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| **Question Paper Name:** | Engineering Sciences 12th Feb 2017 |
| **Subject Name:** | Engineering Sciences |
| **Duration:** | 180 |
| **Total Marks:** | 100 |

# Organizing Institute:
# Indian Institute of Technology Roorkee

If

$$\int_0^{\frac{\pi}{\alpha}} \int_x^{\frac{\pi}{\alpha}} \frac{\sin y}{y} \, dy \, dx = \frac{1}{2}$$

for some $\alpha \geq 1$, then the value of $\alpha$ is _____.

Three fair dice are rolled simultaneously. The probability of getting a sum of 5 is

(A) $\frac{1}{108}$　　　　　　(B) $\frac{1}{72}$　　　　　　(C) $\frac{1}{54}$　　　　　　(D) $\frac{1}{36}$

Suppose $\alpha, \beta, \gamma$ and $\delta$ are constants such that
$$p(x) = \delta + \gamma (x + 1) + \beta x(x + 1) + \alpha x (x + 1)(x - 1)$$
is the interpolating polynomial for the data $(-1, -3), (0,1), (1, -1)$, and $(2, -3)$. Then the value of $\gamma - \beta$ is _____.

Consider the ordinary differential equation
$$y'' + \alpha y' + \beta y = 0,$$
where $\alpha$ and $\beta$ are constants. If $y(x) = x e^x$ is a solution of the above equation, then the value of $\beta - \alpha$ is _____.

Consider the system of linear equations
$$2x_2 + x_3 = 0,$$
$$-2x_1 - x_3 = 0,$$
$$-x_1 + x_2 = 1.$$

The above system has

(A) a unique solution
(B) infinite number of solutions
(C) no solution
(D) only two distinct solutions

**Question Number : 6**  **Correct : 1  Wrong : 0**

Let $C$ be a simple smooth closed curve enclosing the region $R$ in the $xy$-plane. Let $C$ be oriented counterclockwise. If the value of the integral

$$\oint_C (y + e^{x^2})dx + (3x + \cos y)\,dy$$

is 16, then the area of $R$ is _____.

**Question Number : 7**  **Correct : 1  Wrong : -0.33**

Consider the ordinary differential equation
$$x^2 y'' + xy' - y = x, \quad x > 0.$$
In terms of arbitrary constants $c_1$ and $c_2$, the general solution of the above equation is

(A) $y(x) = c_1 x + c_2 x^{-1} + x^3$

(B) $y(x) = c_1 x^2 + c_2 x^{-1} + \dfrac{1}{2}x$

(C) $y(x) = c_1 x + c_2 x^{-1} + \dfrac{1}{2}x \ln x$

(D) $y(x) = c_1 x + c_2 + x^{-1}$

**Question Number : 8**  **Correct : 2  Wrong : -0.66**

Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be defined by
$$f(x) = \begin{cases} x(\sin x)\cos\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x\cos\frac{1}{x}, & x \neq 0 \\ 0, & x = 0, \end{cases}$$
where $\mathbb{R}$ denotes the set of real numbers. Then, at $x = 0$,

(A) $f$ is differentiable but $g$ is NOT differentiable

(B) $f$ is NOT differentiable but $g$ is differentiable

(C) both $f$ and $g$ are differentiable

(D) neither $f$ nor $g$ is differentiable

**Question Number : 9**  **Correct : 2  Wrong : 0**

If $u(x,t) = g(t)\sin x$ is the solution of the wave equation
$$u_{tt} = u_{xx}, \qquad t > 0, \qquad 0 < x < \pi,$$
with the initial conditions
$$u(x,0) = 2\sin x, \qquad u_t(x,0) = 0, \qquad 0 \le x \le \pi,$$
and the boundary conditions
$$u(0,t) = u(\pi,t) = 0, \qquad t \ge 0,$$
then the value of $g(\frac{\pi}{3})$ is _____.

**Question Number : 10**  **Correct : 2  Wrong : 0**

Let
$$I = \int_0^1 \frac{1}{1+t}\,dt + \frac{\pi i}{2}\int_0^1 \frac{e^{\frac{i\pi t}{2}}}{1 + e^{\frac{i\pi t}{2}}}\,dt - i\int_0^1 \frac{1}{1+it}\,dt,$$
where $t$ is a real variable and $i = \sqrt{-1}$. The value of $I$ is _____.

**Question Number : 11**  **Correct : 2  Wrong : -0.66**

Let
$$a_k = 2^{-k}\,k^4 \sin k \qquad \text{and} \qquad b_k = 2^{-k^2}\,k\,\sin^2 k$$
for $k = 1,2,\dots$ Then

(A) $\sum_{k=1}^{\infty} a_k$ converges but $\sum_{k=1}^{\infty} b_k$ does NOT converge
(B) $\sum_{k=1}^{\infty} a_k$ does NOT converge but $\sum_{k=1}^{\infty} b_k$ converges
(C) both $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ converge
(D) neither $\sum_{k=1}^{\infty} a_k$ nor $\sum_{k=1}^{\infty} b_k$ converges

# General Aptitude

**Question Number : 166**                                        **Correct : 1  Wrong : -0.33**

The event would have been successful if you _____ able to come.

(A) are                (B) had been                (C) have been                (D) would have been

**Question Number : 167**                                        **Correct : 1  Wrong : -0.33**

There was no doubt that their work was <u>thorough</u>.

Which of the words below is closest in meaning to the underlined word above?

(A) pretty                (B) complete                (C) sloppy                (D) haphazard

**Question Number : 168**  **Correct : 1  Wrong : -0.33**

Four cards lie on a table. Each card has a number printed on one side and a colour on the other. The faces visible on the cards are 2, 3, red, and blue.

Proposition: If a card has an even value on one side, then its opposite face is red.

The cards which MUST be turned over to verify the above proposition are

(A) 2, red  (B) 2, 3, red  (C) 2, blue  (D) 2, red, blue

**Question Number : 169**  **Correct : 1  Wrong : -0.33**

What is the value of $x$ when $81 \times \left(\frac{16}{25}\right)^{x+2} \div \left(\frac{3}{5}\right)^{2x+4} = 144$ ?

(A) 1  (B) -1  (C) -2  (D) Cannot be determined

**Question Number : 170**  **Correct : 1  Wrong : -0.33**

Two dice are thrown simultaneously. The probability that the product of the numbers appearing on the top faces of the dice is a perfect square is

(A) 1/9  (B) 2/9  (C) 1/3  (D) 4/9

**Question Number : 171**  **Correct : 2  Wrong : -0.66**

Bhaichung was observing the pattern of people entering and leaving a car service centre. There was a single window where customers were being served. He saw that people inevitably came out of the centre in the order that they went in. However, the time they spent inside seemed to vary a lot: some people came out in a matter of minutes while for others it took much longer.

From this, what can one conclude?

(A) The centre operates on a first-come-first-served basis, but with variable service times, depending on specific customer needs.
(B) Customers were served in an arbitrary order, since they took varying amounts of time for service completion in the centre.
(C) Since some people came out within a few minutes of entering the centre, the system is likely to operate on a last-come-first-served basis.
(D) Entering the centre early ensured that one would have shorter service times and most people attempted to do this.

A map shows the elevations of Darjeeling, Gangtok, Kalimpong, Pelling, and Siliguri. Kalimpong is at a lower elevation than Gangtok. Pelling is at a lower elevation than Gangtok. Pelling is at a higher elevation than Siliguri. Darjeeling is at a higher elevation than Gangtok.

Which of the following statements can be inferred from the paragraph above?

    i.    Pelling is at a higher elevation than Kalimpong
    ii.    Kalimpong is at a lower elevation than Darjeeling
    iii.    Kalimpong is at a higher elevation than Siliguri
    iv.    Siliguri is at a lower elevation than Gangtok

(A) Only ii      (B) Only ii and iii      (C) Only ii and iv      (D) Only iii and iv

P, Q, R, S, T and U are seated around a circular table. R is seated two places to the right of Q. P is seated three places to the left of R. S is seated opposite U. If P and U now switch seats, which of the following must necessarily be true?

(A) P is immediately to the right of R
(B) T is immediately to the left of P
(C) T is immediately to the left of P or P is immediately to the right of Q
(D) U is immediately to the right of R or P is immediately to the left of T

Budhan covers a distance of 19 km in 2 hours by cycling one fourth of the time and walking the rest. The next day he cycles (at the same speed as before) for half the time and walks the rest (at the same speed as before) and covers 26 km in 2 hours. The speed in km/h at which Budhan walks is

(A) 1      (B) 4      (C) 5      (D) 6

The points in the graph below represent the halts of a lift for durations of 1 minute, over a period of 1 hour.
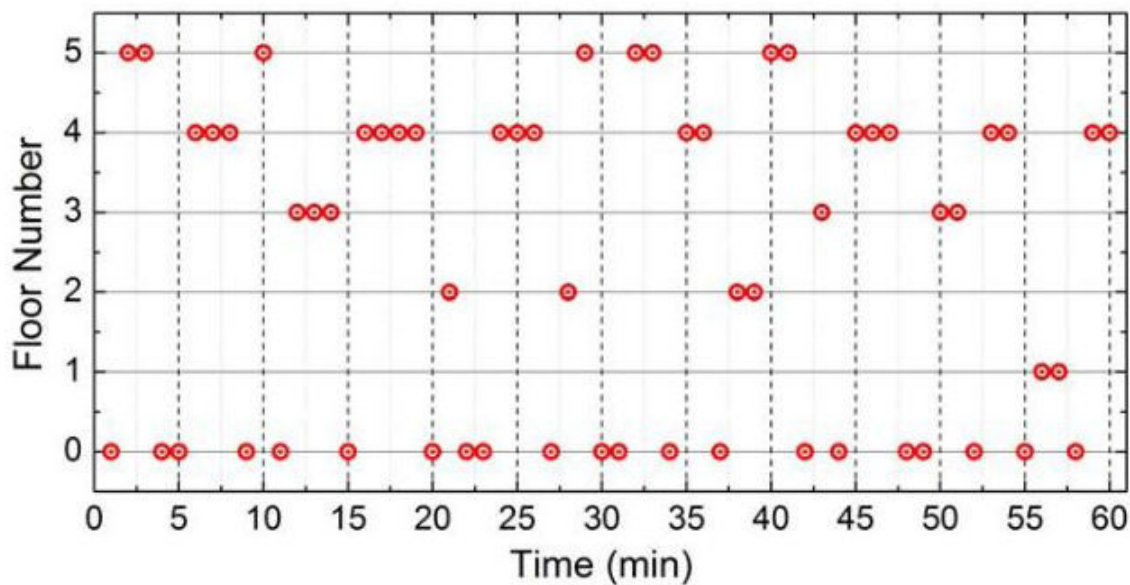


Which of the following statements are correct?

   i.     The elevator never moves directly from any non-ground floor to another non-ground floor over the one hour period

  ii.    The elevator stays on the fourth floor for the longest duration over the one hour period

(A) ) Only i         (B) Only ii         (C) Both i and ii         (D) Neither i nor ii