D–5–A

Total No. of Questions : 29]

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XIIARKDN20
2005–A
MATHEMATICS

Time : 3 Hours]

[Maximum Marks : 100

Section–A

(Multiple Choice Questions) 1 each

1. In a set A = {1, 2, 3, 4, 5, 6, 7} a relation R defined by R = {(*a*, *b*) : both *a* and *b* are either odd or even}, then R is :

(A) Reflexive (B) Symmetric

(C) Transitive (D) None of these

2. If $f : R \to R$ be given by $f(x) = (3 - x^4)^{1/4}$, then $fof(x)$ is :

(A) x (B) x^4

(C) $3^{1/4}$ (D) None of these

3. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to :

(A) A

(B) $I - A$

(C) I

(D) None of these

4. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{2}$, then $\vec{a} \times \vec{b}$ is a unit vector, if angle between $\vec{a}$ and $\vec{b}$ is :

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) None of these

Section–B

(Very Short Answer Type Questions) 2 each

5. Find AB, if $A = \begin{pmatrix} 6 & 9 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 6 & 0 \\ 7 & 9 & 8 \end{pmatrix}$.

6. Prove that :

$$\int \cot x \, dx = \log|\sin x| + C$$

26. Evaluate :

$$\int \frac{x^2}{(x^2+1)(x^2+4)}\,dx$$

Or

Find the area enclosed by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

27. Show that the differential equation :

$$y\,dx + x\log\left(\frac{y}{x}\right)dy - 2x\,dy = 0$$

is homogeneous and solve it.

Or

Find the particular solution of the differential equation :

$$(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}.$$

given that $y = 0$ when $x = 1$.

Or

For the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

show that :

$$A^3 - 6A^2 + 5A + 11I = 0.$$

Hence, find A^{-1}.

25. Find the relation between a and b, so that the function f defined by :

$$f(x) = \begin{cases} ax + 1, & \text{if} \quad x \leq 3 \\ bx + 3, & \text{if} \quad x > 3 \end{cases}$$

is continuous at $x = 3$.

Or

Find $\frac{dy}{dx}$ of the function $xy = e^{(x-y)}$.

16. An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long ?

17. Using differentials find the approximate value upto 3 decimal places of $(25)^{1/3}$

18. Find $\frac{d^2y}{dx^2}$, if $y = x^3 \log x$.

19. Prove that :

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + C$$

20. Using properties of definite integrals, to evaluate :

$$\int_0^{\pi/2} \frac{\sin^{3/2} x \, dx}{\sin^{3/2} x + \cos^{3/2} x}$$

21. Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$.

28. Find the shortest distance between the lines whose vector equations are

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k} \quad \text{and}$$

$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$$

Or

Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$.

29. Find the mean number of heads in three tosses of a fair coin.

Or

There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item ?

7. Evaluate :

$$\int_0^{\pi/4} \sin 2x \, dx$$

8. Find the order and degree of the differential equation :

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

9. Find unit vector in the direction of vector $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$.

10. Define the term optimisation problem.

11. If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, evaluate P(A/B).

12. A family has two children. What is the probability that both the children are boys given that at least one of them is a boy ?

Section–C

(Short Answer Type Questions) 4 each

13. Prove that the function $f : \mathbf{R} \to \mathbf{R}$, given by $f(x) = 2x$, is one-one and onto.

14. If $\sin\left[\sin^{-1}\frac{1}{5} + \cos^{-1}x\right] = 1$, then find the value of x.

15. Find the inverse of $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$.

22. Find the distance of a point (2, 5, −3) from the plane

$\vec{r}\cdot(6\hat{i}-3\hat{j}+2\hat{k})=4$.

23. Determine graphically the minimum value of the objective function

$$Z = -50x + 20y$$

Subject to the constraints

$$2x - y \geq -5$$

$$3x + y \geq 3$$

$$2x - 3y \leq 12$$

$$x \geq 0,\ y \geq 0$$

Section–D

(Long Answer Type Questions) 6 each

24. Using properties of determinants, prove that :

$$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$