

2025 II 24

0930

Seat No.

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Time : 3 Hours

**MATHEMATICS**

**Subject Code**

H	4	7	5	4
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**Total No. of Questions : 36 (Printed Pages : 8)**

**Maximum Marks : 80**

- INSTRUCTIONS :**
- (i) The question paper consists of **36** questions.
  - (ii) Question numbers **1** to **8** are Multiple Choice type of questions of *one* mark each.
  - (iii) Question Numbers **9** to **16** are very short answer type questions of *one* mark each.
  - (iv) Question Numbers **17** to **22** are short answer type-I questions of *two* marks each.
  - (v) Question numbers **23** to **28** are short answer type-II questions of *three* marks each.
  - (vi) Question numbers **29** to **34** are long answer type-I questions of *four* marks each.
  - (vii) Question numbers **35** to **36** are long answer type-II questions of *five* marks each.
  - (viii) There is no overall choice. However an internal choice has been provided in two questions of **4** marks each and **2** questions of **5** marks each.
  - (ix) Use of calculators is not permitted.
  - (x) Graph should be drawn on the answer paper only.

1. If  $A = \begin{bmatrix} 0 & 4 \\ x-5 & 0 \end{bmatrix}$  is skew symmetric matrix, then the value of  $x$  is .....

- 1
- 2
- 3
- 4

2. The relation  $R = \{ (a, a), (b, b), (c, c) \}$  on the set  $A = \{a, b, c\}$  is .....

- Identity
- Reflexive only
- Symmetric only
- Equivalence

3. The value of  $\cos^{-1} \left( \cos \frac{3\pi}{2} \right)$  is .....

- $\frac{\pi}{2}$
- $\frac{3\pi}{2}$
- $\frac{5\pi}{2}$
- $\frac{7\pi}{2}$

4. The integrating factor of differential equation  $\cos x \frac{dy}{dx} + y \sin x = 1$  is .....

- $\sec x + \tan x$
- $\text{Log}(\sec x + \tan x)$
- $e^{\sec x}$
- $\sec x$

5. The derivative of  $x^{2x}$  with respect to  $x$  is .....

- $x^{2x-1}$
- $2x^{2x} \log x$
- $2x^{2x}(1 + \log x)$
- $2x^{2x} (1 - 2 \log x)$

6. The value of  $\frac{dy}{dx}$  at  $(4, 1)$  of  $y^3 - \sqrt{x} = 5$  is .....

- $\frac{5}{4}$
- $\frac{1}{12}$
- $\frac{1}{24}$
- $\frac{1}{3}$

7.  $\int 2^x 3^x dx$  equals to .....

- $\frac{3^x}{\ln 3} + c$
- $\frac{2^x}{\ln 2} + c$
- $\frac{2^x 3^x}{\ln 2 \ln 3} + c$
- $\frac{6^x}{\ln 6} + c$

8. The direction ratios of the line  $6x - 2 = 3y + 1 = 2z - 2$  is .....

- 3, 2, 1
- $\frac{1}{3}, \frac{1}{3}, -1$
- 1, 2, 3
- $\frac{1}{6}, \frac{1}{3}, -1$

9. If  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  are two functions defined by  $f(x) = 2x + 1$  and  $g(x) = x^2 - 2$  respectively, find  $(g \circ f)(x)$ .

10. Find the principal value of  $\sec^{-1}(-2)$ .

11. Find  $\frac{dy}{dx}$  if  $y = \sin(\log x)$ .

12. Evaluate  $\int_{-2}^2 (x^3 + 1) dx$ .

13. Find the vector equation of the line through the points A(3, 4, -7) and B(1, -1, 6).

14. If A and B are two independent events with  $P(A) = \frac{1}{3}$  and  $P(B) = \frac{1}{4}$  find  $P(B'|A)$ .
15. Find the magnitude of  $\bar{a}$ , where  $\bar{a} = (\hat{i} + 3\hat{j} - 2\hat{k}) \times (-\hat{i} + 3\hat{k})$
16. Find the interval in which the function  $f(x) = 2x^2 + 12x + 1$  is increasing.
17. Prove that  $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ ,  $x \in \mathbb{R}$ .
18. Using the First Principle of derivative, find the derivative of  $y = a^{2x+5}$  with respect to  $x$ .
19. Show that the function  $f : \mathbb{R} - \{3\} \rightarrow \mathbb{R} - \{0\}$  defined by  $f(x) = \frac{1}{x-3}$  is bijective.
20. Find the general solution of the differential equation :

$$(x + 2) \frac{dy}{dx} = x^2 + 4x - 9$$

21. Find  $x$ , if  $[2x \ 3] \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$
22. If  $\int_0^x 2t dt + y^2 = 1$ , prove that  $y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 1 = 0$ .
23. Using integration, prove that  $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log|x + \sqrt{x^2 + a^2}| + c$
24. If  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ , prove that  $A^2 = 4A + 5I$

25. Find  $\int \frac{1}{x[1 - 4(\log x)^2]} dx$ .
26. Find the area of the parallelogram whose diagonals are  $3\hat{i} + \hat{j} - 2\hat{k}$  and  $\hat{i} - 3\hat{j} + 4\hat{k}$ . Also find its adjacent sides.
27. Solve the differential equation  $x^2 dy + (xy + y^2) dx = 0$
28. If  $x = t - \frac{1}{t}$  and  $y = t + \frac{1}{t}$ , prove that  $\frac{d^2 y}{dx^2} = 4 \left( \frac{t}{t^2 + 1} \right)^3$ .
29. Solve the following linear programming problem by graphical method  
Maximize  $Z = 6x + 5y$  subject to the constraints

$$2x + y \leq 6$$

$$x + y \leq 5$$

$$x + 3y \geq 3,$$

$$x, y \geq 0$$

30. If  $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ , using adjoint method, find  $A^{-1}$ . Hence solve the system of equations :

$$x - y + z = 4$$

$$2x + y - 3z = 0$$

$$x + y + z = 2$$

31. Discuss the continuity of the function  $f(x)$  at  $x = 0$ , where

$$f(x) = \begin{cases} \frac{(e^{4x} - 1)(1 - \cos x)}{x^3} & \text{if } x < 0 \\ 4 \cos x - 6 & \text{if } x = 0 \\ \frac{\log(1 + x)}{x(\sqrt{1 + x} - 1) \operatorname{cosec} x} & \text{if } x > 0 \end{cases}$$

If the function is discontinuous, state the type of discontinuity at  $x = 0$ .

32. Find  $\int \frac{1}{1 - 2\sin x} dx$

*Or*

32. Find  $\int x^2 \cos^2 x dx$

33. Using integration, find the area enclosed between the parabola  $4y = x^2$  and the straight line  $x = 4y - 2$ .

34. Find the equation of the line passing through the point (1, 2, 3) and parallel to the line of intersection of planes  $x - y + 2z = 6$  and  $3x + y - z = 5$ .

*Or*

34. Find the vector and Cartesian equation of the plane passing through three non-collinear points (0, -1, -1), (4, 5, 1) and (3, 9, 4) and determine whether (-4, 4, 4) lie on the plane.

35. The radius of a solid cylindrical object is increasing at the rate of 2 cm/sec and its altitude is decreasing at the rate of 3 cm/sec. Find the rate of change of total surface area when the radius is 3 cm and altitude is 5 cm.

*Or*

35. A cricket field is of the shape of a rectangle with semicircle at each end. The sports authority wants to construct a running track of 440 m enclosing the cricket field. What is the maximum area of the rectangular cricket field ?
36. The members of a private company rent cars from three rental agencies, 50% from agency X, 30% from agency Y and 20% from agency Z. From the past experience, it is known that 90% of cars from agency X are AC cars, 70% of cars from agency Y are AC cars, 20% of cars from agency Z are AC cars. A rental car is selected and delivered to the company. It is found to be an AC car. Find the probability that the rental car is from agency Z.

*Or*

36. In a group of 30 scientists working on an experiment, 20 never committed error in their work and are reporting results elaborately. Two scientists are selected at random from the group. Find the probability distribution of the number of scientists who committed error in the work and reporting. Also, find the mean of the distribution.