(27) sets

(I)

GOVERNMENT OF KARNATAKA
DEPARTMENT OF PRE-UNIVERSITY EDUCATION

II YEAR P.U.C. EXAMINATION – 2012- March

SCHEME OF VALUATION

Subject Code : 75 Subject : BASIC MATHEMATICS

Qn.No.		Marks
	Part-A	
1.	Negate "He likes to run and she does not like to sit".	
Ans:	He does not like to run or he likes to sit.	-1-
2.	In how many ways can 5 women draw water from 5 taps if no tap is left unused?	
Ans	$5! = 120$ ways.	-1-
3.	If $\|A\| = 7$, find x where $A = \begin{bmatrix} 3 & 1 \\ x & 4 \end{bmatrix}$	
Ans:	$x = 5$	-1-
4	Find x, if $5 : 20 :: 3 : x$	
Ans:	$x = \frac{20 \times 3}{5} = 12$	-1-
5.	The average age of 10 students is 20 years and the average age of another group of 15 students is 10 years. Find the combined average.	

Qn.No.		Marks
Ans:	$\bar{X} = \frac{10(20)+15(10)}{25} = 14$ years.	-1-
6.	Find the present value of Rs. 750 due 4 months hence at 15% p.a.	
Ans:	$P = \frac{F}{1+it} = \frac{750}{1+\left(\frac{4}{12}\right)(0.15)} = 714.29/-$	-1-
7.	Find the equation of the circle whose end points of the diameter are (1,1) and (2,3).	
Ans:	$(x-1)(x-2)+(y-1)(y-3)=0$ OR $x^2+y^2-3x-4y+5=0$	-1-
8.	Evaluate $\lim_{x\to 4} \frac{x^4-256}{x-4}$	
Ans:	$\lim_{x\to 4} \frac{x^4-4^4}{x-4} = (4)(4)^3$ OR $= 256$	-1-
9.	If $y=\log(x^2-2)$, find dy/dx	
Ans:	$\frac{dy}{dx} = \frac{1}{x^2-2} \cdot 2x$	-1-

Qn.No.		Marks
10.	Evaluate $\int \frac{x}{x^2+1} dx$	1
Ans:	$\int \frac{x}{x^2+1} dx = \frac{1}{2} \log(x^2+1) + c$	-1-
	Part-B	
11.	Write the Inverse and the Contra Positive of "If 2 is an Even number then $\sqrt{2}$ is an integer".	
Ans:	Inverse :- "If 2 is not an Even no then $\sqrt{2}$ is not an integer".	-1-
	Contrapositive: "If $\sqrt{2}$ is not an integer then 2 is not an even no"	-1-
12.	Find the no of words that can be formed using all the letters of the word 'FRIEND'? How many of these begin with F and end with D?	
Ans:	$6! = 720$ ways	-1-
	$4! = 24$ ways	-1-

Qn.No.		Marks
13.	If $P(A) = \frac{1}{2}$, $P(\bar{B}) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$, find $P(A \cap B)$.	
Ans:	$P(B) = 1 - P(\bar{B}) = 1 - \frac{5}{8} = \frac{3}{8}$	-1-
	$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ $= \frac{1}{8}$	-1-
14.	Solve by Cramer's Rule: $x + y = 7$, $2x + y = 8$	
Ans:	$\Delta = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1$	
	$\Delta_1 = \begin{vmatrix} 7 & 1 \\ 8 & 1 \end{vmatrix} = -1$ $\Delta_2 = \begin{vmatrix} 1 & 7 \\ 2 & 8 \end{vmatrix} = -6$	
	(Any of the 2 determinants are correct, award one mark)	
	$\Rightarrow x = \frac{\Delta_1}{\Delta} = 1$, $y = \frac{\Delta_2}{\Delta} = 6$	-2-
15.	If $A = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 0 & 2 \end{pmatrix}$ & $B = \begin{pmatrix} 2 & 0 & -3 \\ 3 & -1 & 4 \end{pmatrix}$ find AB'.	
Ans:	$B' = \begin{pmatrix} 2 & 3 \\ 0 & -1 \\ -3 & 4 \end{pmatrix}$	-1-

Qn.No.		Marks
	$AB' = \begin{pmatrix} 5 & -3 \\ -8 & 5 \end{pmatrix}$	-1-
16.	The monthly incomes of A and B are in the ratio 3:4. Their monthly savings are in the ratio 1:2. If each saves Rs. 2000, find their monthly incomes.	
Ans:	Grace of 2 marks to be awarded for any relevant data.	+2-
17.	Find the focus and the equation of the directrix of the parabola $x^2 = 4(y-2)$	
Ans:	Focus $S = (0,3)$	-1-
	Equation of directrix $y = 1$	-1-
18.	Find 'K' if the function;	
	$f(x) = \begin{cases} \frac{e^{2x}-1}{x}, & x \neq 0 \\ K, & x = 0 \end{cases}$ is continuous at $x = 0$.	
Ans:	$\lim_{x \to 0} \frac{e^{2x}-1}{x} = 2$	-1-
	$\lim_{x \to 0} f(x) = f(0) \Rightarrow K = 2$	-1-

Qn.No.		Marks
19.	If $x = \log(1+t)$, $y = \frac{1}{1+t}$, find $\frac{dy}{dx}$.	
Ans:	$\frac{dx}{dt} = \frac{1}{1+t}$, $\frac{dy}{dt} = \frac{-1}{(1+t)^2}$	-1-
	(Any One Correct value award one mark)	
	$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+t)}$	-1-
20.	When Brakes are applied to a moving car, the car travels a distance of S ft in t secs, given by $S = 20t - 4t^2$. When does the car stop?	
Ans:	$V = \frac{dS}{dt} = 20 - 8t$.	-1-
	Car stops when $V = 0 \Rightarrow t = \frac{1}{4}$ sec	-1-
21.	Evaluate $\int x^2 e^x \, dx$	
Ans:	$= x^2 e^x - \int e^x \cdot 2x \cdot dx$	-1-
	$= x^2 e^x - 2x e^x + 2e^x + c$	-1-
22.	Evaluate $\int_1^2 \left(x + \frac{1}{x}\right)^2 dx$	
Ans:	$\int_1^2 \left(x^2 + \frac{1}{x^2} + 2\right) dx = \frac{x^3}{3}\Big|_1^2 - \frac{1}{x}\Big|_1^2 + 2x\Big|_1^2$	-1-
	$= 29/6$	-1-

Qn.No.		Marks
	Part-C	
I		
23.	Verify whether $(p \leftrightarrow q)$ and $(\neg p \vee q) \wedge (\neg q \wedge p)$ are logically equivalent or not.	

Ans:

p	q	$p \rightarrow q$	$\neg p$	$\neg q$	$(\neg p \vee q)$ ①	$(\neg q \wedge p)$ ②	① ∧ ②
T	T	T	F	F	T	F	F
T	F	F	F	T	F	T	F
F	T	F	T	F	T	F	F
F	F	T	T	T	T	F	F
		①	①	①	①	①	

They are not logically equivalent ①

24. From 7 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done, if
i) there are exactly 2 ladies
ii) there are atleast 2 ladies.

Ans: i) $^7C_3 \times {}^4C_2 = 210$ ways -2-

ii) $(^7C_3 \times {}^4C_2) + (^7C_3 \times {}^4C_3) + (^7C_2 \times {}^4C_4)$ -3-
$= 371$ ways

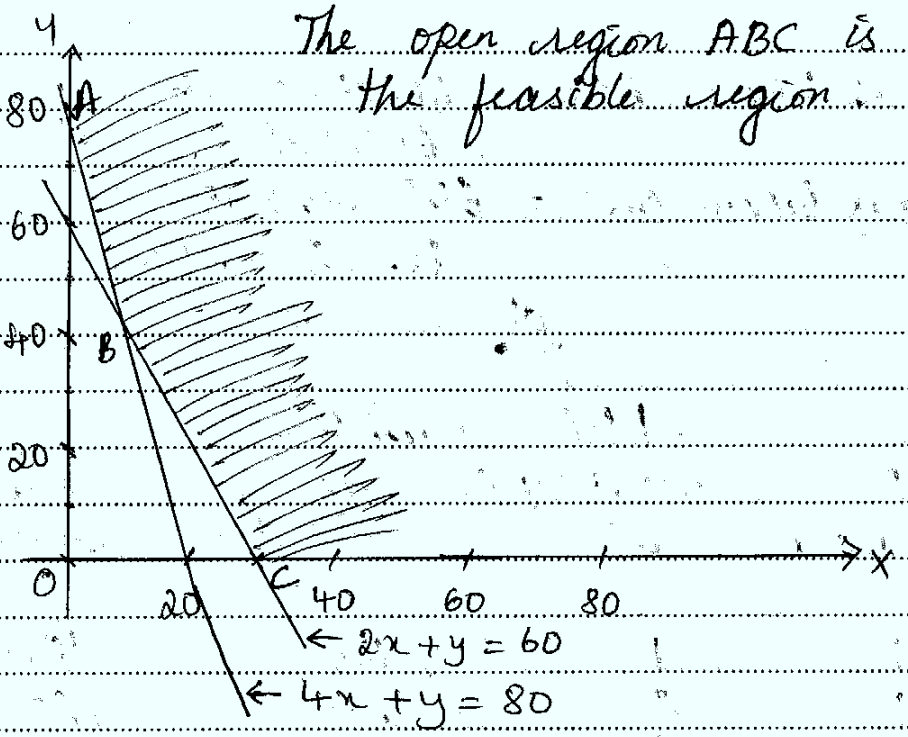
Qn.No.		Marks
25.	Resolve into partial fractions $\frac{x}{(x+1)^2(x+2)}$	
Ans:	$\frac{x}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$	-1-
	$\Rightarrow A = 2$	-1-
	$B = -1$	-1-
	$C = -2$	-1-
	Conclusion	-1-
26.	Solve the following by Matrix method. $x+2y+2z=9$, $3x+2y+z=10$, $x+2y+3z=14$	
Ans:	$\lvert A\rvert = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 4$	-1-
	$\text{adj } A = \begin{bmatrix} 4 & 1 & -3 \\ -8 & 1 & 5 \\ 4 & -1 & -1 \end{bmatrix}$	2
	$A^{-1} = \frac{\text{adj } A}{\lvert A\rvert}$	
	$x=1,\ y=2,\ z=3$	2

Qn.No.		Marks
II 27.	A contractor has 20 men working 8 hours a day to complete a job in a target time of 28 days. At the end of the 18th day, only 1/3rd of the work was completed. He hires more men and all his men work for 9 hours a day. If he wishes to complete the job in scheduled time, find how many more men must he engage?	
Ans:	Hours Days Work Men 8 18 1/3 20 9 10 2/3 20+x	
	8 : 9 :: (20+x) : 20 18 : 10 :: (20+x) : 20 2/3 : 1/3 :: (20+x) : 20	-2-
	$8 \times 18 \times \frac{2}{3} \times 20 = 9 \times 10 \times \frac{1}{3} \times (20+x)$	-1-
	Simplification	-1-
	x = 44 men	-1-
28.	Ramesh has invested Rs. 4300/- partly in 4.5% stock at Rs. 72 and partly in 5% stock at Rs. 95. If the total income from both is Rs. 250, find the investment in both the type of stock.	

Qn.No.		Marks
Ans.	NV MV 100 72 $NI_1 = \frac{100x}{72} \times \frac{4.5}{100}$ x $NI_1 = \frac{4.5x}{72}$	-1-
	NV MV 100 95 $NI_2 = \frac{(4300-x) \times 100}{95} \times \frac{5}{100}$ (4300-x) $NI_2 = \frac{5(4300-x)}{95}$	-1-
	Given $NI_1 + NI_2 = 250$	-1-
	On Solving, Investment in 1st Stock x = 2400/-	-1-
	Investment in 2nd Stock = 1900/-	-1-
	(proportionate marks to be given for alternate method)	
29.	A company requires 100 hours to produce the first 10 units at Rs. 15 per hour. The learning effect is 80%. Find the total labour cost to produce a total of 160 units.	
Ans:	a = 100, x = 16, b = log 80% = −0.3219	-1-
	log y = log a + b log x = log 100 − 0.3219 log 16 = 1.6123	-1-
	y = A(1.6123) = 40.9543	-1-

Qn.No.		Marks
	Total labour hours = 40.9543 × 16 = 655.26	-1-
	Total labour cost = 655.26 × 15 = Rs. 9829.04	-1-
	(OR)	
	1 lot = 10 units	
	Lots produced / Total o/p in lots / Cum. avg time/lot / Total hours	
	1 / 1 / 100 / 100	
	1 / 2 / 80 / 160	
	2 / 4 / 64 / 256	
	4 / 8 / 51.2 / 409.60	
	8 / 16 / 40.96 / 655.36	-4-
	Total Labour cost = 655.36 × 15 = 9830.4/-	-1-
30.	Solve graphically the following LPP:	
	Minimize $Z = 5x + 2y$	
	subject to the constraints	
	$4x + y \geq 80$	
	$2x + y \geq 60$	
	$x, y \geq 0$.	

Qn.No.		Marks



The open region ABC is the feasible region. -2-

A (0, 80) B (10, 40) C (30, 0) are the corner points. -1-

$Z_A = 160$
$Z_B = 130$
$Z_C = 150$ -1-

$\therefore Z_{min} = 130$ @ (10, 40) -1-

III.

31. Find the equation of the circle passing through the points (5, 1) and (3, 4) with it's centre on the x-axis.

Qn.No.		Marks
Ans)	$x^2+y^2+2gx+2fy+c=0$	
	The equations are;	
	$10g+2f+c=-26$ }	
	$6g+8f+c=-25$ }	-1-
	centre on the x-axis $\Rightarrow f=0$	-1-
	$\therefore g=-\frac{1}{4}$ and	-1-
	$c=-\frac{94}{4}$	-1-
	$\therefore$ the equation of the circle is;	
	$x^2+y^2-\frac{x}{2}-\frac{94}{4}=0$	-1-
32.	Prove that the minimum value of $y=x\log x$ is $-\frac{1}{e}$.	
Ans:	$y'=1+\log x$	-1-
	for maxima or minima $y'=0$	-1-
	$\therefore x=-e$	-1-
	$y''=\frac{1}{x}$	-1-
	y'' at $x=-e$ is $-\frac{1}{e}$	-1-
33.	If $y^2+2y=x^2$, show that $y_2=\frac{1}{(1+y)^3}$	

Qn.No.		Marks
Ans:	Given $y^2+2y = x^2$	
	$\Rightarrow 2y \cdot y' + 2y' = 2x$	-1-
	$\Rightarrow y' = \frac{x}{1+y}$	-1-
	$\Rightarrow y'' = \frac{1+y-xy'}{(1+y)^2}$ ∴	-1-
	Substituting for y' and getting the result.	-2-
34.	a) Evaluate : $\int \frac{e^{2x}}{e^x+1}\,dx$	
	b) Evaluate : $\int_2^3 \frac{\log x+1}{x}\,dx$	
Ans:	a) put $e^x+1=t$	-1-
	$e^x dx = dt$, $e^x = t-1$	
	On substituting we get,	
	$\int \frac{t-1}{t} \cdot dt = \int \frac{t}{t} - \frac{1}{t}\,dt$	-1-
	$= (e^x+1) - \log(e^x+1) + c$	-1-
	(b) put $1+\log x = t \Rightarrow \frac{1}{x}\,dx = dt$	-1-
	∴ Integral $= \frac{1}{2}\left[(1+\log 3)^2 - (1+\log 2)^2\right]$	-1-

Qn.No.		Marks
	Part-D	
35a)	In how many ways can 7 Mathematics books, 4 Physics books and 5 chemistry books be arranged in a shelf so that i) Physics books are together. ii) chemistry books are together and Mathematics books are together. iii) no two Mathematics books are together. iv) books of the same subjects are together?	
Ans:-	i) $13! \, 4!$ ways	-1-
	ii) $6! \, 7! \, 5!$ ways	-1-
	iii) $9! \, {}^{10}P_7$ ways	-2-
	iv) $3! \, 7! \, 4! \, 5!$ ways	-1-
b)	Prove that $\lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1}$ for all integral values of n.	
Ans:-	Proving the theorem for $n = 0$	-1-
	Proving the theorem for positive 'n'	-2-
	Proving the theorem for negative 'n'	-2-

Qn.No.		Marks
36) a)	The radius of a circular blot of ink is increasing at the rate of 3cms/min. Find the rate of increase of it's area and the rate of increase of it's circumference when the diameter = 4 cms.	
Ans:	$d = 4 \Rightarrow r = 2$ cms	-1-
	$A = \pi r^2$	
	$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$	-1-
	$\therefore \frac{dA}{dt} = 12\pi$ cm^2/min	-1-
	$C = 2\pi r \Rightarrow \frac{dC}{dt} = 2\pi \frac{dr}{dt}$	-1-
	$\Rightarrow \frac{dC}{dt} = 6\pi$ cm/min	-1-
b)	Find the term independent of x in the expansion of $\left(2x^2 - \frac{3}{x^3}\right)^{25}$.	
Ans:	$T_{r+1} = {}^{n}C_{r}\, x^{n-r} a^{r}$	-1-
	Substitution and simplification	-2-
	$r = 10$	-1-
	$\therefore T_{11} = {}^{25}C_{10} \cdot 2^{15} \cdot (-3)^{10}$	-1-

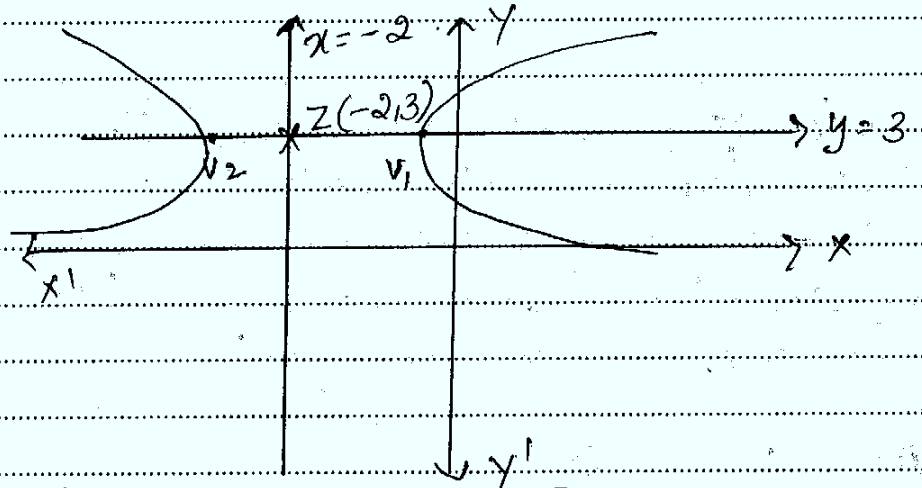
(II)

GOVERNMENT OF KARNATAKA
DEPARTMENT OF PRE-UNIVERSITY EDUCATION

II YEAR P.U.C. EXAMINATION

SCHEME OF VALUATION

Subject Code : 75 Subject : BASIC MATHEMATICS

Qn.No.		Marks
37) a)	Show that $\begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ b+c & c+a & a+b \end{vmatrix} = (a-b)(b-c)(c-a)$	
Ans:	Full 5 marks to be given if any one transformation is mentioned.	-5-
b)	Find the equation of the parabola whose directrix is $x=-2$, axis is $y=3$ and length of the latus rectum $=6$.	
Ans:		
	The required equations are :-	
	$(y-y_1)^2 = \pm 4a(x-x_1)$	-1-
	$4a = 6 \Rightarrow a = \frac{3}{2}$	-1-

Qn.No.		Marks
	for the vertex $V_1 = \left(-\frac{1}{2}, 3\right)$, the equation	
	$(y-3)^2 = 6\left(x+\frac{1}{2}\right)$	
	for the vertex $V_2 = \left(-\frac{7}{2}, 3\right)$, the	
	equation is $(y-3)^2 = -6\left(x+\frac{7}{2}\right)$	-3-
38)a)	Find the area bounded by the curves $y = 2x^2$ and $y = x^2+4$.	
Ans-	Getting the limits of integration as $x = \pm 2$	-1-
	Required Area $= \int_{-2}^{2} 2x^2 - (x^2+4)\, dx$	-1-
	Integrating and applying the limits	-2-
	The required area $= \frac{32}{3}$ sq. units	-1-
b)	A bill of Rs. 5,000 drawn on 10.4.1998 at 3 months was discounted on 1.5.1998 at 12% p.a. For what sum was the bill discounted and how much did the Banker gain?	
Ans:-	$BD = F-P = 150/-$	-1-
	$P = \frac{F}{1+r} = 4854.36/-$	-1-

Scheme of Valuation : -❀- Subject Code :

Qn.No.		Marks
	TD = PW = 145.63/-	-1-
	BG = BD - TD = 4.36/-	-1-
	BPW = F - BD = 4850/-	-1-
	Part - E	
39)	a) Find the value of $(0.99)^5$ upto 4 places of decimal using Binomial Theorem.	
Ans:-	$(0.99)^5 = (1-0.01)^5$	-1-
	Expansion & simplification	-2-
	Answer = 0.9509	-1-
	(b) Two Carpenters A and B earn Rs. 450 and Rs. 500 per week respectively. A can make 5 chairs and 4 tables per week, while B can make 8 chairs and 5 tables per week. How many days shall each work to execute a contract to produce atleast 60 chairs and atleast 32 tables at a minimum labour cost? Formulate the L.P.P.	
Ans:-	Let x denote the no of days A works for. Let y denote the no of days B works for. Objective function:	

Scheme of Valuation : Subject Code :

Qn.No.		Marks
	Minimize $Z = 450x + 500y$	-1-
	subject to the constraints;	
	$5x + 8y \geq 60$	-1-
	$4x + 5y \geq 32$	-1-
	$x \geq 0, y \geq 0$	-1-
c)	Find the total cost and average cost for the marginal cost function $f(x) = 12 + 6x - 6x^2$ (x = production factor) assuming the fixed cost to be Rs. 150.	
Ans:	$TC = \int MC = 12x + 3x^2 - 2x^3 + 150$	-1-
	$AC = \frac{TC}{x} = 12 + 3x - 2x^2 + \frac{150}{x}$	-1-
40)a)	50 candidates appeared in an examination. The average marks obtained by them is 37. The average marks of passed candidates is 39. The average marks of those who failed is 34. Find the number of candidates who passed.	
Ans:	Let x denote the number of candidates who passed and $(50-x)$ denotes the	

Qn.No.		Marks
	number of candidates who failed.	
	$37 = \frac{(39)x + (34)(50-x)}{50}$	-1-
	Simplification	-2-
	Answer $x = 30$	-1-
(b)	The demand function of a firm is $P = 72 - 0.04x$ (P = Price, x = product). If the cost function is $C = 500 + 30x$, at what level of output is the profit maximised? What is the profit at that level?	
Ans:	$TR = Px = 72x - 0.04x^2$	-1-
	Profit = $TR - TC$ =	
	$= 42x - 0.04x^2 - 500$	-1-
	For maximum profit, $\frac{d}{dx}(\text{Profit}) = 0$	
	$\Rightarrow x = 525$ units	-1-
	Maximum profit = Rs. 10,525	-1-
(c)	A bag contains 2 red marbles and 2 green marbles. Two marbles are drawn at random. What is the probability that one is red and the other is green?	

Scheme of Valuation : -0- Subject Code :

Qn.No.		Marks
Ans:	Probability $= \frac{{}^2C_1 \times {}^2C_1}{{}^4C_2}$	-2-
	$\times - \times - \times$	