Series–B

Roll No. ....

(GRAPH PAPER)

Total No. of Questions 34] [Total No. of Printed Pages 16

A–857–B–XII–2325

MATHEMATICS

Time Allowed—3 Hours Maximum Marks—80

Candidates are required to give their answers in their own words as far as practicable.

Marks allotted to each question are indicated against it.

Special Instructions :

(i) You must write Question Paper Series in the circle at the top left side of title page of your Answer-book.

(ii) While answering your Questions, you must indicate on your Answer-book the same Question No. as appears in your Question Paper.

(iii) Do not leave blank page/pages in your Answer book.

(iv) All questions are compulsory.

(v) This question paper contains four sections A, B, C, & D. Each section is compulsory.

(vi) Section–A has 13 MCQ (Multiple Choice Questions) and 3 Assertion, Reason based questions of 1 mark each.

(vii) Section–B has 12 Very Short answer questions of 3 marks each i.e. Question Nos. 17 to 28.

(viii) Section–C has 2 Short answer questions of 4 marks each i.e. Question Nos. 29 & 30.

(ix) Section–D has 4 Long answer questions of 5 marks each i.e. Question Nos. 31 to 34.

(x) Graph paper must be attached in between the Answer-book.

(xi) All questions given in Section–A (Multiple Choice Questions) are to be attempt on OMR sheet provided with Answer book.

SECTION–A

1. Let $f : R \to R$ defined as $f(x) = 1 + x^2$. Choose the correct answer : 1

   (a) f is both one-one and onto

   (b) f is one-one but not onto

   (c) f is onto, but not one-one

   (d) f is neither one-one nor onto.

2. If $\tan^{-1} x = y$, then : 1

   (a) $0 < y < \frac{\pi}{2}$ (b) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

   (c) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ (d) $0 \le y \le \frac{\pi}{2}$.

3. $\sin\left(\tan^{-1} x\right), |x| < 1$ is equal to: 1

   (a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$

   (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$.

4. If the matrix A is both symmetric and skew symmetric, then 1

(a) A is a diagonal matrix

(b) A is a zero matrix

(c) A is a square matrix

(d) None of these.

5. Let A be a non-singular square matrix of order 3×3. Then $|\text{adj}\, A|$ is : 1

(a) $|A|$ (b) $|A|^3$

(c) $|A|^2$ (d) $|3A|$.

6. $\frac{d}{dx}[\sin(\log x)]$ is, $x > 0$ 1

(a) $\frac{\cos(\log x)}{x}$

(b) $\frac{\log x}{x}$

(c) $\cos(\log x)$

(d) $-\cos(\log x)$.

7. If $f(x) = x^2 + 2x - 5$, then f(x) is strictly decreasing in the interval : 1

(a) $(1, \infty)$ (b) $(-\infty, 1)$

(c) $(-\infty, -1)$ (d) None of these.

8. The maximum value of $[x(x-1)+1]^{1/3}$, $0 \le x \le 1$ is : 1

(a) $\left(\frac{1}{3}\right)^{1/3}$ (b) $\frac{1}{2}$

(c) 1 (d) 0.

9. $\int x^2 e^{x^3} dx$ is equal to : 1

(a) $\frac{1}{3}e^{x^3} + C$ (b) $\frac{1}{3}e^{x^2} + C$

(c) $\frac{1}{2}e^{x^3} + C$ (d) $\frac{1}{2}e^{x^2} + C$.

10. The number of arbitrary constants in the general solution of a differential equation of fourth order are : 1

(a) 0 (b) 2

(c) 3 (d) 4.

11. The integrating factor of

$$\frac{dy}{dx} + y\cot x = 2x + x^2\cot x \, (x \neq 0)$$ is : 1

(a) $-\cos x$ (b) $\cos x$

(c) $-\sin x$ (d) $\sin x$.

12. The projection of the vector $2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\hat{i} + 2\hat{j} + \hat{k}$ is : 1

(a) $\frac{8\sqrt{7}}{3}$ (b) $\frac{2\sqrt{3}}{7}$

(c) $\frac{5\sqrt{6}}{3}$ (d) $\frac{3\sqrt{5}}{2}$.

13. If a line makes equal angles with co-ordinate axis, then its direction Cosines are : 1

(a) $\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(b) $\left(\frac{\pm 1}{\sqrt{3}}, \frac{\pm 1}{\sqrt{3}}, \frac{\pm 1}{\sqrt{3}}\right)$

(c) (0, 0, 0)

(d) (1, 1, 1).

Assertion Reason based Questions.

In the following questions, a statement of Assertion(A) is followed by a Reason(R). Choose the correct answer out of the following from (Question No. 14 to 16) :

14. Assertion (A): The three lines with direction Cosines $\left(\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}\right)$, $\left(\frac{4}{13}, \frac{12}{13}, \frac{3}{13}\right)$ & $\left(\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}\right)$ are mutually perpendicular. 1

Reason (R): The line through the point (1, –1, 2) and (3, 4, –2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

Options:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A)

(c) Assertion (A) is true, but Reason (R) is false.

(d) Both Assertion (A) and Reason (R) are false.

15. **Assertion (A):** Two coins are tossed simultaneously the probability of a getting two heads, if it is known that at least one head comes up is $\frac{1}{3}$. 1

Reason (R): **Let E and F be two events with a random experiment, then**

$$P(F/E) = \frac{P(E \cap F)}{P(E)}.$$

Options:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A)

(c) Assertion (A) is true, but Reason (R) is false.

(d) Both Assertion (A) and Reason (R) are false.

16. Assertion (A): If E and F are two events associated with the same sample space of a random experiment, the conditional probability of the events E, given that F has occurred i.e. $P(E/F)$ is given by

$$P(E/F) = \frac{P(E \cap F)}{P(F)}.$$ 1

Reason (R): If $P(E) = \frac{7}{13}$, $P(F) = \frac{9}{13}$ and $P(E \cap F) = \frac{4}{13}$, then the value of $P(E/F) = \frac{5}{9}$.

Options:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A)

(c) Assertion (A) is true, but Reason (R) is false.

(d) Both Assertion (A) and Reason (R) are false.

SECTION–B

17. Show that the relation R in the set $A = \{x \in Z; 0 \le x \le 12\}$ defined as $R = \{(a, b); |a - b| \text{ is multiple of } 4\}$ is an equivalence relation. 3

18. Express the matrix $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ as the sum of a Symmetric and Skew Symmetric matrix. 3

19. Find the area of triangle by using the determinants whose vertices are (2, 7) (1, 1) (10, 8). 3

20. Find all points of discontinuity of 'f' when 'f' is defined by

$$f(x) = \begin{cases} x^3 - 3 & , \text{ if } x \le 2 \\ x^2 + 1 & , \text{ if } x > 2 \end{cases}$$ 3

21. Evaluate :

$$\int \frac{3x - 1}{(x-1)(x-2)(x-3)} dx.$$ 3

22. By using properties of definite integral evaluate :

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}.$$ 3

23. Find the area of the region bounded by ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1.$ 3

24. Solve the differential equation

$$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$$

Or

Find the general solution of differential equation :

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}.$$

3

25. Find a Unit vector perpendicular to each of the vector $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

3

26. Find $|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$.

3

27. An Insurance Company insured 2000 Scooter drivers, 4000 Car drivers and 6000 Truck drivers. The probability of an accident are 0.01, 0.03 and 0.15 respectively. One of the insured

persons meets with an accident. What is the probability that he is a scooter driver? 3

28. A die is thrown three times. Event A and B are defined as below :

A : 4 on the third throw.

B : 6 on the first and 5 on the second throw

Find the probability of A given that B has already occurred.

Or

Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently find the probability :

(a) the problem is solved.

(b) exactly one of them solves the problem. 3

SECTION-C

29. Express :

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x \neq 0$$

in the simplest form.

Or

Prove that :

$$\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{77}{36}\right).$$

30. If $y = Ae^{mx} + Be^{nx}$; show that

$$\frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+mny=0.$$

Or

Find $\frac{dy}{dx}$; if $(\cos x)^y = (\cos y)^x$. 4

SECTION-D

31. Solve the system of linear equation by using matrix method : 5

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3.$$

32. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base.

Or

Find the interval in which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is strictly increasing or strictly decreasing. 5

33. Find the shortest distance between the lines whose vector equations are

$$\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$$

and $\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$.

Or

Find the value of 'p' so that the lines 5

$$\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$$ and

$$\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$$ are at right angles.

34. Solve the following Linear Programming Problem (LPP) graphically : 5

Maximize $Z = 3x + 2y$

Subject to :

$$x + 2y \leq 10$$

$$3x + y \leq 15$$

where $x \geq 0$

$y \geq 0.$