(English Version)

Instructions : 1. The question paper has five Parts namely A, B, C, D and E. Answer **all** the **Parts**.

2. Use the Graph Sheet for the question on Linear Programming problem in Part-E.

PART – A

Answer **all** the **ten** questions : **(10 × 1 = 10)**

1) Let $*$ be the binary operation on N given by $a*b=$ L.C.M of a and b. Find $5*7$.

2) Write the range of the function $y=\sec^{-1}x$.

3) If a matrix has 5 elements, what are the possible orders it can have?

4) Find the values of x for which

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

5) If $y=\tan(\sqrt{x})$, find $\frac{dy}{dx}$.

6) Find $\int(2x^2+e^x)dx$.

7) Define Negative of a vector.

8) If a line makes angles 90°, 135° and 45° with the X, Y and $Z-$axes respectively, find its direction cosines.

9) Define optimal solution in Linear programming problem. 1

10) If $P(A)=\frac{3}{5}$ and $P(B)=\frac{1}{5}$ find $P(A \cap B)$ if A and B are independent events. 1

PART – B

Answer **any ten** questions : **(10 × 2 = 20)**

11) If $f: R \to R$ and $g: R \to R$ are given by $f(x)=\cos x$ and $g(x)=3x^2$. Find $g \circ f$ and $f \circ g$. 2

12) Prove that $\cot^{-1}(-x)=\pi-\cot^{-1}x, \quad \forall x \in R$. 2

13) Find the value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$. 2

14) Find the area of the triangle whose vertices are $(-2,-3)$, $(3, 2)$ and $(-1,-8)$ using determinant method. 2

15) Find $\frac{dy}{dx}$, if $\sin^2 x+\cos^2 y=1$.

16) If $y=x^x$, find $\frac{dy}{dx}$. 2

17) Find the interval in which the function f given by $f(x) = x^2 - 4x + 6$ is strictly decreasing. 2

18) Find $\int \cot x \log(\sin x) dx$.

19) Find $\int x \sec^2 x \, dx$. 2

20) Find the order and degree (if defined) of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$. 2

21) Find the projection of the vector $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$. 2

22) Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$. 2

23) Find the equation of the plane with intercepts 2, 3 and 4 on the X, Y and Z–axes respectively.

24) A random variable X has the following probability distribution. 2

X	0	1	2	3	4
P(X)	0.1	K	2 K	2 K	K

Find the value of K.

PART – C

Answer **any ten** questions : $(10 \times 3 = 30)$

25) Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2\}$ is an equivalence relation. 3

26) Prove that $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$. 3

27) If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then show that $F(x)\,F(y) = F(x+y)$. 3

28) If $x = 2at^2$, $y = at^4$ then find $\frac{dy}{dx}$. 3

29) Verify Mean Value Theorem for the function $f(x) = x^2 - 4x - 3$, $x \in [1, 4]$. 3

30) Use differential to approximate $\sqrt{36.6}$. 3

31) Find $\int \frac{(x-3)e^x}{(x-1)^3}\,dx$.

32) Evaluate : $\int_0^{\pi/2} \cos^2 x \; dx$.

33) Find the area of the region bounded by $x^2=4y$, $y=2$, $y=4$ and the y-axes in the first quadrant.

3

34) Find the equation of a curve passing through the point $(-2,3)$, given that the slope of the tangent to the curve at any point (x,y) is $\frac{2x}{y^2}$.

35) Find a unit vector perpendicular to each of the vector $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$.

3

36) Find x such that the four points $A(3,2,1)$, $B(4,x,5)$, $C(4,2,-2)$ and $D(6,5,-1)$ are coplanar.

2

37) Find the equation of the plane through the intersection of the planes $3x-y+2z-4=0$ and $x+y+z-2=0$ and the point $(2,2,1)$.

3

38) A man is known to speak truth 3 out of 4 times. He throws a dice and reports that it is a six. Find the probability that it is actually a six.

PART – D

Answer **any six** questions : $(6\times 5=30)$

39) Show that the function $f:R\to R$ given by $f(x)=4x+3$ is invertible. Find the inverse of f.

5

40) If $A=\begin{bmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{bmatrix}$, $B=\begin{bmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{bmatrix}$ and $C=\begin{bmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{bmatrix}$ then compute $(A+B)$ and $(B-C)$. Also verify that $A+(B-C)=(A+B)-C$.

5

41) Solve the system of linear equations by matrix method

$$2x+3y+3z=5$$
$$x-2y+z=-4$$
$$3x-y-2z=3$$

4

42) If $y=(\tan^{-1}x)^2$, show that $(x^2+1)^2y_2+2x(x^2+1)y_1=2$.

5

43) Sand is pouring from a pipe at the rate of $12\ \text{cm}^3/\text{s}$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

5

44) Find the integral of $\frac{1}{x^2+a^2}$ w.r.t. x and hence evaluate $\int \frac{1}{x^2+2x+2}dx$.

5

45) Using the method of integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$.

46) Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2\log x$.

47) Derive the equation of a line in space passing through a given point and parallel to a given vector in both Vector and Cartesian form.

5

48) A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize

a) exactly once

b) atleast once?

3

PART – E

Answer **any one** of the following question : (1 × 10 = 10)

49) a) Prove that $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_{0}^{a} f(x)\,dx, & \text{if } f(x) \text{ is an even function} \\ 0, & \text{if } f(x) \text{ is an odd function} \end{cases}$

and hence evaluate $\int_{-1}^{1} \sin^5 x \cos^4 x\,dx$. (6)

b) Show that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$. (4)

50) a) Maximise $z = 4x + y$

Subject to constraints :

$x + y \leq 50$

$3x + y \leq 90$

$x \geq 0$

$y \geq 0$

by graphical method. (6)

5

b) Find the value of K,

if $f(x) = \begin{cases} Kx + 1, & if \quad x \leq \pi \\ \cos x, & if \quad x > \pi \end{cases}$ is continuous at $x = \pi$. 4 (4)