GOVERNMENT OF KARNATAKA

KARNATAKA STATE PRE-UNIVERSITY EDUCATION EXAMINATION BOARD

II YEAR PUC EXAMINATION MARCH – APRIL 2012

## SCHEME OF VALUATION

Subject : **MATHEMATICS** Subject Code : **35**

| Qn.No. | | Marks Allotted |
|---|---|---|
| | **PART – A** | |
| 1 | Write the prime power factorization of the composite number 45. | |
| **Ans :** | $45 = 3^2 \times 5^1$ | **1** |
| | | |
| 2 | Define a diagonal matrix. | |
| **Ans :** | Definition : A square matrix in which all the elements except the principal diagonal elements are zero is called a diagonal matrix. [Note : The diagonal elements may or may not be zero] | **1** |
| | | |
| 3 | Why the usual division is not a binary operation on the set of real numbers ? | |
| **Ans :** | Since $\forall$ $a, b, \in R$ , $\frac{a}{b} \notin R$ when $b = 0$ | **1** |
| | **OR** | |
| | Any counter example like $3, 0 \in R$ but $\frac{3}{0}$ is not defined. | |
| | | |
| 4 | Simplify $(\hat{i} \times \hat{j}) \times \hat{k}$ | |
| **Ans :** | $(\hat{i} \times \hat{j}) \times \hat{k} = \hat{k} \times \hat{k} = \vec{0}$ | **1** |
| | (Vector notation for zero vector is compulsory) | |
| | | |
| | | |

| Qn.No. | | Marks Allotted |
|---|---|---|
| 5 | Find the centre of the circle $3x^2+3y^2-6x+3y-4=0$ | |
| **Ans :** | Writing the equation as, $x^2+y^2-2x+y-\frac{4}{3}=0$<br>$\therefore$ Centre is $\left(1, -\frac{1}{2}\right)$ | **1** |
| | | |
| 6 | For the parabola $2y^2=5x$ what is the length of the latus rectum? | |
| **Ans :** | Writing the equation as $y^2=\frac{5}{2}x$<br>$\therefore$ Length of LR $= 4a = \frac{5}{2}$ | **1** |
| | | |
| 7 | Evaluate $\sin\left[\frac{1}{2}\cos^{-1}\left(\frac{4}{5}\right)\right]$ | |
| **Ans :** | Let $\cos^{-1}\left(\frac{4}{5}\right)=\theta$ $\therefore$ $\cos\theta=\frac{4}{5}$<br>$\therefore$ $\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{4}{5}}{2}} = \frac{1}{\sqrt{10}}$ | **1** |
| | | |
| 8 | Express it in the polar form $z = 2+i(2)$ | |
| **Ans :** | $r=2\sqrt{2}$ , $\theta=\frac{\pi}{4}$ | |
| | $\therefore$ Polar form is $z = 2\sqrt{2}\ \text{cis}\left(\frac{\pi}{4}\right)$ | **1** |
| | | |
| | | |

| Qn.No. | | Marks Allotted |
|---|---|---|
| 9 | If $y = \sin^{-1} x + \cos^{-1} x$ find $\frac{dy}{dx}$ | |
| **Ans:** | $y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$ | **1** |
| | **OR** | |
| | $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \left(\frac{-1}{\sqrt{1-x^2}}\right) = 0$ | |
| | | |
| 10 | Evaluate : $\int_{-\pi/2}^{\pi/2} (x + \sin x)\, dx$ | |
| **Ans:** | Since the integrand is an odd function by a property<br>$\int_{-\pi/2}^{\pi/2} (x + \sin x)\, dx = 0$ | **1** |
| | **OR** | |
| | $\int_{-\pi/2}^{\pi/2} (x + \sin x)\, dx = \left[\frac{x^2}{2} - \cos x\right]_{-\pi/2}^{\pi/2} = 0$ | |
| | **PART -B** | |
| 11 | If (a, b) = 1, (a, c) =1 then prove that (a, bc) = 1 | |
| **Ans:** | $(a, b) = 1 \Rightarrow 1 = ax_1 + by_1$<br>and $(a, c) = 1 \Rightarrow 1 = ax_2 + cy_2$<br>Where $x_1, y_1, x_2, y_2 \in I$<br>(If one of the above equations is correct give one mark) | **1** |
| | $\therefore$ $1 = (ax_1 + by_1)(ax_2 + cy_2)$<br>$1 = a(ax_1x_2 + cx_1y_2 + bx_1y_1) + bc(y_1 y_2)$<br>$1 = ax^1 + bcy^1$ where $x^1, y^1 \in I$<br>$\Rightarrow$ $(a, bc) = 1$ | **1** |

| Qn.No. | | Marks Allotted |
|---|---|---|
| 12 | Solve by Cramer's rule : $3x - y = 4$<br>$5x + 4y = 1$ | |
| **Ans :** | $\Delta = 17$ , $\Delta_1 = 17$ $\therefore$ $x = 1$ | **1** |
| | $\Delta_2 = -17$ $\therefore$ $y = -1$ | **1** |
| | | |
| 13 | Prove that the identity element of a group $(G, *)$ is unique. | |
| **Ans :** | If e and $e^1$ are two different identity elements | |
| | Then $e * e^1 = e^1 * e = e^1$ and<br>$e^1 * e = e * e^1 = e$ | **1** |
| | $\Rightarrow$ $e = e^1$ $\therefore$ Identity element is unique | **1** |
| | | |
| 14 | Find the area of parallelogram whose diagonals are<br>$3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 2\hat{j} + \hat{k}$ | |
| **Ans :** | Let $\vec{d}_1 = (3, 1, -2)$ and $\vec{d}_2 = (1, -2, 1)$ | |
| | $\therefore$ $\vec{d}_1 \times \vec{d}_2 = -3\hat{i} - 5\hat{j} - 7\hat{k}$ | **1** |
| | $\therefore$ Area of the parallelogram $= \frac{1}{2}\left\|\vec{d}_1 \times \vec{d}_2\right\| = \frac{\sqrt{83}}{2}$ sq. units | **1** |
| | | |
| 15 | $3x + 4y = 2$ and $x - y = 3$ are equations of two diameters of a circle whose radius is 5 units. Find the equation of the circle. | |
| **Ans :** | Finding the centre = (2, - 1) | **1** |
| | Writing the equation $(x-2)^2 + (y+1)^2 = 25$ | **1** |
| | | |
| | | |

| Qn.No. | | Marks Allotted |
|---|---|---|
| 16 | Find the centre of the ellipse $25x^2+16y^2-100x+32y-284=0$ | |
| **Ans :** | Reducing to the standard form $\frac{(x-2)^2}{16}+\frac{(y+1)^2}{25}=1$ | **1** |
| | $\therefore$ centre $=(2,-1)$ | **1** |
| | | |
| 17 | If $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}$ then show that $x+y+xy=1$ | |
| **Ans :** | $\tan^{-1}\left(\frac{x+y}{1-xy}\right)=\frac{\pi}{4}$ | **1** |
| | $\therefore \frac{x+y}{1-xy}=\tan\frac{\pi}{4}=1 \Rightarrow x+y+xy=1$ | **1** |
| | | |
| 18 | If $x+\frac{1}{x}=2\cos\theta$ then show that one of the values of $x$ is $e^{i\theta}$ | |
| **Ans :** | Getting $x^2-2x\cos\theta+1=0$ | **1** |
| | $\therefore x=\frac{2\cos\theta\pm2\sqrt{\cos^2\theta-1}}{2}=\cos\theta\pm i\sin\theta,\ e^{i\theta}$ | **1** |
| | | |
| 19 | If $y=\sqrt{\sin 2x+\sqrt{\sin 2x+\sqrt{\sin 2x+\ldots}}}$ upto $\infty$ then show that $\frac{dy}{dx}=\frac{2\cos 2x}{2y-1}$ | |
| **Ans:** | Getting $y=\sqrt{\sin 2x+y}$ | **1** |
| | Squaring and differentiating and getting $\frac{dy}{dx}=\frac{2\cos 2x}{(2y-1)}$ | **1** |
| | | |
| | | |

| Qn.No. | | Marks Allotted |
|---|---|---|
| 20 | Find the y-intercept of the tangent drawn to the curve $y = 3x^2 - 4x$ at (1,-1) on it | |
| **Ans:** | Getting $m = \left(\frac{dy}{dx}\right)_{(1,-1)} = 2$ | **1** |
| | Equation of the tangent is $y = 2x - 3$<br>$\therefore$ the y-intercept of the tangent is – 3 | **1** |
| | | |
| 21 | Evaluate : $\int \frac{\cos x\, dx}{\sqrt{8+\cos^2 x}}$ | |
| **Ans :** | $\int \frac{\cos x}{\sqrt{8+\cos^2 x}} dx = \int \frac{\cos x\, dx}{\sqrt{9-\sin^2 x}}$ | **1** |
| | Put $\sin x = t \Rightarrow \cos x\, dx = dt$<br>$\therefore$ G.I $= \int \frac{dt}{\sqrt{9-t^2}} = \sin^{-1}\left(\frac{\sin x}{3}\right) + C$ | **1** |
| 22 | Find the order and degree of the differential equation<br>$a^2 \frac{d^2y}{dx^2} = \left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{4}}$ | |
| **Ans:** | Order of D.E = 2 | **1** |
| | Degree of D.E = 4 | **1** |
| **I** | **PART - C** | |
| 23 | Find the G.C.D of 111 and 409 and express it in two ways in the form 111 m + 409 n where m, n $\in$ z | |
| **Ans:** | Division | **1** |
| | Writing (111, 409) = 1 | **1** |
| | 1 = 409 (19) + 111 (–70) **OR** 111(–70) + 409(19) | |

| Qn.No. | | Marks Allotted |
|---|---|---|
| | $\therefore$ m = − 70 , n = 19 | 1 |
| | 1 = 111 (−70) + 409 (19) + 111 x 409 − 111 x 409 | 1 |
| | getting m = 339 , n = −92 | 1 |
| | **OR** | |
| | m = − 479 , n = 130 | 1 |
| | | |
| 24. | Solve by matrix method $x+2y+3z=10$ , $2x-3y+z=1$ and $3x+y-2z=9$ | |
| **Ans:** | Writing $X = A^{-1}B$ and $A^{-1} = \frac{AdjA}{\lvert A\rvert}$ | 1 |
| | $\lvert A\rvert = 52$ | 1 |
| | Cofactor matrix = $\begin{bmatrix} 5 & 7 & 11 \\ 7 & -11 & 5 \\ 11 & 5 & -7 \end{bmatrix}$ | 1 |
| | (any four correct co-factors give one mark) | |
| | Adjoint of A = $\begin{bmatrix} 5 & 7 & 11 \\ 7 & -11 & 5 \\ 11 & 5 & -7 \end{bmatrix}$ | 1 |
| | Getting $x = 3$ , y = 2 and z = 1 | 1 |
| | | |
| 25 | Show that the set $G = \left\{ 2^n / n \in z \right\}$ forms an abelian group w.r.t usual multiplication. | |
| **Ans:** | Note : Particular examples do not carry any mark | |
| | Closure law | 1 |
| | Associative law | 1 |

| Qn.No. | | Marks Allotted |
|---|---|---|
| | $2^0 = 1$ , is the identity element | 1 |
| | $\forall\ 2^n \in G, \quad (2^n)^{-1} = 2^{-n} \in G$ | 1 |
| | Commutative law | 1 |
| | | |
| 26 (a) | Prove that $\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right] = 2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ | |
| **Ans:** | Writing L.H.S $= (\vec{a}+\vec{b}) . \left[(\vec{b}+\vec{c}) \times (\vec{c}+\vec{a})\right]$ | 1 |
| | $= (\vec{a}+\vec{b}) . \left[\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{c}+\vec{c}\times\vec{a}\right]$ | 1 |
| | $= \vec{a}.(\vec{b}\times\vec{c}) + \vec{a}.(\vec{b}\times\vec{a}) + \vec{a}.(\vec{c}\times\vec{a})$ $+ \vec{b}.(\vec{b}\times\vec{c}) + \vec{b}.(\vec{b}\times\vec{a}) + \vec{b}.(\vec{c}\times\vec{a})$ | |
| | $= \vec{a}.(\vec{b}\times\vec{c}) + \vec{b}.(\vec{c}\times\vec{a})$ | |
| | $= 2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ | 1 |
| | [ Deduct one mark if $\bullet$ and $\times$ are not mentioned ] | |
| 26 (b) | $\theta$ being angle between $\vec{a} = \hat{i}+\hat{j}-3\hat{k}$ and $\vec{b} = 2\hat{i}+\hat{j}-\hat{k}$ find the value of $\cos\theta$ | |
| **Ans :** | $\cos\theta = \dfrac{\vec{a}.\vec{b}}{\lvert\vec{a}\rvert\ \lvert\vec{b}\rvert}$<br>$= \dfrac{2+1+3}{\sqrt{11}\sqrt{6}} = \sqrt{\dfrac{6}{11}}$<br>(Without the formula if the substitution is correct give one mark) | 1<br>1 |
| **II** | | |
| 27(a) | Find the equation of the tangent to the circle $x^2+y^2+2gx+2fy+c=0$ at a point $P(x_1, y_1)$ on it. | |
| **Ans :** | Correct figure [ No figure or wrong figure carries zero mark] | 1 |
| | Slope of the tangent at $(x_1, y_1) = -\left(\dfrac{x_1+g}{y_1+f}\right)$ | 1 |

| Qn.No. | | Marks Allotted |
|---|---|---|
| | Getting the equation, $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$ | **1** |
| | | |
| 27 (b) | What is the value of k if the circles $x^2 + y^2 + kx + 4y + 2 = 0$ and $2x^2 + 2y^2 - 4x - 3y + k = 0$ cut each other orthogonally ? | |
| **Ans:** | Writing the condition $2g_1 g_2 + 2f_1 f_2 = c_1 + c_2$ | **1** |
| | Getting $k = \frac{-10}{3}$ | **1** |
| | | |
| 28 (a) | Find the equation of the ellipse in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if the distance between foci is 8 and the distance between the directricies is 32. | |
| **Ans :** | Writing $2ae = 8$ and $\frac{2a}{e} = 32$ **OR** $2be = 8$ & $\frac{2b}{e} = 32$ | **1** |
| | Getting $a = 8$ , $e = \frac{1}{2}$ and $b^2 = 48$ \| $b = 8$ , $e = \frac{1}{2}$ and $a^2 = 48$ | **1** |
| | Writing the equation $\frac{x^2}{64} + \frac{y^2}{48} = 1$ \| $\frac{x^2}{48} + \frac{y^2}{64} = 1$ | **1** |
| | | |
| 28 (b) | Find a point on the parabola $y^2 = 2x$ whose focal distance is $\frac{5}{2}$ | |
| | Focal distance $= \|x + a\| = \frac{5}{2}$ | **1** |
| | Getting $a = \frac{1}{2}$ and $x = 2$ , $y = \pm 2$ | |
| | ∴ The point is (2, 2) **OR** (2, −2 ) | **1** |
| | (any one point is sufficient) | |
| | | |

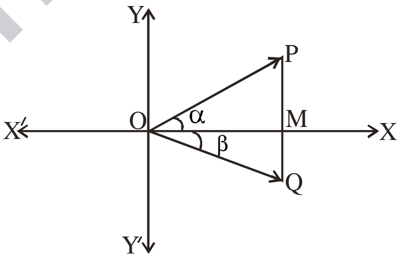
| Qn.No. | | Marks Allotted |
|---|---|---|
| 29 (a) | Solve $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{2}$ | |
| **Ans :** | Let $\sin^{-1} x = A \Rightarrow \sin A = x \quad \therefore \quad \cos A = \sqrt{1-x^2}$ <br> $\sin^{-1} 2x = B \Rightarrow \sin B = 2x \quad \therefore \quad \cos B = \sqrt{1-4x^2}$ <br> and $A + B = \frac{\pi}{2}$ | **1** |
| | Taking cos and getting $\sqrt{1-x^2}\,\sqrt{1-4x^2} - x(2x) = 0$ | **1** |
| | Simplifying and getting $x = \frac{1}{\sqrt{5}}$ | **1** |
| | | |
| 29 (b) | Find the general solution of $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$ | |
| **Ans :** | Getting $\frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta} = \sqrt{3}$ | **1** |
| | Writing $\tan 3\theta = \sqrt{3}$ and $\theta = \frac{n\pi}{3} + \frac{\pi}{9},\ n \in I$ | **1** |
| **III** | | |
| 30 (a) | Differentiate $\tan(ax)$ w.r.t $x$ from first principles. | |
| **Ans :** | Writing $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\tan(ax + a\delta x) - \tan(ax)}{\delta x}$ | **1** |
| | $= \lim_{\delta x \to 0} \frac{\sin(ax + a\delta x)\cos x - \cos(ax + a\delta x)\sin x}{\cos(ax + a\delta x)\cos x}$ | |
| | $= \lim_{\delta x \to 0} \frac{\sin a\delta x}{(\cos ax)\,\delta x\,.\,\cos(ax + a\delta x)}$ | **1** |
| | $\therefore \frac{dy}{dx} = a\sec^2 ax$ | **1** |
| | **OR** | |
| | | |

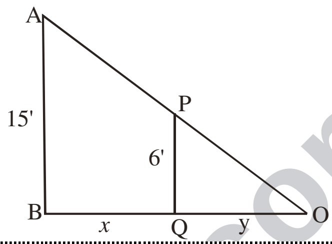
| Qn.No. | | Marks Allotted |
|---|---|---|
| | $\frac{dy}{dx} = \lim\limits_{\delta x \to 0} \frac{a\tan(a\delta x)\left[1+\tan(ax+a\delta x)\tan ax\right]}{a\,\delta x}$ | 1 |
| | $= a\sec^2 ax$ | 1 |
| 30 (b) | Differentiate w.r.t $x$ : $(\cos x)^x$ | |
| **Ans :** | Let $y = (\cos x)^x$ | |
| | $\log y = x\log(\cos x)$ | 1 |
| | Differentiating and getting $\frac{dy}{dx} = (\cos x)^x\left[-x\tan x + \log\cos x\right]$ | 1 |
| | | |
| 31 (a) | If $y = e^{m\tan^{-1}x}$ then show that $(1+x^2)^2 y_2 + 2x(1+x^2)y_1 - m^2 y = 0$ | |
| **Ans :** | $y_1 = me^{m\tan^{-1}x} \cdot \frac{1}{1+x^2}$ | 1 |
| | Again differentiating $(1+x^2)y_2 + y_1 2x = m\,y_1$ | |
| | Substituting for $y_1$ in the R.H.S and getting $(1+x^2)^2 y_2 + 2x(1+x^2)y_1 - m^2 y = 0$ | 1 |
| | | |
| 31 (b) | Differentiate $\cos(3x+2)$ w.r.t $3\cos x$ | |
| **Ans :** | Let $y = \cos(3x+2)$ and $z = 3\cos x$ | |
| | Getting $\frac{dy}{dx} = -\sin(3x+2).\,3$ and $\frac{dz}{dx} = -3\sin x$ | 1 |
| | $\therefore \frac{dy}{dz} = \frac{\sin(3x+2)}{\sin x}$ | 1 |
| | | |
| | | |

| Qn.No. | | Marks Allotted |
|---|---|---|
| 32 (a) | If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$ then show that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$ | |
| **Ans :** | $x\sqrt{1+y} = -y\sqrt{1+x}$ | |
| | $x^2(1+y) = y^2(1+x)$ | |
| | Getting $(x-y)\left[x+y+xy\right] = 0$ | **1** |
| | Writing $y = \frac{-x}{1+x}$ | **1** |
| | $\therefore \frac{dy}{dx} = \frac{-1}{(1+x)^2}$ | **1** |
| | [for correct direct differentiation give one mark] | |
| | | |
| 32 (b) | Evaluate $\int \frac{(x-2)}{\sqrt{x+5}}\,dx$ | |
| **Ans :** | Writing $I = \int \frac{x+5}{\sqrt{x+5}}\,dx - \int \frac{7}{\sqrt{x+5}}\,dx$ | **1** |
| | $= \frac{2}{3}(x+5)^{\frac{3}{2}} - 14\sqrt{x+5} + C$ | **1** |
| | | |
| 33 (a) | Evaluate $\int \left[e^x(1+x)\log_e(xe^x)\right] dx$ | |
| **Ans :** | Put $xe^x = t$ | **1** |
| | $\therefore I = \int \log t\,dt = t\log t - \int t.\frac{1}{t}\,dt$ | **1** |
| | $= t\log t - t + C$ | |
| | | |

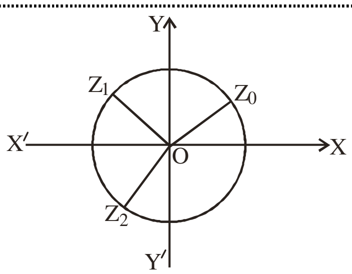
| Qn.No. | | Marks Allotted |
|---|---|---|
| 33 (b) | Evaluate $\int (\tan x + \cot x)^2\, dx$ | |
| **Ans :** | $I = \int (\tan^2 x + \cot^2 x + 2)\, dx$ | **1** |
| | $= \int (\sec^2 x + \operatorname{cosec}^2 x)\, dx \quad = \tan x - \cot x + C$ | **1** |
| | **OR** | |
| | $I = \int \left( \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)^2 dx \quad = \int \frac{4}{\sin^2 2x}\, dx$ | |
| | $= 4\int \operatorname{cosec}^2 2x\, dx$ | **1** |
| | $= -2\cot 2x + C$ | **1** |
| | | |
| 34 | Find the area of the circle $x^2 + y^2 = a^2$ by the method of integration | |
| **Ans :** | Correct figure [ no figure, deduct one mark ] | **1** |
| | Area of the circle $= 4\int_0^a y\, dx = 4\int_0^a \sqrt{a^2 - x^2}\, dx$ | **1** |
| | $= 4\left[ \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} \right]_0^a$ | **1** |
| | $= 4\left[ \frac{a}{2}(0) + \frac{a^2}{2}\sin^{-1}(1) \right] - 4[0 + 0]$ | **1** |
| | $= \pi a^2$ sq. units | **1** |
| | | |
| | **PART – D** | |
| 35 (a) | Define hyperbola as locus of a point and derive its equation in the standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ | |

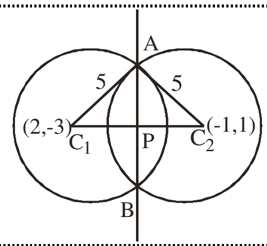
| Qn.No. | | Marks Allotted |
|---|---|---|
| **Ans :** | Definition as locus of a point only | **1** |
| | correct figure | **1** |
| | getting $cs = ae$ | **1** |
| | and $cz = \frac{a}{e}$ | **1** |
| | Writing $ps = e\,pm \quad (e > 1)$ | **1** |
| | Substituting for ps and pm and simplifying to get $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ where $b^2 = a^2(e^2 - 1)$ (No figure or wrong figure carries zero mark) | **1** |
| | | |
| 35 (b) | Prove that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$ | |
| | By $c_1^1 = c_1 + c_2 - c_3$ | |
| | L.H.S $= \begin{vmatrix} 2C & c+a & a+b \\ 2r & r+p & p+q \\ 2z & z+x & x+y \end{vmatrix}$ | **1** |
| | $= 2\begin{vmatrix} c & c+a & a+b \\ r & r+p & p+q \\ z & z+x & x+y \end{vmatrix}$ | **1** |
| | by $c_2^1 = c_2 - c_1$, $= 2\begin{vmatrix} c & a & a+b \\ r & p & p+q \\ z & x & x+y \end{vmatrix}$ <br> by $c_3^1 = c_3 - c_2$, $= 2\begin{vmatrix} c & a & b \\ r & p & q \\ z & x & y \end{vmatrix}$ | **1** |
| | | |

| Qn.No. | | Marks Allotted |
|---|---|---|
| | Interchanging $c_1, c_2$ and $c_2, c_3$ | |
| | $= 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$ | 1 |
| | | |
| 36 (a) | State and prove the De Moivre's theorem for all rational indices | |
| **Ans :** | Statement : If n is an integer | |
| | $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ | |
| | and if n is a fraction then one of the values of $(\cos\theta + i\sin\theta)^n$ is $\cos n\theta + i\sin n\theta$ | 1 |
| | Proving for positive integers | 2 |
| | Proving for negative integers | 2 |
| | Proving for fractions | 1 |
| | | |
| 36 (b) | Prove by vector method : $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ | |
| **Ans :** | Let $XOX'$ and $YOY'$ be the co-ordinate axes Let $\overrightarrow{OP}$, $\overrightarrow{OQ}$ be two unit vectors with $X\hat{O}P = \alpha$ and $X\hat{O}Q = \beta$   | |
| | Correct figure with explanation | 1 |
| | $\overrightarrow{OP} = \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$ and $\overrightarrow{OQ} = \cos\beta\,\hat{i} - \sin\beta\,\hat{j}$ | 1 |
| | $\overrightarrow{OP}\cdot\overrightarrow{OQ} = \lvert\overrightarrow{OP}\rvert\lvert\overrightarrow{OQ}\rvert\cos(\alpha+\beta)$ | 1 |
| | Writing $\overrightarrow{OP}\cdot\overrightarrow{OQ} = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ | |

| Qn.No. | | Marks Allotted |
|---|---|---|
| | and concluding $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ | 1 |
| | | |
| 37 (a) | A man 6 ft tall walks at a rate of 2ft / sec away from the source of light which is hung 15 ft above the horizontal ground.<br>i) How fast is the length of his shadow increasing ?<br>ii) How fast is the tip of his shadow moving ? | |
| **Ans :** | Figure<br>[ no figure carries zero mark]   | 1 |
| | From similar triangles , $\frac{AB}{PQ} = \frac{BO}{QO}$ | 1 |
| | $\Rightarrow \frac{15}{6} = \frac{x+y}{y} \Rightarrow 2x = 3y$ | 1 |
| | $\therefore 2\frac{dx}{dt} = 3.\frac{dy}{dt} \Rightarrow \frac{dy}{dt} = \frac{4}{3}$ feet / sec | 1 |
| | Let $z = x + y$ | 1 |
| | $\therefore \frac{dz}{dt} = \frac{dx}{dt} + \frac{dy}{dt} = \frac{10}{3}$ feet / sec | 1 |
| | | |
| 37 (b) | Find the general solution of $\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$ | |
| **Ans :** | Writing $2\cos 3\theta \cos 2\theta + 2\cos 5\theta \cos 2\theta = 0$ | 1 |
| | $2\cos 2\theta \ [\cos 3\theta + \cos 5\theta] = 0$ | |
| | $\Rightarrow 2\cos 2\theta \ . \ [2\cos 4\theta \ .\cos\theta] = 0$ | |
| | $\Rightarrow \theta = n\pi \pm \frac{\pi}{4}$ | 1 |

| Qn.No. | | Marks Allotted |
|---|---|---|
| | $\theta = \frac{n\pi}{2} \pm \frac{\pi}{8}$ **OR** $\theta = (2n+1)\frac{\pi}{2}$, $\theta = (2n+1)\frac{\pi}{4}$ | 1 |
| | $\theta = 2n\pi \pm \frac{\pi}{2}$ where n is an integer \| $\theta = (2n+1)\frac{\pi}{8}$ | 1 |
| | | |
| 38 (a) | Prove that $\int_0^1 \frac{\log_e(1+x)}{1+x^2}dx = \frac{\pi}{8}\log 2$ | |
| **Ans :** | Put $x = \tan\theta$ and writing $I = \int_0^{\pi/4} \log(1+\tan\theta)\, d\theta$ | 1 |
| | Also $I = \int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right] d\theta$ | 1 |
| | $= \int_0^{\pi/4} \log\left(\frac{2}{1+\tan\theta}\right) d\theta$ | 1 |
| | $= \int_0^{\pi/4} \log 2\, d\theta - \int_0^{\pi/4} \log(1+\tan\theta)\, d\theta$ | 1 |
| | $\therefore\ 2I = \int_0^{\pi/4} \log 2 . d\theta = \log 2 [\theta]_0^{\pi/4}$ | 1 |
| | $\therefore\ I = \frac{\pi}{8}\log 2$ | 1 |
| | | |
| 38 (b) | Find the general solution of $\frac{dy}{dx} = \sqrt{1-x^2-y^2+x^2y^2}$ | |
| **Ans :** | $\frac{dy}{dx} = \sqrt{(1-x^2)(1-y^2)}$ | 1 |
| | $\frac{dy}{\sqrt{1-y^2}} = \sqrt{1-x^2}\, dx$ | 1 |

| Qn.No. | | Marks Allotted |
|---|---|---|
| | $\int \frac{dy}{\sqrt{1-y^2}} = \int \sqrt{1-x^2}\, dx + C$ | 1 |
| | $\sin^{-1}y = \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x + C$ | 1 |
| | (without C deduct one mark) | |
| | **PART – E** | |
| 39 (a) | Find the cube roots of $1+i\sqrt{3}$ and represent them in Argand diagram. | |
| **Ans:** | $r = 2 \qquad \theta = \frac{\pi}{3}$ | 1 |
| | $\therefore \quad 1+i\sqrt{3} = 2\,\text{cis}\left[\frac{\pi}{3}\right]$ | |
| | $= 2\,\text{cis}\left[2n\pi + \frac{\pi}{3}\right]$ | 1 |
| | $= 2\,\text{cis}\left[\frac{6n\pi+\pi}{3}\right]$ | |
| | Writing $z_0 = 2^{\frac{1}{3}}\,\text{cis}\left(\frac{\pi}{9}\right)$, $z_1 = 2^{\frac{1}{3}}\,\text{cis}\left(\frac{7\pi}{9}\right)$ and $z_2 = 2^{\frac{1}{3}}\,\text{cis}\left(\frac{13\pi}{9}\right)$ | 1 |
| | Argand diagram :   | 1 |
| | | |
| | | |

| Qn.No. | | Marks Allotted |
|---|---|---|
| 39 (b) | Find the length of the common chord of the intersecting circles $x^2+y^2-4x+6y-12=0$ and $x^2+y^2+2x-2y-23=0$ | |
| **Ans :** | Figure   | **1** |
| | Getting radical axis $6x-8y-11=0$ | **1** |
| | $C_1P = \frac{5}{2}$ | **1** |
| | $AP = \sqrt{25-\frac{25}{4}} = \frac{5\sqrt{3}}{2}$ | |
| | $\therefore$ length of the chord $AB = 5\sqrt{3}$ unit | **1** |
| | | |
| 39 (c) | Find the number of incongruent solutions and the incongruent solutions of linear congruence $5x \equiv 3 \pmod{6}$ | |
| **Ans :** | $(5, 6) = 1$ $\therefore$ no. of incongruent solution = 1 | **1** |
| | $6 \mid 5x-3$ $\therefore$ $x = 3$ | **1** |
| | | |
| 40 (a) | If $\vec{a}.\vec{b}$ are two unit vectors such that $(\vec{a}+\vec{b})$ is also a unit vector then show that angle between $\vec{a}$ and $\vec{b}$ is $\frac{2\pi}{3}$ and also show that $\|\vec{a}-\vec{b}\| = \sqrt{3}$ | |
| **Ans :** | Given $\|\vec{a}+\vec{b}\| = 1 \Rightarrow \|\vec{a}+\vec{b}\|^2 = 1$ | |
| | $\Rightarrow (\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = 1$ | **1** |
| | Getting $\vec{a}.\vec{b} = -\frac{1}{2}$ and $\theta = \frac{2\pi}{3}$ | **1** |

| Qn.No. | | Marks Allotted |
|---|---|---|
| | Writing $(\vec{a}-\vec{b}).(\vec{a}-\vec{b}) = \|\vec{a}\|^2+\|\vec{b}\|^2-2\vec{a}.\vec{b} = 3$ | 1 |
| | Getting $\|\vec{a}-\vec{b}\|^2 = 3$ and $\|\vec{a}-\vec{b}\| = \sqrt{3}$ | 1 |
| | | |
| 40 (b) | Evaluate $\int \tan^4(3x)\,dx$ | |
| **Ans :** | $I = \int \tan^2 3x \,.\, \tan^2 3x\,dx$ | 1 |
| | $= \int (\sec^2 3x - 1)\tan^2 3x\,dx$ | 1 |
| | $= \int \sec^2 3x \tan^2 3x\,dx - \int (\sec^2 3x - 1)\,dx$ | |
| | $= \frac{\tan^3 3x}{9} - \frac{\tan 3x}{3} + x + C$ | 1+1 |
| | [ If two integrals are correct give one mark] | |
| | | |
| 40 (c) | Differentiate w.r.t $x$ : $\log_5(\log_e^x)$ | |
| **Ans :** | Writing $y = \frac{\log(\log_e^x)}{\log 5}$ | 1 |
| | $\therefore \frac{dy}{dx} = \frac{1}{\log 5}.\frac{1}{\log x}.\frac{1}{x}$ | 1 |

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