

**A-6-A**

Roll No. ....

Total No. of Questions : **29**

[Total No. of Printed Pages : **8**

**12<sup>th</sup>SZARJD22**

**6006-A**

**MATHEMATICS**

Time : **2.30 Hours**

[Maximum Marks : **100**

**Section-A**

(Multiple Choice Questions)

1 each

1. The relation R in the set  $\{1, 2, 3\}$  given by  $R = \{(x, y) \mid x < y, x, y \in A\}$  is :

(A) Reflexive

(B) Symmetric

(C) Transitive

(D) Anti-symmetric

2. The principal value of  $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$  is :

(A)  $\frac{\pi}{4}$

(B)  $\frac{\pi}{2}$

(C)  $\frac{\pi}{3}$

(D)  $\frac{3\pi}{4}$

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3. Two matrices A and B of the same order are said to be equal, if :

(A)  $a_{ij} = 0$

(B)  $b_{ij} = 0$

(C)  $a_{ij} + b_{ij} = 0$

(D)  $a_{ij} = b_{ij}$  for all  $i, j$

4. The direction cosines of a unit vector along  $x$ -axis are :

(A) (1, 0, 0)

(B) (0, 1, 0)

(C) (0, 0, 1)

(D) (1, 1, 1)

### Section-B

(Very Short Answer Type Questions)

2 each

5. Find the values of  $x$ ,  $y$  and  $z$  from the following equation :

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

6. Evaluate :

$$\int (ax^2 + bx + c)dx$$

7. Find the rate of change of area of a circle with respect to its radius when  $r = 5$  cm.

8. Form the differential equation representing the family of curve  $y = mx^2$ , where  $m$  is arbitrary constant.

9. Find the projection of the vector  $\hat{i} + 3\hat{j} + 7\hat{k}$  on the vector  $7\hat{i} - \hat{j} + 8\hat{k}$ .

10. Solve the following L.P.P. graphically :

Maximise :

$$Z = 3x + 4y$$

Subject to constraints :

$$x + y \leq 4,$$

$$x \geq 0, y \geq 0$$

11. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

12. If  $P(A) = \frac{3}{5}$  and  $P(B) = \frac{1}{5}$ , find  $P(A \cap B)$  if A and B are independent events.

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## Section-C

## (Short Answer Type Questions)

4 each

13. Show that  $f : (-1, 1) \rightarrow \mathbb{R}$  given by  $f(x) = \frac{x}{x+2}$  is one-one. Find the inverse of the function  $f : (-1, 1) \rightarrow \text{Range of } f$ .

14. Solve :

$$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$$

15. For the matrix A and B verify that  $(AB)' = B'A'$ ,  $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ ,

$$B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}.$$

16. Find the relationship between  $a$  and  $b$  so that the function  $f$  defined by :

$$f(x) = \begin{cases} ax+1 & \text{if } x \leq 3 \\ bx+3 & \text{if } x > 3 \end{cases}$$

is continuous at  $x = 3$ .

7. Find the intervals in which the function  $f$  given by  $f(x) = 4x^3 - 6x^2 - 72x + 30$  is :

(a) Strictly increasing

(b) Strictly decreasing

18. Find the equations of the tangent and normal to the given curve at the indicated point  $y = x^4 - 6x^3 + 13x^2 - 10x + 5$  at  $(0, 5)$ .

19. Find the general solution of the differential equation :

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

20. Find the area of the region bounded by the ellipse :

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

21. Find  $|\vec{a} \times \vec{b}|$  if  $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$  and  $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ .

22. If  $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$  and  $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$ , then show that  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$  are perpendicular.

23. Solve the following problem graphically :

Minimise and maximise :  $Z = 3x + 9y$

Subject to the linear constraints :

$$x + 3y \leq 60, x \leq y,$$

and  $x + y \geq 10, x \geq 0, y \geq 0$

## Section-D

## (Long Answer Type Questions)

24. By using properties of determinants prove that :

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

Or

If  $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ , then verify that  $A(\text{adj } A) = |A|I$ . Also find  $A^{-1}$ .

25. Find  $\frac{dy}{dx}$  of the function  $x^y + y^x = 1$ .

Or

If  $y = 3 \cos(\log x) + 4 \sin(\log x)$ , show that :

$$x^2 y_2 + x y_1 + y = 0$$

26. Integrate the rational fraction :

$$\frac{2x-3}{(x^2-1)(2x+3)}$$

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Or

Using the properties of definite integrals evaluate :

$$\int_{-5}^5 |x+2| dx$$

27. Find the general solution of the differential equation :

$$x \frac{dy}{dx} + 2y = x^2 \log x$$

Or

Show that the differential equation  $(x^2 - y^2)dx + 2xydy = 0$  is homogeneous and solve it.

28. Find the equation of the plane through the intersection of the planes  $3x - y + 2z - 4 = 0$  and  $x + y + z - 2 = 0$  and the point  $(2, 2, 1)$ .

Or

Find the angle between the line  $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$  and the plane  $10x + 2y - 11z = 3$ .

29. Find the probability distribution of number of doublets in three throws of a pair of dice.

*Or*

Two balls are drawn at random with replacement from a box containing 10 black balls and 8 red balls. Find the probability that :

- (a) Both balls are red
- (b) First ball is black and second is red
- (c) One of them is black and other is red