

22

QUESTION PAPER  
SERIES CODE  
**A**

Registration No. :

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Centre of Exam. :

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Name of Candidate :

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Signature of Invigilator

**ENTRANCE EXAMINATION, 2017**

MASTER OF COMPUTER APPLICATIONS

[ Field of Study Code : MCAM (224) ]

Time Allowed : 3 hours

Maximum Marks : 100

**INSTRUCTIONS FOR CANDIDATES**

Candidates must read carefully the following instructions before attempting the Question Paper :

- (i) Write your Name and Registration Number in the space provided for the purpose on the top of this Question Paper and in the Answer Sheet.
- (ii) **Please darken the appropriate Circle of Question Paper Series Code on the Answer Sheet.**
- (iii) All questions are compulsory.
- (iv) Answer all the 100 questions in the Answer Sheet provided for the purpose by darkening the correct choice, i.e., (a) or (b) or (c) or (d) with BALLPOINT PEN only against the corresponding circle. Any overwriting or alteration will be treated as wrong answer.
- (v) Each correct answer carries 1 mark. **There will be negative marking and 0.25 mark will be deducted for each wrong answer.**
- (vi) Answer written by the candidates inside the Question Paper will not be evaluated.
- (vii) Pages at the end have been provided for Rough Work.
- (viii) Return the Question Paper and Answer Sheet to the Invigilator at the end of the Entrance Examination. **DO NOT FOLD THE ANSWER SHEET.**

**INSTRUCTIONS FOR MARKING ANSWERS**

- 1. Use only Blue/Black Ballpoint Pen (do not use pencil) to darken the appropriate Circle.
- 2. Please darken the whole Circle.
- 3. Darken ONLY ONE CIRCLE for each question as shown in the example below :

Wrong	Wrong	Wrong	Wrong	Correct
<input type="radio"/> (a) <input type="radio"/> (b) <input type="radio"/> (c) <input type="radio"/> (d)	<input checked="" type="radio"/> (a) <input type="radio"/> (b) <input type="radio"/> (c) <input type="radio"/> (d)	<input checked="" type="radio"/> (a) <input type="radio"/> (b) <input type="radio"/> (c) <input checked="" type="radio"/> (d)	<input type="radio"/> (a) <input checked="" type="radio"/> (b) <input type="radio"/> (c) <input type="radio"/> (d)	<input type="radio"/> (a) <input checked="" type="radio"/> (b) <input type="radio"/> (c) <input type="radio"/> (d)

- 4. Once marked, no change in the answer is allowed.
- 5. Please do not make any stray marks on the Answer Sheet.
- 6. Please do not do any rough work on the Answer Sheet.
- 7. Mark your answer only in the appropriate space against the number corresponding to the question.
- 8. **Ensure that you have darkened the appropriate Circle of Question Paper Series Code on the Answer Sheet.**



1. Find the median of the data given in the table :

Income (in \$)	1000	1100	1200	1300	1400	1500
No. of persons	14	26	21	18	28	15

- (a) 1300  
(b) 1250  
(c) 1200  
(d) None of the above
2. Determine the set
- $$B = \left\{ x \in R : \frac{2x+1}{x+2} < 1 \right\}$$
- where,  $R$  is the set of real numbers
- (a)  $B = (-2, 1)$   
(b)  $B = (1, 2)$   
(c)  $B = (-\infty, -1) \cup \left(\frac{1}{2}, \infty\right)$   
(d) None of the above
3. The odds against  $A$  solving the problem is 4 is to 3 and the odds in favour of  $B$  solving the problem is 7 is to 5. What is the probability that the problem will not be solved?
- (a)  $\frac{4}{21}$   
(b)  $\frac{16}{21}$   
(c)  $\frac{63}{84}$   
(d)  $\frac{69}{84}$
4. Let  $\{f_n\}$  be the Fibonacci sequence of numbers defined by

$$f_1 = 1, f_2 = 1 \text{ and } f_n = f_{n-1} + f_{n-2} \text{ for } n > 2$$

It is defined that

$$x_n = \frac{f_n}{f_{n+1}} \text{ for } n \geq 1$$

Then, the sequence  $\{x_n\}$

- (a) diverges  
(b) converges to  $\frac{1}{2}(-1 + \sqrt{5})$   
(c) converges to  $\frac{1}{2}(1 + \sqrt{5})$   
(d) None of the above

5. Let  $a, b$  be integers. A necessary and sufficient condition for  $(a^2 - b^2)$  to be an odd number is
- (a) both  $a, b$  are even
  - (b) both  $a, b$  are odd
  - (c)  $a$  even and  $b$  odd or  $a$  odd and  $b$  even
  - (d) None of the above
6. The following system of linear equations
- $$\begin{aligned}2x + 3y - z &= 5 \\x - 2y + 3z &= 7 \\x + 5y - 4z &= 0\end{aligned}$$
- has
- (a) a unique solution
  - (b) no solution
  - (c) infinitely many solutions
  - (d) None of the above
7. The variance of the first  $n$  natural numbers is
- (a)  $n^2/4$
  - (b)  $(n^2 + 1)/8$
  - (c)  $(n^2 - 1)/12$
  - (d) None of the above
8. The number of  $2 \times 2$  matrices whose entries are 1's and -1's will be equal to
- (a) 8
  - (b) 16
  - (c) 32
  - (d) None of the above

9. In a certain language, MACHINE is coded as LBBIHOD. Which one of the following words will be coded as SLTMFNB?

- (a) RKSLEMA
- (b) TKULGMC
- (c) RMSNEOA
- (d) TMUNGMC

10. Find the rank of the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 6 \end{bmatrix}$$

- (a) 1
- (b) 2
- (c) 3
- (d) None of the above

11. When Gauri was born, her mother was 25 years older than her sister and her father was 32 years older than her brother. If Gauri's brother is 6 years older than her and her mother is 3 years younger than her father, how old was Gauri's sister when Gauri was born?

- (a) 10 years
- (b) 8 years
- (c) 7 years
- (d) 5 years

12. The first four moments of a distribution about the value 4 of the variable are -1.5, 17, -30 and 108. The variance is

- (a) 21.0
- (b) 19.5
- (c) 14.75
- (d) None of the above

13. If

$$6\cos 2\theta + 2\cos^2(\theta/2) + 2\sin^2 \theta = 0, \quad 0 < \theta < \pi/2$$

then the value of  $\theta$  is equal to

- (a)  $\pi/3$
- (b)  $\pi/4$
- (c)  $\pi/6$
- (d) None of the above

14. The three words, Historian : Scholar : Researcher are related in some way. Which one of the following options exhibit the same relationship?

- (a) Professor : Lecturer : Teacher
- (b) Story : History : Book
- (c) Morning : Day : Night
- (d) Novel : Book : Epic

15. The vectors  $2\hat{i} + \hat{j} - 2\hat{k}$ ,  $\hat{i} + \hat{j} + 3\hat{k}$  and  $x\hat{i} + \hat{j}$  are coplanar when  $x$  is equal to

- (a)  $8/13$
- (b)  $5/8$
- (c)  $13/8$
- (d)  $8/5$

16. Find the determinant of the skew-symmetric matrix  $A$  defined by

$$A = \begin{pmatrix} \sin x & -\sin(x - \pi/4) & \tan(x - \pi/4) \\ \sin(x - \pi/4) & 0 & \log(x/y) \\ -\tan(x - \pi/4) & \log(y/x) & \tan x \end{pmatrix}$$

- (a)  $\sin x + \tan x$
- (b) 1
- (c) 0
- (d) None of the above

17. An anti-aircraft gun can take a maximum of three shots at an enemy plane moving away from it. The probabilities of hitting the plane in the first, second and third shot are 0.3, 0.2 and 0.1 respectively. The probability that gun hits the plane is

- (a) 0.423
- (b) 0.481
- (c) 0.496
- (d) None of the above

18. Let  $X$  be a random variable such that  $E[|X|] < \infty$ . Then

$$E[|X - c|]$$

is minimized if we choose  $c$  is equal to

- (a) the variance of  $X$
- (b)  $E[X]$
- (c) the median of  $X$
- (d) None of the above

19. Priya walked 10 meters in front and 10 meters to her right. Then every time turning to her left, she walked 5, 15 and 15 meters respectively. How far is she from her starting point?

- (a) 55 meters
- (b) 35 meters
- (c) 25 meters
- (d) 5 meters

20. If  $x$  is real, then the maximum value of

$$\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$$

is

- (a) 40
- (b) 48
- (c) 51
- (d) None of the above

21. The number of equivalence relations on the set

$$S = \{a, b, c\}$$

is

- (a) 6
- (b) 8
- (c) 9
- (d) None of the above

22. Complete the series

$$2, 5, 9, 19, 37, \underline{\hspace{2cm}}.$$

- (a) 76
- (b) 75
- (c) 74
- (d) 72

23. Let  $a, b, c$  be real numbers. Consider the following equalities :

(i)  $\max\{a, b\} = \frac{1}{2}(a + b + |a - b|)$

(ii)  $\min\{a, b\} = \frac{1}{2}(a + b - |a - b|)$

(iii)  $\min\{a, b, c\} = \min\{\min\{a, b\}, c\}$

Among these, identify the number of correct statements

- (a) 0
- (b) 1
- (c) 2
- (d) 3

24. Statements :

All trolleys are pulleys.  
Some pulleys are chains.  
All chains are bells.

Conclusions :

- (i) Some bells are trolleys.
- (ii) No bell is trolley.
- (iii) Some pulleys are bells.
- (iv) All chains are pulleys.

From the statements given above which of the conclusions logically follow?

- (a) Either (i) or (ii)
- (b) Only (iii) and (iv)
- (c) Either (i) and (iii), or (ii) and (iv)
- (d) Either (i) or (ii), and (iii)

25. Among the following series defined as

(i)  $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

(ii)  $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$

(iii)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

Identify the converging series

- (a) Only (i) and (ii)
- (b) Only (i) and (iii)
- (c) Only (ii) and (iii)
- (d) None of the above

26. Given that  $(4373)^2 + 1 = 2(9561, 565)$ . Write the number (9561, 565) as the sum of two squares

- (a)  $(2185)^2 + (2184)^2$
- (b)  $(2180)^2 + (2183)^2$
- (c)  $(2188)^2 + (2189)^2$
- (d)  $(2185)^2 + (2187)^2$

27. Let

$$A = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

and  $C = AB$  be  $2 \times 2$  matrices. Then  $C^k$  will be equal to

(a)  $\begin{bmatrix} 1 & 0 \\ 0 & (-1)^k \end{bmatrix}$

(b)  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(c)  $\begin{bmatrix} (-1)^k & 0 \\ 0 & 0 \end{bmatrix}$

(d) None of the above

28. Let  $\hat{a}$ ,  $\hat{b}$ ,  $\hat{c}$  be vectors such that

$$\hat{a} + \hat{b} + \hat{c} = \hat{0} \quad \text{and} \quad |\hat{a}| = 3, \quad |\hat{b}| = 5, \quad |\hat{c}| = 7$$

Then the angle between  $\hat{a}$  and  $\hat{b}$  is

(a)  $\pi/6$

(b)  $\pi/3$

(c)  $2\pi/3$

(d)  $5\pi/3$

29. An object is moving in the clockwise direction around the unit circle  $x^2 + y^2 = 1$ . As it passes through the point  $(1/2, \sqrt{3}/2)$ , its  $y$ -coordinate is decreasing at the rate of 3 units per second. The rate at which the  $x$ -coordinate changes at this point is

(a)  $\sqrt{3}$

(b)  $2\sqrt{3}$

(c) 3

(d)  $3\sqrt{3}$

30. Let  $\mathcal{R} = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (2, 3), (3, 1)\}$  be a binary relation on the set  $S = \{1, 2, 3\}$ . Then the relation  $\mathcal{R}$  is

- (a) reflexive
- (b) symmetric
- (c) transitive
- (d) equivalence

31. Find all the eigenvalues of the matrix

$$\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

- (a) 1, -1
- (b) 0, -1
- (c) 1, 0
- (d) 0, 0

32. Find the greatest common divisor of 1800 and 756.

- (a) 36
- (b) 32
- (c) 24
- (d) None of the above

33. Consider the Fibonacci function  $F: N \rightarrow N$ , where  $N$  is the set of natural numbers defined by

$$1 \rightarrow F(1) = F_1 = 1$$

$$2 \rightarrow F(2) = F_2 = 1$$

and

$$n \rightarrow F(n) = F_n = F_{n-1} + F_{n-2} \quad \text{for } n > 2$$

Then the Fibonacci function is

- (a) one-to-one
- (b) onto
- (c) both one-to-one and onto
- (d) None of the above

- 34.** The stripes measuring 1 ml in a 1 litre cylindrical kitchen measuring jug is 1 mm wide, if the radius is with an error of no more than 1%. What is the radius of the cylinder?
- (a) 50 mm
  - (b) 60 mm
  - (c) 75 mm
  - (d) 100 mm
- 35.** In a cricket match, five batsmen  $A$ ,  $B$ ,  $C$ ,  $D$  and  $E$  scored an average of 36 runs.  $D$  scored 5 more than  $E$ ,  $E$  scored 8 fewer than  $A$ ,  $B$  scored as much as the combined score of  $D$  and  $E$ , and  $B$  and  $C$  together scored 107. How many runs did  $E$  score?
- (a) 62
  - (b) 45
  - (c) 28
  - (d) 20
- 36.** How many numbers in the range 1000 – 9999 do not have any repeated digits?
- (a) 4653
  - (b) 4435
  - (c) 4365
  - (d) 4536
- 37.** A student must answer exactly eight questions out of ten on a final examination. In how many ways can she choose the questions to answer if she must answer at least three of the last five questions and at most four of the first five?
- (a) 30
  - (b) 32
  - (c) 35
  - (d) 38

38. Among the following statements, identify the true statement(s) :

- (i) Any subgroup of a cyclic group is cyclic.
- (ii) Let  $(H, *)$  be a subgroup of a group  $(G, *)$ . Let  $N = \{x \mid x \in G, xHx^{-1} = H\}$ , where  $x^{-1}$  is the inverse of the element  $x$ . Then  $(N, *)$  is a subgroup of  $(G, *)$ .
- (iii) Let  $(G, *)$  be a group. Let  $H = \{x \in G \mid x*y = y*x \text{ for all } y \in G\}$ . Then  $(H, *)$  is a subgroup of  $(G, *)$ .
- (a) Only (i) and (ii) are true
- (b) Only (i) and (iii) are true
- (c) Only (ii) and (iii) are true
- (d) All of the above are true

39. Find the following sum :

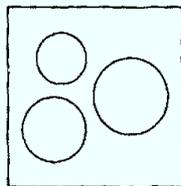
$$\sum_{x=0}^n \binom{M}{x} \binom{N-M}{n-x}$$

- (a)  $\binom{M}{N}$
- (b)  $\binom{N}{M}$
- (c)  $\binom{N}{n}$
- (d) None of the above

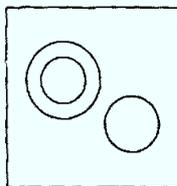
40. Which Venn diagrams best illustrate the relationship among the given two sets of items?

Set 1 : Yak, Zebra, Bear

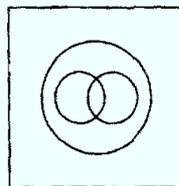
Set 2 : Sun, Moon, Stars



(i)



(ii)



(iii)

- (a) Only (i) and (iii)
- (b) Only (ii) and (iii)
- (c) Only (i) and (ii)
- (d) Only (i)

41. Let the equation of two circles be given as

$$x^2 + y^2 + 2g_1x + 2f_1y = 0 \quad \text{and} \quad x^2 + y^2 + 2g_2x + 2f_2y = 0$$

If they touch each other, then

(a)  $f_1^2 + g_1^2 = f_2^2 + g_2^2$

(b)  $f_2g_1 = f_1g_2$

(c)  $f_1f_2 = g_1g_2$

(d)  $f_1^2 + g_2^2 = f_2^2 + g_1^2$

42. Among the five groups of letters given below, two are different from the remaining three groups of letters :

(i) LEVEL

(ii) FRETFUL

(iii) DRUID

(iv) UELOPE

(v) CALORIC

Which one of the given is true?

(a) Only (i) and (ii)

(b) Only (ii) and (iii)

(c) Only (ii) and (iv)

(d) Only (iv) and (v)

43. If  $ca \equiv cb \pmod{n}$  and  $\gcd(c, n) = d$ , then

(a)  $a \equiv b \pmod{n/d}$

(b)  $a \equiv b \pmod{n}$

(c)  $a \equiv b \pmod{d/n}$

(d)  $a \equiv b \pmod{nd}$

44. The total work done in moving a particle in a force field given by

$$\mathbf{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$$

along the curve  $x = t^2 + 1$ ,  $y = 2t^2$ ,  $z = t^3$  from  $t = 1$  to  $t = 2$  is

- (a) 101  
(b) 202  
(c) 303  
(d) 330
45. Find the root of the equation
- $$\frac{1}{x+1} + \frac{1}{x+5} = \frac{1}{x+2} + \frac{1}{x+4}$$
- (a) -2  
(b) -1/2  
(c) 1/3  
(d) -3
46. For any positive integer  $n > 1$ , the canonical form for the number  $n$  is
- (a)  $n = p_1^{k_1} p_2^{k_2} \dots p_r^{k_r}$ , where  $p_i$ 's are primes and  $k_i$ 's are positive integers  
(b)  $n = p_1 p_2 \dots p_r$ , where  $p_i$ 's are primes  
(c)  $n = 2p + 1$ , where  $p$  is a prime  
(d) None of the above
47. In a certain code, 15789 is written as EGKPT and 2346 is written as ALUR. How is 23549 written in that code?
- (a) ALGUT  
(b) ALEUT  
(c) ALGTU  
(d) ALGRT

48. The vectors  $\mathbf{X}$ ,  $\mathbf{Y}$ ,  $\mathbf{Z}$  issue from a common point and have their heads in a plane. The vector perpendicular to this plane is

- (a)  $(\mathbf{X} \times \mathbf{Y} \times \mathbf{Z})$
- (b)  $(\mathbf{X} \times \mathbf{Y}) + (\mathbf{Z} \times \mathbf{X})$
- (c)  $(\mathbf{X} \times \mathbf{Y}) \times (\mathbf{Y} \times \mathbf{Z}) \times (\mathbf{Z} \times \mathbf{X})$
- (d)  $(\mathbf{X} \times \mathbf{Y}) + (\mathbf{Y} \times \mathbf{Z}) + (\mathbf{Z} \times \mathbf{X})$

49. Let  $z$  be a complex number such that  $|z|=1$  and  $z \neq \pm 1$ . Then all the values of

$$\frac{z}{1-z^2}$$

are

- (a) on the  $X$ -axis
- (b) on the  $Y$ -axis
- (c) not on the  $X$ -axis but on a line parallel to the  $X$ -axis
- (d) None of the above

50. Six fruits—an apple, an orange, a guava, a banana, a papaya and a kiwi are placed in two rows, three fruits in each of the rows. Consider the following information :

- (i) Papaya is not at the end of any row.
- (ii) Banana is second to the left of kiwi.
- (iii) Guava, placed next to papaya is diagonally opposite to banana.
- (iv) Orange is beside kiwi.

Which one of the following statements is true regarding the fruit arrangement?

- (a) Apple and orange are placed diagonally opposite to each other.
- (b) Apple, guava and papaya are placed in the same row.
- (c) Banana is placed opposite to guava.
- (d) None of the above is true.

51. Evaluate

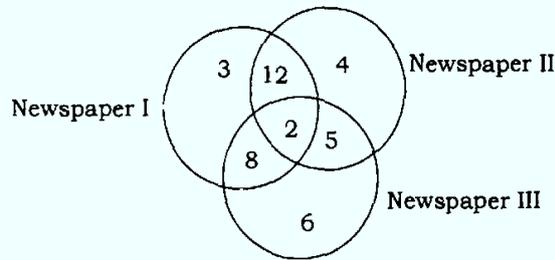
$$\left( e \cdot \lambda \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} \right)$$

- (a)  $e^\lambda$
- (b)  $\lambda$
- (c)  $\lambda e^\lambda$
- (d) None of the above

52. Suppose  $a$  is a rational number and  $b$  is an irrational number. Then  $a + b$  will become

- (a) integer
- (b) rational
- (c) irrational
- (d) complex

53. Consider the Venn diagram below :



The number in the Venn diagram indicates the number of persons reading the newspapers. The diagram is drawn after surveying 50 persons. In a population of 10000, how many can be expected to read at least two newspapers?

- (a) 5000
- (b) 5400
- (c) 4400
- (d) 4000

54. Let  $X = \{a, b, c\}$  and  $Y = \{1, 2, 3, 4\}$  be sets. How many one-to-one functions are there from  $X$  to  $Y$ ?

- (a) 14
- (b) 17
- (c) 20
- (d) None of the above

55. Among the following sequences

(i)  $\lim_{n \rightarrow \infty} \left( \frac{2^n}{n!} \right)$

(ii)  $\lim_{n \rightarrow \infty} \left( \frac{\sin x}{n} \right)$

(iii)  $\lim_{n \rightarrow \infty} ((-1)^n n^2)$

find the converging sequences

(a) Only (i) and (ii)

(b) Only (i) and (iii)

(c) Only (ii) and (iii)

(d) All converge

56. In the series

6 4 1 2 2 8 7 4 2 1 5 3 8 6 2 1 7 1 4 1 3 2 8 6

How many pairs of alternate numbers have a difference of 2?

(a) Five

(b) Four

(c) Three

(d) Two

57. Let  $A, B, C$  be square matrices. Assume that

$$AB = I \text{ and } BC = I$$

where,  $I$  is the identity matrix. Then, the matrix  $A$  should be equal to

(a)  $B$

(b)  $C$

(c) the zero matrix

(d) the identity matrix

58. The points  $(a, b + c)$ ,  $(b, c + a)$  and  $(c, a + b)$

- (a) are vertices of an equilateral triangle
- (b) are vertices of a right angled triangle
- (c) lie on a circle
- (d) None of the above

59. If a line  $OP$  through the origin  $O$  makes angles  $\alpha$ ,  $45^\circ$  and  $60^\circ$  with  $X$ -,  $Y$ - and  $Z$ -axis respectively, then the value of  $\cos\alpha$  is

- (a)  $1/2$
- (b)  $\sqrt{3}/2$
- (c)  $1/\sqrt{2}$
- (d)  $1$

60. Find the value of the trigonometric sum

$$\tan 203^\circ + \tan 22^\circ + \tan 203^\circ \tan 22^\circ$$

- (a)  $-1$
- (b)  $0$
- (c)  $1$
- (d) None of the above

61. How many elements are in the power set of the power set of the empty set?

- (a)  $0$
- (b)  $1$
- (c)  $2$
- (d) None of the above

62. Consider the following limits :

(i)  $\lim_{x \rightarrow 0} f(x)$ , where  $f(x) = \begin{cases} x & \text{when } x \text{ is rational} \\ 0 & \text{when } x \text{ is irrational} \end{cases}$

(ii)  $\lim_{x \rightarrow 0} x \sin(1/x)$

(iii)  $\lim_{x \rightarrow 0} \left( x + \frac{x}{|x|} \right)$

Then

- (a) all of them do not exist
- (b) exactly two of them do not exist
- (c) exactly one of them does not exist
- (d) all of them exist

63. If  $-$  means  $+$ ,  $+$  means  $\times$ ,  $\times$  means  $-$ , and  $\div$  means  $+$ , then which one of the following equations is incorrect?

- (a)  $52 + 4 + 5 \times 8 - 2 = 36$
- (b)  $36 \times 4 - 12 + 5 + 3 = 2$
- (c)  $45 \times 5 + 15 + 8 - 4 = 19$
- (d)  $36 - 3 \times 12 + 4 \div 6 = 54$

64. The value of  $573_8$  is equal to

- (a)  $319_{10}$
- (b)  $359_{10}$
- (c)  $389_{10}$
- (d)  $379_{10}$

65. Let  $T: R^3 \rightarrow R^3$  be a transformation defined by  
for  $(u_1, v_2, v_3)$  in  $R^3$

$$T(u_1, v_2, v_3) = (v_2, v_3, u_1)$$

Then  $T^{100}(u_1, v_2, v_3)$  is equal to

- (a)  $(u_1, v_2, v_3)$   
(b)  $(v_2, v_3, u_1)$   
(c)  $(v_3, u_1, v_2)$   
(d) None of the above
66. Let  $a_1, a_2, \dots$  be the sequence of numbers defined by

$$a_1 = 1, a_2 = 0$$

and

$$a_n = 4a_{n-1} - 4a_{n-2} \text{ for } n > 2$$

Then  $a_{10}$  will become

- (a) -4096  
(b) -4224  
(c) 4672  
(d) 4360
67. A man, a woman, a boy, a girl, a dog and a cat are walking down a long road one after the other. In how many ways can this happen if only the dog is between the man and the boy?
- (a) 40  
(b) 42  
(c) 46  
(d) None of the above
68. In a row of boys, Aryan is eighth from the right and Nilesh is twelfth from the left. When Aryan and Nilesh interchange positions, Nilesh becomes twenty-first from the left. Which will be Aryan's position from the right?
- (a) Seventeenth  
(b) Nineteenth  
(c) Twenty-first  
(d) Thirtieth

69. Let

$$A = \begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix}$$

where  $\omega (\neq 1)$  is a cubic root of unity. Then, the matrix  $A^2$  will become

(a) zero matrix

(b) identity matrix

(c)  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(d) None of the above

70. What is the probability of picking either a spade or a queen from a well-shuffled pack of playing cards?

(a)  $4/13$

(b)  $11/52$

(c)  $7/26$

(d)  $22/52$

71. The value of the integral

$$\int_{-\infty}^{\infty} \exp[-(x-7)^2/32] dx$$

is equal to

(a)  $2\sqrt{\pi}$

(b)  $4\sqrt{2\pi}$

(c)  $7\sqrt{2}$

(d) None of the above

72. How many natural numbers are lying between 20000 and 60000, the sum of the digits being even?

(a) 19998

(b) 19999

(c) 39998

(d) 39999

73. The probability density function (pdf) of a continuous random variable  $X$  is

$$f(x) = \begin{cases} |x|, & |x| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

The variance of  $X$  is

- (a)  $\frac{1}{2}$
- (b) 1
- (c) 2
- (d) 0

74. Which one of the following numeral groups is odd one out?

- (a) 12-144
- (b) 15-180
- (c) 18-198
- (d) 21-252

75. Let  $u$  and  $v$  be differentiable functions of the variables  $x$ ,  $y$  and  $z$ . Show that a necessary and sufficient condition that  $u$  and  $v$  are functionally related by the equation  $F(u, v) = 0$  is

- (a)  $\nabla u \times \nabla v = 0$
- (b)  $\nabla u \cdot \nabla v = 0$
- (c) Both (a) and (b) should be satisfied
- (d) None of the above

76. If every alternate letter starting from  $B$  of the given alphabet is written in small letters, and rest all in capital letters, how will the month of September be written?

- (a) SepteMbeR
- (b) SEptEMbEr
- (c) SEptembER
- (d) sePTeMbeR

77. A fair coin is flipped until head appears for the first time. If this occurs on the  $k$ th trial, the player gets  $2^k$  amount. The expected gain from this game is
- (a) 0
  - (b) 2
  - (c)  $\infty$
  - (d) None of the above

78. Find the number of solutions to the equation

$$x_1 + x_2 + x_3 + x_4 = 13$$

so that  $x_1, x_2, x_3, x_4$  are non-negative integers

- (a) 560
  - (b) 520
  - (c) 490
  - (d) 356
79. For any two integers  $a, b$ , when  $a$  divides  $b$  we denote it by  $a|b$ . For integers  $a, b, c$  and  $d$  consider the following statements :
- (i) If  $a|b$  and  $c|b$ , then  $a \times c|b$
  - (ii) If  $a|b$  and  $c|d$ , then  $a \times c|b \times d$
  - (iii) If  $a|b$  and  $c|\left(\frac{b}{a}\right)$ , then  $c|b$  and  $a|\left(\frac{b}{c}\right)$

Among them, identify the true statements.

- (a) Only (i) and (ii)
- (b) Only (i) and (iii)
- (c) Only (ii) and (iii)
- (d) (i), (ii) and (iii)

80. Suppose  $f : A \rightarrow B$  and  $g : B \rightarrow C$  are functions where  $A, B, C$  are non-empty sets. Let  $g \circ f$  denote the composite function such that  $(g \circ f)(x) = g(f(x))$ . If  $g \circ f$  is one-to-one and  $f$  is onto, then

- (a)  $g$  is one-to-one
- (b)  $g$  is onto
- (c)  $g$  is both one-to-one and onto
- (d) None of the above

81. Consider the four matrices given below :

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 4 & 6 \\ 1 & 5 & 9 \end{bmatrix}, \begin{bmatrix} 2 & 0 & -3 \\ 3 & 1 & 2 \\ -4 & 0 & 6 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

Find the number of matrices whose determinant is zero.

- (a) 1
- (b) 2
- (c) 3
- (d) 4

82. The result of adding the binary numbers

$$11011 \text{ and } 10011$$

will be

- (a) 110010
- (b) 101100
- (c) 101010
- (d) 101110

83. Let  $A, B, C$  be sets. Identify the number of true statements from below :

- (i)  $A \cup B = A \cup C \Rightarrow B = C$
  - (ii)  $A \cup B \subseteq A \cap B \Rightarrow A = B$
  - (iii)  $A \cap B = A \cap C \Rightarrow B = C$
- (a) 0
  - (b) 1
  - (c) 2
  - (d) 3

84. Let  $\{x_n\}$  be the sequence, defined by

$$x_1 = 1, x_2 = 2 \text{ and } x_n = \frac{1}{2}(x_{n-2} + x_{n-1}) \text{ for } n > 2$$

It is given that  $\lim_{n \rightarrow \infty} x_n = x$ .

Then

- (a)  $1 < x < 2$
- (b)  $x > 2$
- (c)  $x = \infty$
- (d)  $x < 0$

85. Define the function  $f : Z \rightarrow Z$  by

$$f(x) = 3x^3 - x$$

where  $Z$  denotes the set of integers. Then  $f$  is

- (a) injective
- (b) surjective
- (c) bijective
- (d) None of the above

86. Let ' $\sim$ ' be an equivalence relation on the Euclidian plane  $R^2$  defined by for  $(x_1, y_1), (x_2, y_2)$  in  $R^2$

$$(x_1, y_1) \sim (x_2, y_2) \text{ if and only if } x_1^2 - y_1^2 = x_2^2 - y_2^2$$

Then the equivalence class of the point  $(0, 0)$  will be

- (a) a pair of straight lines
- (b) a parabola
- (c) an ellipse
- (d) a hyperbola

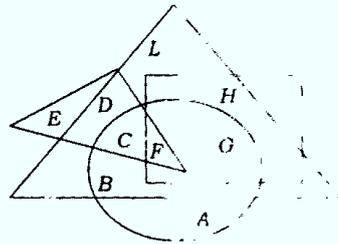
87. Among the following statements, identify the number of the true statements :

- (i) If all diagonal entries of a square matrix  $A$  are zero, then  $A$  is singular.
  - (ii) If  $U$  is an invertible upper triangular matrix, then  $(U^{-1})^T$  is a lower triangular matrix.
  - (iii) If  $A, B$  are invertible matrices, then  $(A + B)$  is always invertible.
- (a) 0
  - (b) 1
  - (c) 2
  - (d) 3

88. Let  $f: R \rightarrow (1, \infty)$  be the function defined by  $f(x) = 3^{2x} + 1$ , where  $R$  is the set of real numbers. Then, the function  $g: (1, \infty) \rightarrow R$  be the inverse of  $f$  when:

- (a)  $g(x) = \log_3(x - 1)$
- (b)  $g(x) = 2\log_3(x - 1)$
- (c)  $g(x) = 3\log_2(x - 1)$
- (d) None of the above

89. In the following figure, the smaller triangle represents the teachers, the big triangle the politicians; the circle, the graduates and the rectangle, the members of parliament:



Who among the following are graduates or teachers but not politicians?

- (a) B, G
  - (b) G, H
  - (c) E, F
  - (d) A, E
90. How many integers between 1 and 300 (inclusive) are divisible by 3 and by 5 but not by 7?
- (a) 16
  - (b) 20
  - (c) 22
  - (d) 18

91. In a group of 100 people, several will have their birthdays in the same month. At least how many must have their birthdays in the same month?

- (a) 7
- (b) 9
- (c) 11
- (d) None of the above

92. Let  $z$  and  $w$  be two non-zero complex numbers such that

$$|z| = |w| \text{ and } \arg(z) + \arg(w) = \pi$$

If  $\bar{w}$  is the complex conjugate of  $w$ , then  $z$  will be equal to

- (a)  $w$
- (b)  $\bar{w}$
- (c)  $-w$
- (d)  $-\bar{w}$

93. A coin is tossed four times. How many times would you expect it falls heads?

- (a) 4
- (b) 3
- (c) 2
- (d) 1

94. Let  $z$  be a complex number such that  $|z|=1$ . Let

$$w = \frac{z-1}{z+1} \text{ (where } z \neq -1\text{)}$$

Then the real part of the complex number  $w$  is equal to

- (a) 0
- (b) 1
- (c)  $\sqrt{2}$
- (d) None of the above

95. Consider the following limits :

(i)  $\lim_{x \rightarrow 0} \frac{1}{x}$

(ii)  $\lim_{x \rightarrow 0} \sin(1/x)$

(iii)  $\lim_{x \rightarrow 0} \frac{x}{|x|}$

Then, the limits of

- (a) all of them do not exist
- (b) exactly two of them do not exist
- (c) exactly one of them does not exist
- (d) all of them exist

96. Let  $a, b$  be real numbers. Consider the inequalities :

(i)  $\left(\frac{1}{2}(a+b)\right)^2 \leq \frac{1}{2}(a^2+b^2)$

(ii)  $a < b \Rightarrow a < \sqrt{ab} < b$

(iii)  $ab > 0 \Leftrightarrow |a+b| = |a| + |b|$

Among them, the correct statements are

- (a) only (i) and (ii)
- (b) only (i) and (iii)
- (c) only (ii) and (iii)
- (d) None of the above

97. For  $k = 1, 2, 3, 4$ , let

$$z_k = \cos\left(\frac{k\pi}{10}\right) + i \sin\left(\frac{k\pi}{10}\right)$$

be complex numbers. Then the product  $z_1 z_2 z_3 z_4$  will be

- (a) 1
- (b) -1
- (c) 0
- (d) None of the above

98. Let the complex numbers  $z_1$ ,  $z_2$  and  $z_3$  be vertices of a parallelogram  $ABCD$ . Then, its fourth vertex is

- (a)  $z_1 + z_2 + z_3$
- (b)  $z_1 + z_2 - z_3$
- (c)  $z_1 - z_2 + z_3$
- (d) None of the above

99. A bus starts from city  $X$ . The number of women in the bus is half of the number of men. In city  $Y$ , as 10 men leave the bus and 5 women enter, the number of men and women in the bus is equal. From city  $X$ , how many passengers started the journey?

- (a) 15
- (b) 24
- (c) 36
- (d) 45

100. Let a point undergoes the following three transformations successively :

- (i) Reflection about the line  $y = x$
- (ii) Translation through a distance of 2 units along the positive direction of the  $x$ -axis
- (iii) Rotation through the angle  $90^\circ$  about the origin in the anticlockwise direction

What is the final position if the given point is  $(4, 1)$ ?

- (a)  $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$
- (b)  $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$
- (c)  $(-\sqrt{2}, 7\sqrt{2})$
- (d)  $(\sqrt{2}, 7\sqrt{2})$

SPACE FOR ROUGH WORK

SPACE FOR ROUGH WORK

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