2019 III 13 **1000** **Seat No.** | | | | | |

Time : 2½ Hours **MATHEMATICS**

Subject Code

H	7	5	4

Total No. Of Questions : 30 **(Printed Pages : 8)** **Maximum Marks : 80**

INSTRUCTIONS :

(*i*) *All* questions are compulsory.

(*ii*) The question paper consists of **30** questions divided into five sections A, B, C, D and E.

(*iii*) Section A contains **7** questions of **1** mark each, which are multiple choice type questions. Section B contains **7** questions of **2** marks each, Section C contains **7** questions of **3** marks each, Section D contains **7** questions of **4** marks each and Section E contains **2** questions of **5** marks each.

(*iv*) There is no overall choice in the paper. However internal choice is provided in **2** questions of **3** marks each, **2** questions of **4** marks each and **2** questions of **5** marks each. In questions with choices only one of the choices is to be attempted.

(*v*) Use of calculator is not allowed.

Section A

Question Nos. **1** to **7** carry **1** mark each. In each question, *four* options are provided out of which *one* is correct. Write the correct option.

1. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$, then AB is :

- a zero matrix
- an identity matrix
- matrix A
- matrix B

2. $\bar{a} \times \bar{b}$ is a vector which is :

- parallel to $\bar{a}$
- parallel to $\bar{b}$
- perpendicular to both $\bar{a}$ and $\bar{b}$
- perpendicular to only $\bar{a}$

3. $\cot\left(\pi - \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right)$ is equal to :

- $\frac{1}{\sqrt{3}}$
- $\frac{1}{2}$
- $\sqrt{3}$
- 2

4. If $y = 2\log\left(e^{3x}\right) + 5$, then $\frac{dy}{dx} = \ldots\ldots\ldots\ldots$

- 2
- 3
- 5
- 6

5. If $f(x) = \sqrt{2 - x^2}$, then $f o f(x) = \ldots\ldots\ldots\ldots$

- x
- x^2
- $2 - x^2$
- $\sqrt{x}$

6. $y = 5e^{x} + 2e^{-x} + x$ is a solution of the differential equation :

- $\frac{d^2y}{dx^2} + \frac{dy}{dx} = y$
- $\frac{d^2y}{dx^2} + x = y$
- $\frac{d^2y}{dx^2} + y = x$
- $\frac{d^2y}{dx^2} - \frac{dy}{dx} = x$

7. The direction cosines of two perpendicular lines are l_1, m_1, n_1 and l_2, m_2, n_2 respectively. Then the direction cosines of a line which is perpendicular to both these lines are :

- $l_1 + l_2, m_1 + m_2, n_1 + n_2$
- $l_1 - l_2, m_1 - m_2, n_1 - n_2$
- $m_1n_2 - n_1m_2, n_1l_2 - l_1n_2, l_1m_2 - l_2m_1$
- $m_1n_1 - m_2n_2, l_1n_1 - l_2n_2, l_1m_2 - l_2m_1$

Section B

Question Nos. **8** to **14** carry **2** marks each.

8. Prove that $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ for $a < c < b$.

9. Find the value of 'x' if the vectors $\hat{i} - 2\hat{j} + k$, $x\hat{i} - 5\hat{j} + 3\hat{k}$ and $5\hat{i} - 9\hat{j} + 4\hat{k}$ are coplanar.

10. The probability that a student is not a swimmer is $\frac{1}{3}$. What is the probability that out of 5 students, at least one is a swimmer ?

11. Find matrices X and Y if :

$$2X + Y\begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix} \text{ and } X - Y = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}.$$

12. The cartesian equations of a line are $4x - 2 = 3y - 1 = 2 - 2z$. Find the vector equation of the line and also find its direction ratios.

13. Form the differential equation representing the family of curves $y = A(x + B)^2$ by eliminating arbitrary constants A and B.

14. A particle moves along the curve $y = 2x^3 + 1$. Find the points on the curve at which the y-coordinate changes six times as fast as the x-coordinate.

Section C

Question Nos. **15** to **21** carry **3** marks each.

15. Using integration prove that :

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log |x + \sqrt{x^2 - a^2}| + c.$$

16. Prove that :

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x - y}{1 + xy}\right), xy > -1.$$

17. Find 'adj.A' if $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$. Hence find A(adj. A).

18. A unit vector $\overline{a}$ makes angle $\frac{\pi}{3}$ with $\overline{b}$ and vector $\overline{a} + 3\overline{b}$ is perpendicular to vector $2\overline{a} - \overline{b}$. Find the magnitude of $\overline{b}$.

19. Show that the function $f : R - \{3\} \rightarrow R - \{1\}$ defined by $f(x) = \frac{x - 2}{x - 3}$ is invertible.

20. Solve the differential equation :

$$x^2 dy + y(x + y)\, dx = 0.$$

Or

Find a particular solution of the differential equation :

$$xe^x\,dy = (x^3 + 2ye^x)dx$$

given that $y = 0$ when $x = 1$.

21. Find the equation of a plane passing through the points (1, 0, 0) and (0, 2, 0) which is at a distance of $\frac{6}{7}$ units from the origin.

Or

Find the distance of the point (1, –2, 3) from the plane $x - y + z = -1$ measured along a line parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$.

Section D

Question Nos. **22** to **28** carry **4** marks each.

22. Find the values of 'A' and 'B' if the function defined below is continuous in its domain :

$$f(x) = \begin{cases} \dfrac{Ax^3 + 3x^2 + Bx}{e^{6x} - 1}; & -1 \le x < 0 \\[2ex] \dfrac{1}{2}; & x = 0 \\[2ex] \dfrac{\sqrt{Ax + 4} - 2}{\log(3 + 6x) - \log 3}; & 0 < x \le 1 \end{cases}$$

23. Solve the following linear programming problem graphically :

Minimise :

$$Z = 50x + 60y$$

Subject to the constraints :

$$3x + 4y \leq 24$$

$$x + y \geq 5$$

$$x + 4y \geq 8$$

$$x, y \geq 0.$$

24. Evaluate :

$$\int_0^{\frac{\pi}{2}} (2 \log (\sin x) - \log(\sin 2x))\, dx.$$

25. An electronic component consists of 3 parts A, B and C. The probabilities that each part perform satisfactorily are 0.4, 0.3 and 0.2 respectively. The component fails if two or more parts do not perform satisfactorily. Assuming that the parts perform independently, find the probability that the component does not perform satisfactorily.

Or

Four defective bolts are accidently mixed with six good ones. If two bolts are drawn successively without replacement from this lot, find the probability distribution of the number of defective bolts. Also find the mean of the number of defective bolts.

26. Using integration, find the area of the region :

$$\{(x, y) : y^2 \leq 3x \text{ and } x^2 + y^2 \leq 4\}.$$

27. If the sum of the lengths of the hypotenuse and one of the sides of a right-angled triangle is constant, then find the angle between that side and the hypotenuse so that the area of triangle is maximum.

Or

Find the points on the curve $9y^2 = x^3$ where normal to the curve makes equal intercepts with the coordinate axes.

28. If a, b, c are positive and unequal such that :

$$\begin{vmatrix} a & a^3 & a^4 \\ b & b^3 & b^4 \\ c & c^3 & c^4 \end{vmatrix} = 0$$

Then using properties of determinants prove that $ab + bc + ca = 0$.

Section E

Question Nos. **29** to **30** carry **5** marks each.

29. Find $\frac{d^2y}{dx^2}$ if $x = \tan^{-1}\left[\frac{t^2}{1+\sqrt{1-t^4}}\right]$ and $y = \sin\left[2\cot^{-1}\sqrt{\frac{1+t^2}{1-t^2}}\right]$ where 't' is the parameter.

Or

If $(x-a)^2 + (y-b)^2 = c^2$ prove that

$$\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}$$

is a constant independent of 'a' and 'b'.

30. Find $\int \frac{e^{3x} + 2e^x}{e^{3x} - 2e^{2x} + e^x - 2}dx.$

Or

Find $\int \frac{\sqrt{x^4+1}\{\log(x^4+1) - 4\log x\}}{x^7}dx.$