MATHEMATICS

1. $7^{2Log_7 5}$ is equal to

    1) 5
    2) $Log_7 35$
    3) $Log_7 25$
    4) 25

2. In the group $(G \otimes_{15})$, where $G = \{3, 6, 9, 12\}$, $\otimes_{15}$ is multiplication modulo 15, the identity element is

    1) 6
    2) 3
    3) 9
    4) 12

3. A group $(G *)$ has 10 elements. The minimum number of elements of G, which are their own inverses is

    1) 1
    2) 2
    3) 0
    4) 9

4. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is

    1) 60^0
    2) 120^0
    3) 30^0
    4) 90^0

5. $\dfrac{3x^2+1}{x^2-6x+8}$ is equal to

    1) $\dfrac{49}{2(x-4)} - \dfrac{13}{2(x-2)}$

    2) $3 + \dfrac{49}{2(x-4)} - \dfrac{13}{2(x-2)}$

    3) $\dfrac{49}{2(x-4)} + \dfrac{13}{2(x-2)}$

    4) $\dfrac{-49}{2(x-4)} + \dfrac{13}{2(x-2)}$

(Space for Rough Work)

6. If $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-5\hat{k}$, $\vec{c}=3\hat{i}+5\hat{j}-\hat{k}$, then a vector perpendicular to $\vec{a}$ and in the plane containing $\vec{b}$ and $\vec{c}$ is

1) $17\hat{i}+21\hat{j}-123\hat{k}$

2) $-17\hat{i}+21\hat{j}-97\hat{k}$

3) $-17\hat{i}-21\hat{j}-97\hat{k}$

4) $-17\hat{i}-21\hat{j}+97\hat{k}$

7. $\overrightarrow{OA}$ and $\overrightarrow{BO}$ are two vectors of magnitudes 5 and 6 respectively. If $\underline{|BOA}=60^0$, then $\overrightarrow{OA}\cdot\overrightarrow{OB}$ is equal to

1) 15

2) 0

3) $15\sqrt{3}$

4) -15

8. A vector perpendicular to the plane containing the points $A(1,-1,2)$, $B(2,0,-1)$, $C(0,2,1)$ is

1) $8\hat{i}+4\hat{j}+4\hat{k}$

2) $4\hat{i}+8\hat{j}-4\hat{k}$

3) $\hat{i}+\hat{j}-\hat{k}$

4) $3\hat{i}+\hat{j}+2\hat{k}$

9. $\dfrac{1}{2.5}+\dfrac{1}{5.8}+\dfrac{1}{8.11}+\cdots\cdots\cdots\dfrac{1}{(3n-1)(3n+2)}=$

1) $\dfrac{n}{6n+3}$

2) $\dfrac{n}{6n-4}$

3) $\dfrac{n+1}{6n+4}$

4) $\dfrac{n}{6n+4}$

10. The ninth term of the expansion $\left(3x-\dfrac{1}{2x}\right)^8$ is

1) $\dfrac{-1}{512x^9}$

2) $\dfrac{1}{512x^9}$

3) $\dfrac{1}{256\cdot x^8}$

4) $\dfrac{-1}{256\cdot x^8}$

(Space for Rough Work)

11. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$ and B is the inverse of A, then the value of α is

1) 0

2) 2

3) 4

4) 5

12. If $A = \begin{bmatrix} 0 & x & 16 \\ x & 5 & 7 \\ 0 & 9 & x \end{bmatrix}$ is singular, then the possible values of x are

1) 0, 1, −1

2) 0, +12, −12

3) 0, 5, −5

4) 0, 4, −4

13. If $A = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$, then $A \cdot \text{adj}(A)$ is equal to

1) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

2) $\begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

3) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

4) $\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

14. If $f : R \to R$ is defined by $f(x) = |x|$, then,

1) $f^{-1}(x) = \dfrac{1}{|x|}$

2) $f^{-1}(x) = -x$

3) $f^{-1}(x) = \dfrac{1}{x}$

4) The function $f^{-1}(x)$ does not exist.

15. The value of $\begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix}$ is

1) $(x - p)(x - q)(x + p + q)$

2) $x(x - p)(x - q)$

3) $pq(x - p)(x - q)$

4) $(p - q)(x - q)(x - p)$

(Space for Rough Work)

16. The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y - 24 = 0$ is,

     1)   4   2)   3

     3)   1   4)   2

17. If $3x + y + k = 0$ is a tangent to the circle $x^2 + y^2 = 10$, the values of k are,

     1) ± 5 2) ± 7

     3) ± 9 4) ± 10

18. The negation of the proposition "If 2 is prime, then 3 is odd" is

     1)   2 is prime and 3 is not odd   2)   If 2 is not prime then 3 is not odd

     3)   If 2 is not prime then 3 is odd   4)   2 is not prime and 3 is odd.

19. The equation to two circles which touch the Y-axis at $(0, 3)$ and make an intercept of 8 units on X-axis are

     1) $x^2 + y^2 \pm 6x - 10y + 9 = 0$ 2) $x^2 + y^2 \pm 10x - 6y + 9 = 0$

     3) $x^2 + y^2 + 10x \pm 6y + 9 = 0$ 4) $x^2 + y^2 - 8x \pm 10y + 9 = 0$

20. The orthocentre of the triangle with vertices $A(0, 0)$, $B\left(0, \frac{3}{2}\right)$, $C(-5, 0)$ is

     1) $\left(-\frac{5}{2}, \frac{3}{4}\right)$ 2) $\left(\frac{5}{2}, \frac{3}{4}\right)$

     3) $(0, 0)$ 4) $\left(-5, \frac{3}{2}\right)$

(Space for Rough Work)

21. $x^2 + y^2 - 6x - 6y + 4 = 0$, $x^2 + y^2 - 2x - 4y + 3 = 0$, $x^2 + y^2 + 2kx + 2y + 1 = 0$ If the Radical centre of the above three circles exists, then which of the following cannot be the value of k ?

    1) 1   2) 2

    3) 4   4) 5

22. If the circles $x^2 + y^2 - 2x - 2y - 7 = 0$ and $x^2 + y^2 + 4x + 2y + k = 0$ cut orthogenally, then the length of the common chord of the circles is

    1) 2   2) $12/\sqrt{13}$

    3) 8   4) 5

23. The co-ordinates of the foot of the perpendicular drawn from the point $(3, 4)$ on the line $2x + y - 7 = 0$ is

    1) $(1, 5)$ 2) $\left(\dfrac{9}{5}, \dfrac{17}{5}\right)$

    3) $(1, -5)$ 4) $(-5, 1)$

24. The area enclosed by the pair of lines $xy = 0$, the line $x - 4 = 0$ and $y + 5 = 0$ is

    1) 10 sq. units.   2) 20 sq. units

    3) 0 sq. units.   4) $\dfrac{5}{4}$ sq. units.

25. If the area of the auxillary circle of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $(a > b)$ is twice the area of the ellipse, then the eccentricity of the ellipse is

    1) $\dfrac{\sqrt{3}}{2}$ 2) $\dfrac{1}{\sqrt{2}}$

    3) $\dfrac{1}{2}$ 4) $\dfrac{1}{\sqrt{3}}$

(Space for Rough Work)

26. A graph G has 'm' vertices of odd degree and 'n' vertices of even degree. Then which of the following statements is necessarily true ?

    1) $m + n$ is an even number   2) $m + n$ is an odd number

    3) $m + 1$ is an odd number   4) $n + 1$ is an even number

27. If p is any point on the ellipse $\dfrac{x^2}{36} + \dfrac{y^2}{16} = 1$, and S and S' are the foci, then $PS + PS' =$

    1)   8   2)   4

    3)   12   4)   10

28. The value of $Sin\left[2Cos^{-1} \dfrac{\sqrt{5}}{3} \right]$ is

    1) $\dfrac{2\sqrt{5}}{3}$ 2) $\dfrac{\sqrt{5}}{3}$

    3) $\dfrac{2\sqrt{5}}{9}$ 4) $\dfrac{4\sqrt{5}}{9}$

29. If $\dfrac{x^2}{36} - \dfrac{y^2}{k^2} = 1$ is a hyperbola, then which of the following statements can be true ?

    1) $(3,\ 1)$ lies on the hyperbola   2) $(-3,\ 1)$ lies on the hyperbola

    3) $(5,\ 2)$ lies on the hyperbola   4) $(10,\ 4)$ lies on the hyperbola

30. The focus of the parabola is

    1) $\left(\dfrac{1}{3},\ \dfrac{-3}{2} \right)$ 2) $\left(\dfrac{-1}{3},\ \dfrac{3}{2} \right)$

    3) $\left(\dfrac{1}{3},\ \dfrac{-1}{2} \right)$ 4) $\left(\dfrac{1}{3},\ \dfrac{3}{2} \right)$

(Space for Rough Work)

31. The solution of $Tan^{-1}x + 2Cot^{-1}x = \dfrac{2\pi}{3}$ is

1) $\dfrac{1}{\sqrt{3}}$ 2) $-\dfrac{1}{\sqrt{3}}$

3) $\sqrt{3}$ 4) $-\sqrt{3}$

32. $Sin^2 17.5^0 + Sin^2 72.5^0$ is equal to

1) $Tan^2 45^0$ 2) $Cos^2 90^0$

3) $Sin^2 45^0$ 4) $Cos^2 30^0$

33. The conjugate of the complex number $\dfrac{(1+i)^2}{1-i}$ is
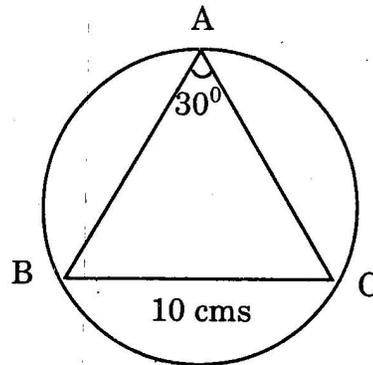
1) $1+i$ 2) $1-i$

3) $-1-i$ 4) $-1+i$

34. ABC is a triangle with $\lfloor A = 30^0$ $BC = 10\,cms$
The area of the circum-circle of the triangle is

1) 5 sq. cms.   2) 100π sq. cms.

3) $\dfrac{100\pi}{3}$ sq. cms.   4) 25 sq. cms.

35. If $Sin\,3\theta = Sin\,\theta$, how many solutions exist such that $-2\pi < \theta < 2\pi$?

1) 9   2) 8

3) 7   4) 5

(Space for Rough Work)

36. The imaginary part of i^i is

     1)   1   2)   0

     3) -1 4)   2

37. The amplitude of $(1+i)^5$ is

     1) $\dfrac{-3\pi}{4}$ 2) $\dfrac{3\pi}{4}$

     3) $\dfrac{5\pi}{4}$ 4) $\dfrac{-5\pi}{4}$

38. ABC is a tringle. G is the centroid. D is the mid point of BC. If $A = (2,\ 3)$ and $G = (7,\ 5)$, then the point D is

     1) $\left(\dfrac{19}{2},\ 6\right)$ 2) $\left(\dfrac{9}{2},\ 4\right)$

     3) $\left(8,\ \dfrac{13}{2}\right)$ 4) $\left(\dfrac{11}{2},\ \dfrac{11}{2}\right)$

39. $\displaystyle \lim_{x \to 1} \dfrac{Tan\left(x^2-1\right)}{x-1}$ is equal to

     1) $\dfrac{1}{2}$ 2)   2

     3) $\dfrac{-1}{2}$ 4) -2

40. If $y = 2^{Log\,x}$, then $\dfrac{dy}{dx}$ is

     1) $2^{Log\,x} \cdot Log\,2$ 2) $\dfrac{2^{Log\,x}}{Log\,2}$

     3) $\dfrac{2^{Log\,x} \cdot Log\,2}{x}$ 4) $\dfrac{2^{Log\,x}}{x}$

(Space for Rough Work)

41. If $Sec^{-1}\left(\dfrac{1+x}{1-y}\right)=a$, then $\dfrac{dy}{dx}$ is

    1) $\dfrac{y+1}{x-1}$ 2) $\dfrac{y-1}{x+1}$

    3) $\dfrac{x-1}{y+1}$ 4) $\dfrac{x-1}{y-1}$

42. If $y=Cos^2\dfrac{3x}{2}-Sin^2\dfrac{3x}{2}$, then $\dfrac{d^2y}{dx^2}$ is

    1) $9y$ 2) $-3\sqrt{1-y^2}$

    3) $3\sqrt{1-y^2}$ 4) $-9y$

43. If the function $f(x)=\begin{cases}\dfrac{1-Cos\,x}{x^2} & for\ x\neq 0 \\ k & for\ x=0\end{cases}$ is continuous at $x=0$, then the value

of k is

    1) 0 2) 1

    3) -1 4) $\dfrac{1}{2}$

44. If $1, w, w^2$ are the cube roots of unity then $(1+w)\left(1+w^2\right)\left(1+w^4\right)\left(1+w^8\right)$ is equal to

    1) 0 2) 1

    3) w 4) w^2

45. If $x^x=y^y$ then $\dfrac{dy}{dx}$ is

    1) $-\dfrac{x}{y}$ 2) $-\dfrac{y}{x}$

    3) $\dfrac{1+Log\,x}{1+Log\,y}$ 4) $1+Log\left(\dfrac{x}{y}\right)$

(Space for Rough Work)

46. The point on the curve $y^2 = x$, the tangent at which makes an angle 45^0 with X-axis is

    1) $\left(\frac{1}{2}, \frac{1}{4}\right)$ 2) $\left(\frac{1}{4}, \frac{1}{2}\right)$

    3) $\left(\frac{1}{2}, \frac{1}{2}\right)$ 4) $\left(\frac{1}{2}, \frac{-1}{2}\right)$

47. The length of the subtangent to the curve $x^2 y^2 = a^4$ at $(-a, a)$ is

    1) $2a$ 2) $\frac{a}{2}$

    3) $\frac{a}{3}$ 4) a

48. The number of positive divisors of 252 is
    1) 5   2) 9
    3) 10   4) 18

49. The remainder obtained when 5^{124} is divided by 124 is
    1) 0   2) 5
    3) 1   4) 2

50. Which of the following is not a group with respect to the given operation ?
    1) The set of odd integers under additon.
    2) The set of even integers under addition.

    3) $\{1, -1\}$ under multiplication.

    4) $\{0\}$ under addition.

(Space for Rough Work)

51. The range in which $y = -x^2 + 6x - 3$ is increasing is

    1) $x > 3$ 2) $x < 3$

    3) $5 < x < 6$ 4) $7 < x < 8$

52. The value of the integral $\int_0^{\pi/2} \left(Sin^{100}x - Cos^{100}x \right) dx$ is

    1) $\dfrac{100\,!}{(100)^{100}}$ 2) $\dfrac{1}{100}$

    3) 0 4) $\dfrac{\pi}{100}$

53. OA and OB are two roads enclosing an angle of 120^0. X and Y start from 'O' at the same time. X travels along OA with a speed of 4 km/hour and Y travels along OB with a speed of 3 km/hour. The rate at which the shortest distance between X and Y is increasing after 1 hour is

    1) 37 km/hour   2) $\sqrt{37}$ km/hour

    3) $\sqrt{13}$ km/hour   4) 13 km/hour

54. If $k\int_0^1 x \cdot f(3x)dx = \int_0^3 t \cdot f(t)dt$, then the value of k is

    1) 3   2) 9

    3) $\dfrac{1}{3}$ 4) $\dfrac{1}{9}$

55. The value of $\int \dfrac{1}{1 + Cos\,8x}\, dx$ is

    1) $\dfrac{Tan\,8x}{8} + C$ 2) $\dfrac{Tan\,2x}{8} + C$

    3) $\dfrac{Tan\,4x}{8} + C$ 4) $\dfrac{Tan\,4x}{4} + C$

(Space for Rough Work)

56. The value of $\int e^x \left(x^5 + 5x^4 + 1 \right) \cdot dx$ is

1) $e^x \cdot x^5 + e^x + C$

2) $e^x \cdot x^5$

3) $5x^4 \cdot e^x$

4) $e^{x+1} \cdot x^5 + C$

57. The value of $\int \dfrac{x^2 + 1}{x^2 - 1} dx$ is

1) $Log\left(\dfrac{x+1}{x-1} \right) + C$

2) $Log\left(\dfrac{x-1}{x+1} \right) + C$

3) $Log\left(x^2 - 1 \right) + C$

4) $x + Log\left(\dfrac{x-1}{x+1} \right) + C$

58. The area bounded by the curve $x = 4 - y^2$ and the Y-axis is

1) 32 sq. units

2) 16 sq. units

3) $\dfrac{16}{3}$ sq. units

4) $\dfrac{32}{3}$ sq. units

59. The differential equation of the family of straight lines whose slope is equal to y-intercept is

1) $(x+1)\dfrac{dy}{dx} + y = 0$

2) $(x+1)\dfrac{dy}{dx} - y = 0$

3) $\dfrac{dy}{dx} = \dfrac{x+1}{y+1}$

4) $\dfrac{dy}{dx} = \dfrac{x-1}{y-1}$

60. The order and degree of the differential equation $\left[1 + \left(\dfrac{dy}{dx} \right)^5 \right]^{\frac{1}{3}} = \dfrac{d^2y}{dx^2}$ are respectively

1) 2, 1

2) 1, 5

3) 2, 3

4) 2, 5

(Space for Rough Work)