

**B-12-X**

Roll No. ....

Total No. of Questions : 29]

[Total No. of Printed Pages : 8

**XIIARJKUT23**

**9112-X**

**MATHEMATICS**

[Maximum Marks : 100

Time : 3 Hours]

**SECTION-A**

**(MULTIPLE CHOICE QUESTIONS)**

1 each

1. Range of the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined as  $f(x) = x^2$  is :

(A)  $(0, \infty)$

(B)  $(-\infty, 0)$

(C)  $[0, \infty)$

(D)  $(-\infty, 0]$

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2.  $\sin^{-1} x + \cos^{-1} x, x \in [-1, 1]$  is equal to :
- (A)  $\frac{\pi}{2}$
- (B)  $\frac{\pi}{3}$
- (C)  $\frac{\pi}{4}$
- (D)  $\pi$
3. If  $A$  is a square matrix of order  $n$ , then  $A(\text{adj } A) = \dots\dots\dots$
- (A)  $|A|$
- (B)  $I$
- (C)  $|A| I$
- (D) None of these
4. If  $\vec{a}$  and  $\vec{b}$  are two unit vectors, then  $\vec{a} \cdot \vec{b} = \dots\dots\dots$
- (A)  $\cos \theta$
- (B)  $\sin \theta$
- (C)  $ab \cos \theta$
- (D)  $ab \sin \theta$

## SECTION-B

(VERY SHORT ANSWER TYPE QUESTIONS) 2 each

5. If  $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ , find the values of  $x$ .

6. Examine the continuity of the function  $f(x) = 2x^2 - 1$  at  $x = 3$ .

7. Differentiate  $\sin(x^2 + 5)$  with respect to  $x$ .

8. Find :

$$\int \frac{x^3 + 5x^2 - 4}{x^2} dx$$

9. A coin is tossed three times. Find  $P(E/F)$ , where  $E$  is the event "head on third toss" and  $F$  is the event "heads on first two tosses".

10. Compute  $P(A \cap B)$ , where  $P(A) = 0.8$ ,  $P(B) = 0.5$  and  $P(B/A) = 0.4$ .

11. Find the vector in the direction of vector  $5\hat{i} - \hat{j} + 2\hat{k}$  and having magnitude of 8 units.

12. Define Linear Constraints.

## SECTION-C

## (SHORT ANSWER TYPE QUESTIONS)

4 each

13. Find  $g \circ f$  and  $f \circ g$  if  $f(x) = |x|$  and  $g(x) = |5x - 2|$ .

14. If  $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$ , find the value of  $x$ .

15. If :

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

prove that  $A^3 - 6A^2 + 7A + 2I = 0$ .

16. Find local maxima and local minima if any of the function :

$$f(x) = x^3 - 6x^2 + 9x + 15$$

17. Find general solution of differential equation :

$$x \frac{dy}{dx} - y + x \sin\left(\frac{y}{x}\right) = 0$$

18. Find  $\frac{dy}{dx}$  if  $x^3 + x^2y + xy^2 + y^3 = 81$ .

19. Prove that :

$$\int \sqrt{a^2 - x^2} dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) + C$$

20. If  $y = (\tan^{-1} x)^2$ , show that :

$$(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2$$

21. If  $\vec{a}, \vec{b}, \vec{c}$  are unit vectors such that  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ , find the value

of  $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ .

22. Find the Vector and Cartesian equations of the line that passes through the points (3, -2, -5) and (3, -2, 6).

23. Solve the following graphically :

Minimise :

$$Z = x + 2y$$

Subject to the constraints :

$$2x + y \geq 3$$

$$x + 2y \geq 6$$

$$x, y \geq 0$$

## SECTION-D

## (LONG ANSWER TYPE QUESTIONS)

6 each

24. Using properties of determinants show that :

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = (a - b)(b - c)(c - a)(ab + bc + ca)$$

Or

Solve the system of linear equations using matrix method :

$$2x + y + z = 1$$

$$x - 2y - z = \frac{3}{2}$$

$$3y - 5z = 9$$

25. Find  $\frac{dy}{dx}$  if  $x = a \left( \cos t + \log \tan \frac{t}{2} \right)$ ,  $y = a \sin t$ .

Or

Find  $\frac{dy}{dx}$  if  $(\cos x)^y = (\cos y)^x$ .

26. Find  $\int_0^{\pi/2} \sqrt{\sin \phi} \cos^5 \phi d\phi$ .

Or

Using integration, find the area of region bounded by triangle whose vertices are  $(-1, 0)$ ,  $(1, 3)$ ,  $(3, 2)$ .

27. If  $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ ,  $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ ,  $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ , find the vector  $\vec{d}$  which is perpendicular to both  $\vec{a}$  and  $\vec{b}$  and  $\vec{c} \cdot \vec{d} = 15$ .

Or

Find the shortest distance between the lines  $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$  and

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \quad \text{https://www.jkboseonline.com}$$

28. Find the intervals in which the function  $f(x) = -2x^3 - 9x^2 - 12x + 1$  is strictly increasing or decreasing.

Or

Evaluate :

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

( 8 )

29. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item.

*Or*

Find the mean and variance of the number obtained on a throw of an unbiased die.