

**B-5-C**

Roll No.....

Total No. of Questions : 29 ]

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**XIIKDAR21**  
**5005-C**  
**MATHEMATICS**

**Time : 3 Hours ]****[ Maximum Marks : 100****Section-A****(Multiple Choice Questions)**

1 each

1. Let R be the relation in the set N given by  $R = \{(a, b) \mid a = b - 2, (b > 6)\}$ :

(A)  $(2, 4) \in R$

(B)  $(3, 8) \in R$

(C)  $(6, 8) \in R$

(D)  $(8, 7) \in R$

(Choose the correct answer)

2.  $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$  is equal to :

(A)  $\frac{7\pi}{6}$

(B)  $\frac{5\pi}{6}$

(C)  $\frac{\pi}{3}$

(D)  $\frac{\pi}{6}$

(Choose the correct answer)

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3. A and B are symmetric matrices of same order, then  $AB - BA$  is a :

(A) Skew symmetric matrix

(B) Symmetric matrix

(C) Zero matrix

(D) Identity matrix

(Choose the correct answer)

4. The value of :

$$\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{j} \times \hat{i})$$

is :

(A) 0

(B) -1

(C) 1

(D) 3 (Choose the correct answer)

### Section-B

(Very Short Answer Type Questions)

5. Evaluate :

$$\int \frac{\sin(\tan^{-1} x) dx}{1+x^2}$$

6. Evaluate :

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

7. Find the values of  $x$ ,  $y$  and  $z$  from the equation :

$$\begin{bmatrix} x + y + z \\ x + z \\ y + z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

8. Find the order and degree of differential equation :

$$\left(\frac{ds}{dt}\right)^4 + 3s \frac{d^2s}{dt^2} = 4$$

9. If a line has direction ratios 2, -1, 2, determine its direction cosines.

10. Define objective function and optimal solution of L.P.P.

11.  $P(A) = \frac{6}{11}$ ,  $P(B) = \frac{5}{11}$ ,  $P(A \cup B) = \frac{7}{11}$ , find  $P(A \cap B)$ .

12. If  $P(A) = \frac{3}{5}$  and  $P(B) = \frac{1}{5}$ , find  $(A \cap B)$  if A and B are independent events.

### Section-C

(Short Answer Type Questions)

4 each

13. If :

$$f(x) = \frac{4x+3}{6x-4} \quad x \neq \frac{2}{3}$$

Show that  $f \circ f(x) = x$  for all  $x \neq \frac{2}{3}$ . What is the inverse of  $f$  ?

14. Write in the simplest form :

$$\tan^{-1} \left( \frac{\cos x - \sin x}{\cos x + \sin x} \right)$$

15. Express the matrix  $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & 3 \end{bmatrix}$  as the sum of a symmetric and a skew symmetric matrix.

16. Find the equation of tangent and normal to the curve at the indicated point  $y = x^4 - 6x^3 + 13x^2 - 10x + 5$  at  $(0, 5)$ .

17. Use differentials, find the approximate value up to 3 places of decimal  $(25)^{1/3}$ .

18. Integrate the function  $x \tan^{-1} x$ .

19. Find the relationship between  $a$  and  $b$  such that the function  $f$  defined by :

$$f(x) = \begin{cases} ax + 1 & x \leq 3 \\ bx + 3 & x > 3 \end{cases}$$

is continuous at  $x = 3$ .

20. If  $y = \sin^{-1} x$ , show that :

$$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = 0$$

21. Find the unit vector perpendicular to each of the vectors  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$  where  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ .

22. Find the angle between two planes :

$$2x + y - 2z = 5$$

and

$$3x - 6y - 2z = 7$$

using vector method.

23. Solve graphically (L.P.P.) :

Maximize :

$$Z = 4x + y$$

Subject to the constraints :

$$x + y \leq 50$$

$$3x + y \leq 90$$

$$x \geq 0, y \geq 0$$

**Section-D****(Long Answer Type Questions)**

6 each

24. Using the properties of determinants. Prove that :

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

*Or*

Solve the system of equations by matrix method :

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

25. If  $y = x^{\sin x} + (\sin x)^{\cos x}$ , find  $\frac{dy}{dx}$ .*Or*

Show that of all the rectangles inscribed in a given fixed circle the square has the maximum area.

26. Find :

$$\int \frac{(3 \sin x - 2) \cos x \, dx}{5 - \cos^2 x - 4 \sin x}$$

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Or

Find the area of the region bounded by the ellipse :

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

27. Find the general solution of the differential equation given by :

$$x \frac{dy}{dx} + 2y = x^2 \log x$$

Or

Find the general solution of the differential equation :

$$\frac{dy}{dx} = \frac{x+1}{2-y}, \quad y \neq 2$$

28. Find the equation of the plane passing through three points (1, 1, 0)

(1, 2, 1), (-2, 2, -1).

Or

Find the shortest distance between the lines whose vector equations

are :

$$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$$

and

$$\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

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29. Find the probability distribution of the number of doublets in three throws of a pair of dice.

*Or*

A die is thrown 6 times if "getting an odd number" is a success.

What is the probability of :

- (i) 5 success
- (ii) at least 5 success
- (iii) at most 5 success