2)

Q. 5. Find the principal solutions of  $\cot \theta = 0$  (2)

Q. 6. Find the value of  $k$ , if  $2x + y = 0$  is one of the lines represented by  $3x^2 + kxy + 2y^2 = 0$  (2)

Q. 7. Find the cartesian equation of the plane passing through  $A(1, 2, 3)$  and the direction ratios of whose normal are 3, 2, 5. (2)

Q. 8. Find the cartesian co-ordinates of the point whose polar co-ordinates are  $\left(\frac{1}{2}, \frac{\pi}{3}\right)$ . (2)

Q. 9. Find the equation of tangent to the curve  $y = 2x^3 - x^2 + 2$  at  $\left(\frac{1}{2}, 2\right)$ . <https://www.maharashtrastudy.com> (2)

Q. 10. Evaluate:  $\int_0^{\frac{\pi}{4}} \sec^4 x \, dx$  (2)

Q. 11. Solve the differential equation  $y \frac{dy}{dx} + x = 0$  (2)

Q. 12. Show that function  $f(x) = \tan x$  is increasing in  $\left(0, \frac{\pi}{2}\right)$ . (2)

Q. 13. Form the differential equation of all lines which makes intercept 3 on x-axis. (2)

Q. 14. If  $X \sim B(n, p)$  and  $E(X) = 6$  and  $\text{Var}(X) = 4.2$ , then find  $n$  and  $p$ . (2)

### SECTION – C

Attempt any EIGHT of the following questions : [24]

Q. 15. If  $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$ , then find the value of  $x$ . (3)

Q. 16. If angle between the lines represented by  $ax^2 + 2hxy + by^2 = 0$  is equal to the angle between the lines represented by  $2x^2 - 5xy + 3y^2 = 0$ , then show that  $100(h^2 - ab) = (a + b)^2$ . (3)

Q. 17. Find the distance between the parallel lines  $\frac{x}{2} = \frac{y}{-1} = \frac{z}{2}$  and

$$\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z-1}{2} \quad (3)$$

Q. 18. If A(5, 1, p), B(1, q, p) and C(1, -2, 3) are vertices of a triangle and  $G\left(r, \frac{-4}{3}, \frac{1}{3}\right)$  is its centroid, then find the values of  $p, q, r$  by vector method. (3)

Q. 19. If  $A(\bar{a})$  and  $B(\bar{b})$  be any two points in the space and  $R(\bar{r})$  be a point on the line segment AB dividing it internally in the ratio  $m : n$  then prove that  $\bar{r} = \frac{m\bar{b} + n\bar{a}}{m + n}$ . (3)

Q. 20. Find the vector equation of the plane passing through the point  $A(-1, 2, -5)$  and parallel to the vectors  $4\hat{i} - \hat{j} + 3\hat{k}$  and  $\hat{i} + \hat{j} - \hat{k}$ . (3)

Q. 21. If  $y = e^{m \tan^{-1} x}$ , then show that  $(1 + x^2) \frac{d^2 y}{dx^2} + (2x - m) \frac{dy}{dx} = 0$  (3)

Q. 22. Evaluate:  $\int \frac{dx}{2 + \cos x - \sin x}$  (3)

Q. 23. Solve  $x + y \frac{dy}{dx} = \sec(x^2 + y^2)$  (3)

Q. 24. A wire of length 36 meters is bent to form a rectangle. Find its dimensions if the area of the rectangle is maximum. (3)

Q. 25. Two dice are thrown simultaneously. If  $X$  denotes the number of sixes, find the expectation of  $X$ . (3)

Q. 26. If a fair coin is tossed 10 times. Find the probability of getting at most six heads. (3)

### SECTION - D

Attempt any FIVE of the following questions :

[20]

Q. 27. Without using truth table prove that

$$(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv p \vee q \quad (4)$$

Q. 28. Solve the following system of equations by the method of inversion

$$x - y + z = 4, 2x + y - 3z = 0, x + y + z = 2 \quad (4)$$

Q. 29. Using vectors prove that the altitudes of a triangle are concurrent. (4)

Q. 30. Solve the L. P. P. by graphical method,

Minimize  $z = 8x + 10y$

Subject to  $2x + y \geq 7,$

$2x + 3y \geq 15,$

$y \geq 2, x \geq 0$

(4)

0 7 6 2

**Q. 31.** If  $x = f(t)$  and  $y = g(t)$  are differentiable functions of  $t$  so that  $y$  is

differentiable function of  $x$  and  $\frac{dx}{dt} \neq 0$ , then prove that :

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Hence find  $\frac{dy}{dx}$  if  $x = \sin t$  and  $y = \cos t$ . (4)

**Q. 32.** If  $u$  and  $v$  are differentiable functions of  $x$ , then prove that :

$$\int uv \, dx = u \int v \, dx - \int \left[ \frac{du}{dx} \int v \, dx \right] dx$$
 (4)

Hence evaluate  $\int \log x \, dx$

**Q. 33.** Find the area of region between parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$ . (4)

**Q. 34.** Show that :  $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) \, dx = \frac{\pi}{8} \log 2$  (4)



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