

**Instructions :** 1. The question paper has five parts namely A, B, C, D and E.  
Answer all the Parts.

2. Part A has 15 multiple choice questions, 5 fill in the blank questions.

3. Use the graph sheet for question on linear programming problem in Part E.

**PART -A**

**I. Answer all the multiple choice questions : 15 x 1 = 15**

1. The relation R in the set { 1,2,3 } given by { (1,2) ,(2,1) } is
 

a) reflexive	b) symmetric
c) transitive	d) equivalence relation
2. If  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined as  $f(x) = x^4$ , then the function f is
 

a) one-one and onto	b) many-one and onto
c) one-one but not onto	d) neither one-one nor onto
3. The principal value branch of  $\cot^{-1} x$  is
 

a) $[-\frac{\pi}{2}, \frac{\pi}{2}]$	b) $(-\frac{\pi}{2}, \frac{\pi}{2})$
c) $[0, \pi]$	d) $(0, \pi)$
4. The number of all possible matrices of order 3 x 3 with each entry 0 or 1 is
 

a) 27	b) 18	c) 81	d) 512
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5. Let A be a nonsingular matrix of order 3 x 3 and  $|\text{adj } A| = 25$ , then a possible value of  $|A|$  is
 

a) 625	b) 25	c) 5	d) 125
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6. Which of the following x belongs to domain of the greatest integer function  $f(x) = [x]$ ,  $0 < x < 3$  is not differentiable
 

a) 2 and 3	b) 1 and 2	c) 0 and 2	d) 1 and 3
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7. If  $y = \log_7 2x$ , then  $\frac{dy}{dx}$  is
 

1) $\frac{1}{x \log 7}$	b) $\frac{1}{7 \log x}$
c) $\frac{\log x}{7}$	d) $\frac{7}{\log x}$
8. The point of inflection of the function  $y = x^3$  is
 

a) ( 2, 8 )	b) ( 1, 1 )	c) ( 0, 0 )	d) ( -3, -27 )
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9.  $\int \sin 2x \, dx$  is
 

a) $-\frac{\sin 2x}{2} + c$	b) $-\frac{\cos 2x}{2} + c$
c) $\frac{\cos 2x}{2} + c$	d) $\frac{\sin 2x}{2} + c$
10.  $\int e^x \left( \frac{1}{x} - \frac{1}{x^2} \right) dx$  is
 

a) $e^{-x} \left( \frac{1}{x} \right) + c$	b) $e^{-x} \left( \frac{1}{x^2} \right) + c$	c) $e^x \left( \frac{1}{x} \right) + c$	d) $e^x \left( \frac{1}{x^2} \right) + c$
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11. If  $\theta$  is the angle between any two vectors  $\vec{a}$  and  $\vec{b}$ , then  $\vec{a} \cdot \vec{b} = |\vec{a} \times \vec{b}|$ , when  $\tan\theta$  is equal to,  
 a) 1                                      b)  $\frac{1}{\sqrt{3}}$                                       c)  $\sqrt{3}$                                       d) 0
12. Unit vector in the direction of  $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$  is  
 a)  $\frac{2\hat{i} + 3\hat{j} + \hat{k}}{14}$                                       b)  $\frac{2\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{14}}$   
 c)  $\frac{2\hat{i} + 3\hat{j} + \hat{k}}{\sqrt{14}}$                                       d)  $\frac{2\hat{i} + 3\hat{j} - \hat{k}}{14}$
13. If the direction cosines  $l, m, n$  of a line are  $0, \frac{1}{2}, \frac{\sqrt{3}}{2}$  then the angle made by the line with the positive direction of  $y$  - axis is  
 a)  $60^\circ$                                       b)  $30^\circ$                                       c)  $90^\circ$                                       d)  $45^\circ$
14. In a Linear programming problem, the objective function is always  
 a) a cubic function                                      b) a quadratic function  
 c) a linear function                                      d) a constant function
15. If  $A$  and  $B$  are two non empty events such that  $P(A/B) = P(B/A)$  and  $P(A \cap B) \neq \emptyset$  then  
 a)  $A \subset B$  but  $A \neq B$                                       b)  $A = B$   
 c)  $B \subset A$  but  $A \neq B$                                       d)  $P(A) = P(B)$

**II. Fill in the blanks by choosing the appropriate answer from those given in the bracket** **5 x 1 = 5**

$$\left(0, 1, 4, \frac{1}{36}, 7, \frac{1}{6}\right)$$

16. The value of  $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$  is \_\_\_\_\_
17. A square matrix  $A$  is a singular matrix if  $|A|$  is \_\_\_\_\_
18. The order of the differential equation  $\frac{d^4y}{dx^4} + \sin(y''') = 0$  is \_\_\_\_\_
19. The lines  $\frac{x-5}{k} = \frac{y+2}{-5} = \frac{z}{1}$  and  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$  are perpendicular, then  $k$  is \_\_\_\_\_
20. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is \_\_\_\_\_

**PART -B**

**Answer any six questions**

**6 x 2 = 12**

21. Prove that  $2 \sin^{-1} \frac{3}{5} = \tan^{-1} \frac{24}{7}$
22. Find the equation of line joining  $(1, 2), (3, 6)$  using determinant method
23. Find  $\frac{dy}{dx}$ , if  $y + \sin y = \cos x$
24. Find the rate of change of the area of a circle with respect to its radius  $r$  when  $r = 3$  cm
25. Find the local minimum value of the function  $f$  given by  $f(x) = 3 + |x|$ ,  $x \in \mathbb{R}$
26. Find  $\int \frac{dx}{(x+1)(x+2)}$
27. Evaluate  $\int_0^{\frac{\pi}{2}} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}\right) dx$
28. Find the projection of the vector  $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$  on the vector  $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

29. Find the angle between the pair of lines given by  
 $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + (\hat{i} + 2\hat{j} + 2\hat{k})$  and  $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$
30. A fair die is rolled. Consider events  $E = \{1, 3, 5\}$ ,  $F = \{2, 3\}$ , find  $P(E/F)$
31. If A and B two events such that  $P(A) = \frac{1}{4}$ ,  $P(B) = \frac{1}{2}$  and  $P(A \cap B) = \frac{1}{8}$ ,  
 find  $P(\text{not A and not B})$

### PART - C

**Answer any six questions**

**6 x 3 = 18**

32. Show that the relation R in the set  $A = \{1, 2, 3, 4, 5\}$  given by  
 $R = \{(a, b) : |a - b| \text{ is even}\}$  is an equivalence relation
33. Write in the simplest form  $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ ,  $x \neq 0$
34. Express  $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$  as the sum of a symmetric and a skew symmetric matrix.
35. Differentiate  $\sin^2 x$  with respect to  $e^{\cos x}$
36. Differentiate  $x^{\sin x}$ ,  $x > 0$  with respect to  $x$
37. Find the interval in which the function  $f(x) = 10 - 6x - 2x^2$  is strictly increasing
38. Find  $\int x \sin^{-1} x \, dx$
39. Find the equation of curve passing through the point  $(-2, 3)$ , given that the slope of the tangent to the curve at any point  $(x, y)$  is  $\frac{2x}{y^2}$
40. Show that the position vector of the point P, which divides the line joining the points A and B having position vectors  $\vec{a}$  and  $\vec{b}$  internally in the ratio  $m : n$  is  $\frac{m\vec{b} + n\vec{a}}{m + n}$
41. Find a unit vector perpendicular to each of the vectors  $(\vec{a} + \vec{b})$  and  $(\vec{a} - \vec{b})$ , where  $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$
42. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn at random from the bag and it is found to be red. Find the probability that the ball is drawn from first bag?

### PART - D

**Answer any four questions**

**4 x 5 = 20**

43. Let  $f : N \rightarrow Y$  be a function defined as  $f(x) = 4x + 3$ , where  $Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$ . Show that  $f$  is invertible. Find the inverse of  $f$ .
44. If  $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$  and  $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$  then calculate  $AC$ ,  $BC$  and  $(A + B)C$ . Also verify  $(A + B)C = AC + BC$
45. Solve the system of linear equations by matrix method  
 $2x - 3y + 5z = 11$ ,  $3x + 2y - 4z = -5$ ,  $x + y - 2z = -3$
46. If  $y = 3 \cos(\log x) + 4 \sin(\log x)$ , show that  $x^2 y_2 + xy_1 + y = 0$
47. Find the integral of  $\frac{1}{x^2 - a^2}$  with respect to  $x$  and hence evaluate  $\int \frac{dx}{x^2 - 16}$
48. Find the area of the region bounded by the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$  using integration.

49. Find the general solution of the differential equation  
 $x \frac{dy}{dx} + 2y = x^2 \log x, (x \neq 0)$
50. Derive the equation of a line in space through a given point and parallel to a vector both in the vector and Cartesian form

**PART – E**

**Answer the following questions**

51. P.T.  $\int_{-a}^a f(x)dx = \begin{cases} 2 \int_0^a f(x)dx, & \text{if } f(x) \text{ is an even function} \\ 0 & \text{if } f(x) \text{ is an odd function} \end{cases}$

and hence evaluate  $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$

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**OR**

Solve the following linear programming problem graphically

Minimise  $Z = 200x + 500y$ ,

subject to the constraints :  $x + 2y \geq 10, 3x + 4y \leq 24, x \geq 0, y \geq 0$

52. Show that the matrix  $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$  satisfies the equation  $A^2 - 4A + I = O$ ,

where I is  $2 \times 2$  identity matrix and O is  $2 \times 2$  zero matrix.

Using this equation, find  $A^{-1}$ .

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**OR**

Find the value of k so that the function  $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x} & \text{if } x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases}$

is continuous at  $x = \frac{\pi}{2}$

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