



(English Version)

- Instructions :**
1. The question paper has five Parts namely A, B, C, D and E. Answer **all** the **Parts**.
 2. Use the Graph Sheet for the question on Linear Programming problem in Part-E.

PART – A

Answer **all** the **ten** questions :

(10 × 1 = 10)

- 1) Let * be the binary operation on N given by $a*b = \text{L.C.M of } a \text{ and } b$. Find $5*7$. ✓
- 2) Write the range of the function $y = \sec^{-1} x$. ✓
- 3) If a matrix has 5 elements, what are the possible orders it can have? ✓
- 4) Find the values of x for which

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

- 5) If $y = \tan(\sqrt{x})$, find $\frac{dy}{dx}$. ✓
- 6) Find $\int (2x^2 + e^x) dx$. ✓
- 7) Define Negative of a vector. ✓
- 8) If a line makes angles 90° , 135° and 45° with the X, Y and Z – axes respectively, find its direction cosines.

35 (NS)



9) Define optimal solution in Linear programming problem.

10) If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$ find $P(A \cap B)$ if A and B are independent events.

PART - B

Answer **any ten** questions :

(10 × 2 = 20)

11) If $f: R \rightarrow R$ and $g: R \rightarrow R$ are given by $f(x) = \cos x$ and $g(x) = 3x^2$. Find $g \circ f$ and $f \circ g$.

12) Prove that $\cot^{-1}(-x) = \pi - \cot^{-1} x, \forall x \in R$.

13) Find the value of $\sin^{-1}\left(\sin \frac{3\pi}{5}\right)$.

14) Find the area of the triangle whose vertices are $(-2, -3)$, $(3, 2)$ and $(-1, -8)$ using determinant method.

15) Find $\frac{dy}{dx}$, if $\sin^2 x + \cos^2 y = 1$.

16) If $y = x^x$, find $\frac{dy}{dx}$.



17) Find the interval in which the function f given by $f(x) = x^2 - 4x + 6$ is strictly decreasing. ✓ 2

18) Find $\int \cot x \log(\sin x) dx$.

19) Find $\int x \sec^2 x dx$. ✓ 2

20) Find the order and degree (if defined) of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0.$$
 ✓ 2

21) Find the projection of the vector $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$. ✓ 2

22) Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$. ✓ 2

23) Find the equation of the plane with intercepts 2, 3 and 4 on the X, Y and Z-axes respectively.

24) A random variable X has the following probability distribution.

X	0	1	2	3	4
P(X)	0.1	K	2K	2K	K

 ✓ 2

Find the value of K .



PART - C

Answer any ten questions :

(10 × 3 = 30)

25) Show that the relation R defined in the set A of all triangles as
$$R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2\}$$
 is an equivalence relation. ✓ 3
26) Prove that $2 \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$. ✓ 327) If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then show that $F(x)F(y) = F(x+y)$. ✓ 328) If $x = 2at^2$, $y = at^4$ then find $\frac{dy}{dx}$. ✓ 329) Verify Mean Value Theorem for the function $f(x) = x^2 - 4x - 3$, $x \in [1, 4]$. ✓ 330) Use differential to approximate $\sqrt{36.6}$. ✓ 331) Find $\int \frac{(x-3)e^x}{(x-1)^3} dx$.32) Evaluate: $\int_0^{\pi/2} \cos^2 x dx$.



33) Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y -axes in the first quadrant. ✓ 3

34) Find the equation of a curve passing through the point $(-2, 3)$, given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$.

35) Find a unit vector perpendicular to each of the vector $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$. ✓ 3

36) Find x such that the four points $A(3, 2, 1)$, $B(4, x, 5)$, $C(4, 2, -2)$ and $D(6, 5, -1)$ are coplanar. ✓ 2

37) Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point $(2, 2, 1)$. ✓ 3

38) A man is known to speak truth 3 out of 4 times. He throws a dice and reports that it is a six. Find the probability that it is actually a six. ✓ 3

PART - D

(6 × 5 = 30)

Answer any six questions :

39) Show that the function $f: R \rightarrow R$ given by $f(x) = 4x + 3$ is invertible. Find the inverse of f . ✓ 5

40) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$ then compute $(A + B)$ and $(B - C)$. Also verify that $A + (B - C) = (A + B) - C$. ✓ 5

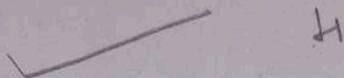


- 41) Solve the system of linear equations by matrix method

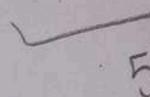
$$2x + 3y + 3z = 5$$

$$x - 2y + z = -4$$

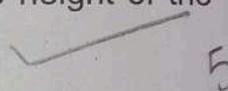
$$3x - y - 2z = 3$$



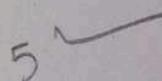
- 42) If $y = (\tan^{-1} x)^2$, show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$.



- 43) Sand is pouring from a pipe at the rate of $12 \text{ cm}^3/\text{s}$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?



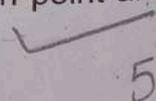
- 44) Find the integral of $\frac{1}{x^2 + a^2}$ w.r.t. x and hence evaluate $\int \frac{1}{x^2 + 2x + 2} dx$.



- 45) Using the method of integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

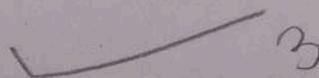
- 46) Find the general solution of the differential equation $x \frac{dy}{dx} + 2y = x^2 \log x$.

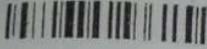
- 47) Derive the equation of a line in space passing through a given point and parallel to a given vector in both Vector and Cartesian form.



- 48) A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize

- a) exactly once
b) atleast once?





PART - E

Answer any one of the following question :

(1 × 10 = 10)

49) a) Prove that $\int_{-a}^a f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(x) \text{ is an even function} \\ 0, & \text{if } f(x) \text{ is an odd function} \end{cases}$

and hence evaluate $\int_{-1}^1 \sin^5 x \cos^4 x dx$. (6)

b) Show that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$. (4)

50) a) Maximise $z = 4x + y$

Subject to constraints :

$x + y \leq 50$

$3x + y \leq 90$

$x \geq 0$

$y \geq 0$

by graphical method. (6)

b) Find the value of K ,

if $f(x) = \begin{cases} Kx + 1, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$ is continuous at $x = \pi$. (4)