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[Q. Booklet Number]

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MATHEMATICS QUESTIONS & ANSWERS

1. If C is the reflecton of A (2, 4) in x-axis and B is the reflection of C in y-axis, then |AB| is

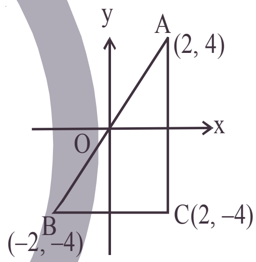
(A) 20 (B) $2\sqrt{5}$ (C) $4\sqrt{5}$ (D) 4

Ans : (C)

Hints : $A \equiv (2,4); C \equiv (2,-4); B \equiv (-2,-4)$

$|AB| = \sqrt{(2-(-2))^2 + (4-(-4))^2} = \sqrt{4^2+8^2}$

$= \sqrt{16+64} = \sqrt{80} = \sqrt{16\times 5} = 4\sqrt{5}$



2. The value of $\cos 15^\circ \cos 7\frac{1^\circ}{2} \sin 7\frac{1^\circ}{2}$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{16}$

Ans : (B)

Hints : $\cos 15^\circ \cos 7\frac{1}{2}^0 \sin 7\frac{1}{2}^0 = \frac{1}{2}\left(2\sin 7\frac{1}{2}^0 \cos 7\frac{1}{2}^0\right).(\cos 15^\circ)$

$\frac{1}{2}(\sin 15^\circ)(\cos 15^\circ) = \frac{1}{4}(2\sin 15^\circ \cos 15^\circ) = \frac{1}{4}\times \sin 30^0 = \frac{1}{8}$

3. The value of integral $\int_{-1}^{1} \frac{|x+2|}{x+2}dx$ is

(A) 1 (B) 2 (C) 0 (D) –1

Ans : (B)

Hints : $I = \int_{-1}^{1} \frac{|x+2|}{x+2}dx$, $x+2 = v \Rightarrow dx = dv$

$\therefore I = \int_{1}^{3} \frac{|v|}{v}dv = \int_{1}^{3} \frac{v}{v}dv = \int_{1}^{3} dv = 2$

4. The line $y = 2t^2$ intersects the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ in real points if

(A) $|t| \le 1$ (B) $|t| < 1$ (C) $|t| > 1$ (D) $|t| \ge 1$

Ans : (A)

Hints : $\frac{x^2}{9}+\frac{y^2}{4}=1$; $y = 2t^2$

$\frac{x^2}{9}+\frac{4t^4}{4}=1 \Rightarrow \frac{x^2}{9}+t^4=1 \Rightarrow x^2=9(1-t^4)$

$x^2 \ge 0 \Rightarrow 9(1-t^4) \ge 0 \Rightarrow t^4-1 \le 0$

$\Rightarrow (t^2-1)(t^2+1) \le 0$

$\Rightarrow t^2-1 \le 0 \;\; (\because t^2+1>0)$

$\Rightarrow |t| \le 1$

5. General solution of $\sin x + \cos x = \min\limits_{a\in IR}\{1, a^2-4a+6\}$ is

(A). $\frac{n\pi}{2}+(-1)^n\frac{\pi}{4}$ (B) $2n\pi+(-1)^n\frac{\pi}{4}$ (C) $n\pi+(-1)^{n+1}\frac{\pi}{4}$ (D) $n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4}$

Ans : (D)

Hints : $\sin x+\cos x=\min\limits_{a\in IR}\{1, a^2-4a+6\}$

$a^2-4a+6=(a-2)^2+2 \;\therefore\; \min\limits_{a\in IR}(a^2-4a+6)=2$

$\therefore \min\limits_{a\in IR}\{1, a^2-4a+6\}=\min\{1,\;2\}=1$

$\sin x+\cos x=1 \Rightarrow \frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x=\frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(x+\frac{\pi}{4}\right)=\sin\frac{\pi}{4}$, $\Rightarrow x+\frac{\pi}{4}=n\pi+(-1)^n.\frac{\pi}{4}$

$\Rightarrow x=n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4}$

6. If A and B square matrices of the same order and AB = 3I, then A^{-1} is equal to

(A) 3B (B) $\frac{1}{3}B$ (C) $3B^{-1}$ (D) $\frac{1}{3}B^{-1}$

Ans : (B)

Hints : $AB=3I,\; A^{-1}.AB=3.A^{-1}I \Rightarrow B=3A^{-1} \Rightarrow A^{-1}=\frac{1}{3}B$

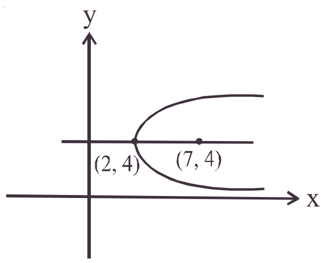
7. The co-ordinates of the focus of the parabola described parametrically by $x = 5t^2 + 2$, $y = 10t + 4$ are

(A) (7, 4) (B) (3, 4) (C) (3, –4) (D) (–7, 4)

Ans : (A)

Hints : $x = 5t^2 + 2$; $y = 10t + 4$, $\left(\frac{y-4}{10}\right)^2 = \left(\frac{x-2}{5}\right)$

or, $(y-4)^2 = 20(x-2)$



8. For any two sets A and B, A – (A – B) equals

(A) B (B) A – B (C) $A \cap B$ (D) $A^C \cap B^C$

Ans : (C)

Hints : $A - (A - B) = A - (A \cap B^c) = A \cap (A \cap B^c)^c = A \cap (A^c \cup B) = (A \cap A^c) \cup (A \cap B) = A \cap B$

9. If $a = 2\sqrt{2}$, $b = 6$, $A = 45^o$, then

(A) no triangle is possible (B) one triangle is possible

(C) two triangle are possible (D) either no triangle or two triangles are possible

Ans : (A)

Hints : $a = 2\sqrt{2}$; $b = 6$; $A = 45^0$

$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \sin B = \frac{b}{a}\sin A$

$\Rightarrow \sin B = \frac{6}{2\sqrt{2}}\sin 45^o = \frac{3}{\sqrt{2}}.\frac{1}{\sqrt{2}} = \frac{3}{2} \Rightarrow$ No triangle is possible since $\sin B > 1$

10. A Mapping from IN to IN is defined as follows :

$f : IN \rightarrow IN$

$f(n) = (n + 5)^2$, $n \in IN$

(IN is the set of natural numbers). Then

(A) f is not one-to-one (B) f is onto

(C) f is both one-to-one and onto (D) f is one-to-one but not onto

Ans : (D)

Hints : $f : IN \rightarrow IN$; $f(n) = (n + 5)^2$

$(n_1 + 5)^2 = (n_2 + 5)^2$

$\Rightarrow (n_1 - n_2)(n_1 + n_2 + 10) = 0$

$\Rightarrow n_1 = n_2 \rightarrow$ one-to-one

There does not exist $n \in IN$ such that $(n + 5)^2 = 1$

Hence f is not onto

11. In a triangle ABC if $\sin A \sin B = \frac{ab}{c^2}$, then the triangle is

(A) equilateral (B) isosceles (C) right angled (D) obtuse angled

Ans : (C)

Hints : $\sin A \sin B = \frac{ab}{c^2}$

$$\Rightarrow c^2 = \frac{ab}{\sin A \sin B} = \left(\frac{a}{\sin A}\right)\left(\frac{b}{\sin B}\right)$$

$$\Rightarrow c^2 = \left(\frac{c}{\sin C}\right)^2 \Rightarrow \sin^2 C = 1 \Rightarrow \sin C = 1 \Rightarrow C = 90^\circ$$

12. $\int \frac{dx}{\sin x + \sqrt{3}\cos x}$ equals

(A) $\frac{1}{2}\ln\left|\tan\left(\frac{x}{2} - \frac{\pi}{6}\right)\right| + c$ (B) $\frac{1}{2}\ln\left|\tan\left(\frac{x}{4} - \frac{\pi}{6}\right)\right| + c$ (C) $\frac{1}{2}\ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)\right| + c$ (D) $\frac{1}{2}\ln\left|\tan\left(\frac{x}{4} + \frac{\pi}{3}\right)\right| + c$

where c is an arbitrary constant

Ans : (C)

Hints : $\int \frac{dx}{\sin x + \sqrt{3}\cos x} = \int \frac{dx}{2\left(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right)} = \frac{1}{2}\int \frac{dx}{\sin\left(x + \frac{\pi}{3}\right)}$

$$= \frac{1}{2}\int \operatorname{cosec}\left(x + \frac{\pi}{3}\right)dx = \frac{1}{2}\log\left|\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)\right| + c$$

$$= \frac{1}{2}\ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)\right| + c$$

13. The value of $\left(1 + \cos \pi/6\right)\left(1 + \cos \pi/3\right)\left(1 + \cos 2\pi/3\right)\left(1 + \cos 7\pi/6\right)$ is

(A) $\frac{3}{16}$ (B) $\frac{3}{8}$ (C) $\frac{3}{4}$ (D) $\frac{1}{2}$

Ans : (A)

Hints : $\left(1 + \cos\frac{\pi}{6}\right)\left(1 + \cos\frac{\pi}{3}\right)\left(1 + \cos\frac{2\pi}{3}\right)\left(1 + \cos\frac{7\pi}{6}\right)$

$$= \left(1 + \frac{\sqrt{3}}{2}\right)\left(1 + \frac{1}{2}\right)\left(1 - \frac{1}{2}\right)\left(1 - \frac{\sqrt{3}}{2}\right) = \left(1 - \frac{3}{4}\right)\left(1 - \frac{1}{4}\right) = \frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$$

14. If $P=\frac{1}{2}\sin^2\theta+\frac{1}{3}\cos^2\theta$ then

(A) $\frac{1}{3}\le P\le\frac{1}{2}$ (B) $P\ge\frac{1}{2}$ (C) $2\le P\le 3$ (D) $-\frac{\sqrt{13}}{6}\le P\le\frac{\sqrt{13}}{6}$

Ans : (A)

Hints : $P=\frac{1}{2}\sin^2\theta+\frac{1}{3}\cos^2\theta=\frac{1}{2}\sin^2\theta+\frac{1}{3}\left(1-\sin^2\theta\right)=\frac{1}{3}+\frac{1}{6}\sin^2\theta$

$0\le\sin^2\theta\le 1\Rightarrow\frac{1}{3}\le\frac{1}{3}+\frac{1}{6}\sin^2\theta\le\frac{1}{3}+\frac{1}{6}$

$\Rightarrow\frac{1}{3}\le P\le\frac{1}{2}$

15. A positive acute angle is divided into two parts whose tangents are $\frac{1}{2}$ and $\frac{1}{3}$. Then the angle is

(A) $\pi/4$ (B) $\pi/5$ (C) $\pi/3$ (D) $\pi/6$

Ans : (A)

Hints : Angle $\theta=\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}}\right)$

$=\tan^{-1}\left(\frac{5/6}{5/6}\right)=\tan^{-1}(1)=\pi/4$

16. If $f(x)=f(a-x)$ then $\int_0^a x\,f(x)dx$ is equal to

(A) $\int_0^a f(x)dx$ (B) $\frac{a^2}{2}\int_0^a f(x)dx$ (C) $\frac{a}{2}\int_0^a f(x)dx$ (D) $-\frac{a}{2}\int_0^a f(x)dx$

Ans : (C)

Hints : $f(x)=f(a-x),\ I=\int_0^a xf(x)dx=\int_0^a (a-x)f(a-x)dx$

$=\int_0^a (a-x)f(x)dx=a\int_0^a f(x)dx-I$

$\therefore 2I=a\int_0^a f(x)dx\Rightarrow I=\frac{a}{2}\int_0^a f(x)dx$

17. The value of $\int_0^{\infty} \frac{dx}{(x^2+4)(x^2+9)}$ is

(A) $\frac{\pi}{60}$ (B) $\frac{\pi}{20}$ (C) $\frac{\pi}{40}$ (D) $\frac{\pi}{80}$

Ans : (A)

Hints : $\int_0^{\infty} \frac{dx}{(x^2+4)(x^2+9)} = \int_0^{\pi/2} \frac{\sec^2\theta}{(\tan^2\theta+4)(\tan^2\theta+9)} d\theta$ (putting $x = \tan\theta$)

$$= \frac{1}{5}\int_0^{\pi/2} \frac{\{(9+\tan^2\theta)-(4+\tan^2\theta)\}\sec^2\theta}{(\tan^2\theta+4)(\tan^2\theta+9)} d\theta$$

$$= \frac{1}{5}\left[\int_0^{\pi/2} \frac{\sec^2\theta}{4+\tan^2\theta} d\theta - \int_0^{\pi/2} \frac{\sec^2\theta}{9+\tan^2\theta} d\theta\right]$$

$$= \frac{1}{5}\left[\frac{1}{2}\tan^{-1}\left(\frac{\tan\theta}{2}\right)\Big|_0^{\pi/2} - \frac{1}{3}\tan^{-1}\left(\frac{\tan\theta}{3}\right)\Big|_0^{\pi/2}\right]$$

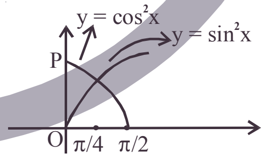
$$= \frac{1}{5}\left[\frac{1}{2}.\frac{\pi}{2} - \frac{1}{3}.\frac{\pi}{2}\right] = \left(\frac{\pi}{2}\right)\left(\frac{1}{5}\right)\left(\frac{1}{2}-\frac{1}{3}\right) = \frac{\pi}{2}.\frac{1}{5}.\frac{1}{6} = \frac{\pi}{60}$$

18. If $I_1 = \int_0^{\pi/4} \sin^2 x dx$ and $I_2 = \int_0^{\pi/4} \cos^2 x dx$, then,

(A) $I_1 = I_2$ (B) $I_1 < I_2$ (C) $I_1 > I_2$ (D) $I_2 = I_1 + \pi/4$

Ans : (B)

Hints : $I_1 = \int_0^{\pi/4} \sin^2 x dx$; $I_2 = \int_0^{\pi/4} \cos^2 x dx$



In $\left(0, \frac{\pi}{4}\right)$, $\cos^2 x > \sin^2 x$ $\therefore \int_0^{\pi/4} \cos^2 x dx > \int_0^{\pi/4} \sin^2 x dx$

$I_2 > I_1$ i.e. $I_1 < I_2$

19. The second order derivative of $a \sin^3 t$ with respect to $a \cos^3 t$ at $t = \frac{\pi}{4}$ is

(A) 2 (B) $\frac{1}{12a}$ (C) $\frac{4\sqrt{2}}{3a}$ (D) $\frac{3a}{4\sqrt{2}}$

Ans : (C)

Hints : $y = a \sin^3 t$; $x = a \cos^3 t$

$\frac{dy}{dt} = 3a\sin^2 t \cos t$; $\frac{dx}{dt} = -3a\cos^2 t \sin t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3a\sin^2 t\cos t}{-3a\cos^2 t\sin t} = -\frac{\sin t}{\cos t} = -\tan t$$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}(-\tan t)=\frac{d}{dt}(-\tan t).\frac{dt}{dx}$$

$$=(-\sec^2 t)\frac{1}{-3a\cos^2 t\sin t}\quad=\frac{1}{+3a\cos^4 t\sin t}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\pi/4}=\frac{1}{3a\left(\frac{1}{\sqrt{2}}\right)^4\cdot\left(\frac{1}{\sqrt{2}}\right)}\quad=\frac{\left(\sqrt{2}\right)^5}{3a}=\frac{4\sqrt{2}}{3a}$$

20. The smallest value of $5\cos\theta+12$ is

(A) 5 (B) 12 (C) 7 (D) 17

Ans : (C)

Hints : $5\cos\theta+12,\ -1\le\cos\theta\le1$

$\Rightarrow -5\le 5\cos\theta\le5$

$\therefore\ 5\cos\theta+12\ \ge -5+12 \Rightarrow 5\cos\theta+12\ \ge 7$

21. The general solution of the differential equation $\frac{dy}{dx}=e^{y+x}+e^{y-x}$ is

(A) $e^{-y}=e^{x}-e^{-x}+c$ (B) $e^{-y}=e^{-x}-e^{x}+c$ (C) $e^{-y}=e^{x}+e^{-x}+c$ (D) $e^{y}=e^{x}+e^{-x}+c$

where c is an arbitrary constant

Ans : (B)

Hints : $e^{-y}dy=(e^{x}+e^{-x})dx$ Integrate

$-e^{-y}=e^{x}-e^{-x}+c,\quad e^{-y}=e^{-x}-e^{+x}+c$

22. Product of any r consecutive natural numbers is always divisible by

(A) r ! (B) (r + 4) ! (C) (r + 1) ! (D) (r + 2) !

Ans : (A)

Hints : (n + 1) (n + 2) .... (n + r)

$$=\frac{(n+r)!}{n!}$$

$$=\frac{(n+r)!}{n!\,r!}\ r!=r!\ {}^{n+r}C_n$$

23. The integrating factor of the differential equation $x\log x\frac{dy}{dx}+y=2\log x$ is given by

(A) e^x (B) log x (C) log (log x) (D) x

Ans : (B)

Hints : $\frac{dy}{dx}+\frac{1}{x\log x}.y=\frac{2}{x}$

$$\text{If}=e^{\int\frac{1}{x\log x}dx}=e^{\int\frac{1/x}{\log x}dx}$$

$$=e^{\log(\log x)}=\log x$$

24. If $x^2 + y^2 = 1$ then

(A) $yy'' - (2y')^2 + 1 = 0$ (B) $yy'' + (y')^2 + 1 = 0$ (C) $yy'' - (y')^2 - 1 = 0$ (D) $yy'' + (2y')^2 + 1 = 0$

Ans : (B)

Hints : $2x + 2yy' = 0$

$x + yy' = 0$

$1 + yy'' + (y')^2 = 0$

25. If $c_0, c_1, c_2, ..., c_n$ denote the co-efficients in the expansion of $(1 + x)^n$ then the value of $c_1 + 2c_2 + 3c_3 + ... + nc_n$ is

(A) $n.2^{n-1}$ (B) $(n + 1)2^{n-1}$ (C) $(n + 1) 2^n$ (D) $(n + 2) 2^{n-1}$

Ans. (A)

Hints : $(1+x)^n = c_0 + xc_1 + x^2c_2 + ...x^nc_n$

$n(1+x)^{n-1} = c_1 + 2xc_2 + ...nx^{n-1}c_n$

Put x = 1

$n(2)^{n-1} = c_1 + 2c_2 + 3c_3 ....\ .... + nc_n$

26. A polygon has 44 diagonals. The number of its sides is

(A) 10 (B) 11 (C) 12 (D) 13

Ans : (B)

Hints : ${}^nC_2 - n = 44$

$\frac{n(n-1)}{2} - n = 44$

$n\left[\frac{n-1}{2} - 1\right] = 44$

$n(n-3) = 88$

$n(n-3) = 11 \times 8$

$n = 11$

27. If α, β be the roots of $x^2 - a(x - 1) + b = 0$, then the value of $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} + \frac{2}{a+b}$

(A) $\frac{4}{a+b}$ (B) $\frac{1}{a+b}$ (C) 0 (D) –1

Ans : (C)

Hints : $x^2 - ax = a + 3$ $\alpha\beta = a + b$

$\alpha + \beta = a$

$\alpha^2 - a\alpha = -(a+b)$

$\beta^2 - a\alpha = -(a+b)$

$-\frac{1}{a+b} - \frac{1}{a+b} + \frac{2}{a+b} = 0$

28. The angle between the lines joining the foci of an ellipse to one particular extremity of the minor axis is 90º. The eccentricity of the ellipse is

(A) $\frac{1}{8}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\sqrt{\frac{2}{3}}$ (D) $\sqrt{\frac{1}{2}}$

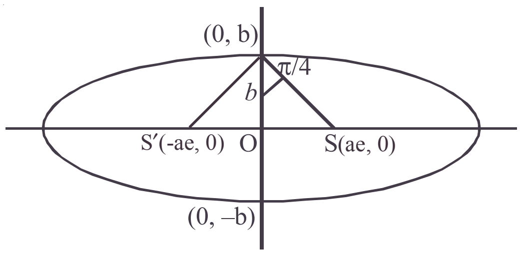
Ans : (D)

Hints : $\frac{b}{ae} = \tan\frac{\pi}{4}$

$b = ae \Rightarrow \frac{b}{a} = e$

$e^2 = 1 - \frac{b^2}{a^2}$

$e^2 = 1 - e^2$



$e^2 = \frac{1}{2} \Rightarrow e = \frac{1}{\sqrt{2}}$

29. The order of the differential equation $\frac{d^2y}{dx^2} = \sqrt{1 - \left(\frac{dy}{dx}\right)^2}$ is

(A) 3 (B) 2 (C) 1 (D) 4

Ans : (B)

30. The sum of all real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$

(A) 7 (B) 4 (C) 1 (D) 5

Ans : (B)

Hints : Put $1x - 21 = y$

$y^2 + y - 2 = 0$

$(y-1)(y+2) = 0$

$y = 1$ $\quad$ $y = -2$

$|x-2| = 1$ $\quad$ (Not possible)

$x - 2 = \pm 1$

$x = 2 \pm 1$

$x = 3, 1$

Sum $= 4$

31. If $\int_{-1}^{4} f(x)dx = 4$ and $\int_{2}^{4} \{3 - f(x)\}dx = 7$ then the value of $\int_{-1}^{2} f(x)dx$

(A) –2 (B) 3 (C) 4 (D) 5

Ans : (D)

Hints : $\int_{-1}^{4} f(x)\,dx = 4$

$3(4-2) - \int_{2}^{4} f(x)dx = 7$

$\int_{2}^{4} f(x)dx = -1$

$\int_{-1}^{2} f(x)dx = \int_{-1}^{4} f(x)dx + \int_{4}^{2} f(x)dx = 4 - \int_{2}^{4} f(x)dx = 4 - (-1) = 5$

32. For each $n \in N$, $2^{3n} - 1$ is divisible by

(A) 7 (B) 8 (C) 6 (D) 16

where N is a set of natural numbers

Ans : (A)

Hints : $2^{3n} = (8)^n = (1+7)^n = 1 + {}^nC_1 7 + {}^nC_2 7^2 \ldots + {}^nC_n 7^n$

$2^{3n} - 1 = 7[{}^nC_1 + {}^nC_2 7 + \ldots]$

33. The Rolle's theorem is applicable in the interval $-1 \le x \le 1$ for the function

(A) $f(x) = x$ (B) $f(x) = x^2$ (C) $f(x) = 2x^3 + 3$ (D) $f(x) = |x|$

Ans : (B)

Hints : $f(x) = x^2$ | and $f(1) = f(-1)$ for $f(x) = |x|$ but at $x = 0$, $f(x) = |x|$ is not differentiable hence (B) is the correct option.

$f(1) = 1 = f(-1)$

34. The distance covered by a particle in t seconds is given by $x = 3 + 8t - 4t^2$. After 1 second velocity will be

(A) 0 unit/second (B) 3 units/second (C) 4 units/second (D) 7 units/second

Ans : (A)

Hints : $v = \frac{dx}{dt} = 8 - 8t$

$t = 1, v = 8 - 8 = 0$

35. If the co-efficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ be same, then the value of 'a' is

(A) $\frac{3}{7}$ (B) $\frac{7}{3}$ (C) $\frac{7}{9}$ (D) $\frac{9}{7}$

Ans : (D)

Hints : $(3+ax)^9 = {}^9C_0 3^9 + {}^9C_1 3^8 (ax) + {}^9C_2 3^7 (ax)^2 + {}^9C_3 3^6 (ax)^3$

${}^9C_2 3^7 a^2 = {}^9C_3 3^6 a^3$

$\frac{9}{7} = a$

36. The value of $\left(\frac{1}{\log_3 12} + \frac{1}{\log_4 12}\right)$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2

Ans : (C)

Hints : $\log_{12} 3 + \log_{12} 4 = \log_{12} 12 = 1$

37. If $x = \log_a bc$, $y = \log_b ca$, $z = \log_c ab$, then the value of $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$ will be

(A) $x + y + z$ (B) 1 (C) $ab + bc + ca$ (D) abc

Ans : (B)

Hints : $1 + x = \log_a a + \log_a bc = \log_a abc$

$\frac{1}{1+x} = \log_{abc} a$, Similarly $\frac{1}{1+y} = \log_{abc} b$

$\frac{1}{1+z} = \log_{abc} c$, Ans. $= \log_{(abc)} abc = 1$

38. Using binomial theorem, the value of $(0.999)^3$ correct to 3 decimal places is

(A) 0.999 (B) 0.998 (C) 0.997 (D) 0.995

Ans : (C)

Hints : ${}^3C_0 - {}^3C_1(.001) + {}^3C_2(.001)^2 - {}^3C_3(.001)^3$

$= 1 - .003 + 3(.000001) - (.000000001) = 0.997$

39. If the rate of increase of the radius of a circle is 5 cm/.sec., then the rate of increase of its area, when the radius is 20 cm, will be

(A) 10π (B) 20π (C) 200π (D) 400π

Ans : (C)

Hints : $A = \pi r^2$ $\quad \frac{dr}{dt} = 5$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt} = 2\pi\, 20\,(5)$

$= 200\,\pi$

40. The quadratic equation whose roots are three times the roots of $3ax^2 + 3bx + c = 0$ is

(A) $ax^2 + 3bx + 3c = 0$ (B) $ax^2 + 3bx + c = 0$ (C) $9ax^2 + 9bx + c = 0$ (D) $ax^2 + bx + 3c = 0$

Ans : (A)

Hints : $3a\alpha^2 + 3b\alpha + c = 0$

$x = 3\alpha \Rightarrow \alpha = \frac{x}{3}$

$3a\frac{x^2}{9} + 3b.\frac{x}{3} + c = 0$

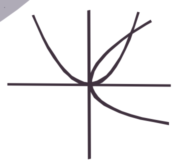
$ax^2 + 3bx + 3c = 0$

41. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

(A) $2\tan^{-1}\left(\frac{3}{4}\right)$ (B) $\tan^{-1}\left(\frac{4}{3}\right)$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

Ans : (C)

Hins : Angle between axes (since co-ordinate axes are the tangents for the given curve).



42. In triangle ABC, a = 2, b = 3 and $\sin A = \frac{2}{3}$, then B is equal to

(A) 30º (B) 60º (C) 90º (D) 120º

Ans : (C)

Hints : $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\sin B = \frac{b}{a}.\sin A = \frac{3}{2}\cdot\frac{2}{3} = 1$

$B = \frac{\pi}{2}$

43. $\int_0^{1000} e^{x-[x]}$ is equal to

(A) $\frac{e^{1000}-1}{e-1}$ (B) $\frac{e^{1000}-1}{1000}$ (C) $\frac{e-1}{1000}$ (D) $1000\,(e-1)$

Ans : (D)

Hins : $I = 1000\int_0^1 e^{x-[x]}$

$= 1000\int_0^1 e^x dx = 1000(e^x)_0^1 = 100(e-1)$

Period of function is 1

44. The coefficient of x^n, where n is any positive integer, in the expansion of $(1 + 2x + 3x^2 + \ldots\ldots\ldots \infty)^{½}$ is

(A) 1 (B) $\frac{n+1}{2}$ (C) $2n + 1$ (D) $n + 1$

Ans : (A)

Hints :

$$\begin{aligned} s &= 1 + 2x + 3x^2 \ldots\ldots\ldots\ldots\infty \\ xs &= x + 2x^2 + \ldots\ldots\ldots\ldots\infty \\ \hline s(1-x) &= 1 + x + x^2 + \ldots\ldots\ldots\infty \end{aligned}$$

$s = \frac{1}{(1-x)^2}$

$f(x) = \frac{1}{1-x}$, $f(x) = (1-x)^{-1} = 1 + x + x^2 + x^3 \ldots\ldots\ldots\ldots\ldots = 1$

45. The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = a^2$ intersect at two distinct points if

(A) $a < 2$ (B) $2 < a < 8$ (C) $a > 8$ (D) $a = 2$

Ans. (B)

Hints : $C_1(5, 0)$ $r_1 = \sqrt{25-16} = 3$

$C_2(0, 0)$ $r_2 = a$

$r_1 \,\&\, r_2 < C_1C_2 < r_1 + r_2$

$|a-3| < \sqrt{25} < a + 3$

$|a-3| < 5 < a + 3$

$-5 < a - 3 < 5$ $\quad 2 < a$

$-2 < a < 8$

$2 < a < 8$

46. $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$ is equal to

(A) $\log(\sin^{-1} x) + c$ (B) $\frac{1}{2}(\sin^{-1} x)^2 + c$ (C) $\log\left(\sqrt{1-x^2}\right) + c$ (D) $\sin(\cos^{-1} x) + c$

where c is an arbitrary constant

Ans : (B)

Hints : $I = \int t\,dt$ $\qquad \sin^{-1} x = t$

$= \frac{1}{2}t^2 + c$ $\qquad \frac{1}{\sqrt{1-x^2}} dx = dt$

$= \frac{1}{2}(\sin^{-1} x)^2 + c$

47. The number of points on the line $x + y = 4$ which are unit distance apart from the line $2x + 2y = 5$ is

(A) 0 (B) 1 (C) 2 (D) Infinity

Ans : (A)

Hints : $x + y = 4$

$x + y = \frac{5}{2}$

$PQ = \frac{4 - 5/2}{\sqrt{2}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$

48. Simplest form of $\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4x}}}}$ is

(A) $\sec\frac{x}{2}$ (B) $\sec x$ (C) $\operatorname{cosec} x$ (D) 1

Ans : (A)

Hints : $\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2.2\cos^2 2x}}}} = \frac{2}{\sqrt{2+\sqrt{2+2\cos 2x}}} = \frac{2}{\sqrt{2+\sqrt{2.2\cos^2 x}}}$

$= \frac{2}{\sqrt{2+2\cos x}} = \frac{2}{2\cos\frac{x}{2}} = \sec\frac{x}{2}$

49. If $y = \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ is

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) –1

Ans : (A)

Hints : $y = \tan^{-1}\sqrt{\dfrac{1-\cos\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}}$

$$= \tan^{-1}\sqrt{\frac{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}} = \tan^{-1}\left|\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right| = \left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$\frac{dy}{dx} = -\frac{1}{2}$$

50. If three positive real numbers a, b, c are in A.P. and $abc = 4$ then minimum possible value of b is

(A) $2^{3/2}$ (B) $2^{2/3}$ (C) $2^{1/3}$ (D) $2^{5/2}$

Ans : (B)

Hints : $(b - d)\, b\, (b + d) = 4$

$(b^2 - d^2)\, b = 4$

$b^3 = 4 + d^2 b$

$b^3 \geq 4 \Rightarrow b \geq (2)^{2/3}$

51. If $5\cos 2\theta + 2\cos^2\frac{\theta}{2} + 1 = 0$, when $(0 < \theta < \pi)$, then the values of θ are :

(A) $\frac{\pi}{3} \pm \pi$ (B) $\frac{\pi}{3}, \cos^{-1}\left(\frac{3}{5}\right)$ (C) $\cos^{-1}\left(\frac{3}{5}\right) \pm \pi$ (D) $\frac{\pi}{3}, \pi - \cos^{-1}\left(\frac{3}{5}\right)$

Ans : (D)

Hints : $5\cos 2\theta + 1 + \cos\theta + 1 = 0$

$5(2\cos^2\theta - 1) + \cos\theta + 2 = 0$

$10\cos^2\theta + \cos\theta - 3 = 0$

$(5\cos\theta + 3)(2\cos\theta - 1) = 0$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

$\cos\theta = -\frac{3}{5}$

$\theta = \cos^{-1}\left(-\frac{3}{5}\right)$

$= \pi - \cos^{-1}\left(\frac{3}{5}\right)$

52. For any complex number z, the minimum value of $|z| + |z-1|$ is

(A) 0 (B) 1 (C) 2 (D) –1

Ans : (B)

Hints : $1 = |z - (z-1)|$

$1 \leq |z| + |z-1|$

53. For the two circles $x^2 + y^2 = 16$ and $x^2 + y^2 - 2y = 0$ there is / are

(A) one pair of common tangents (B) only one common tangent

(C) three common tangents (D) no common tangent

Ans : (D)

Hints : $C_1\,(0, 0)$ $r_1 = 4$

$C_2\,(0, 1)$ $r_2 = \sqrt{0+1} = 1$

$C_1C_2 = \sqrt{0+1} = 1$

$r_1 - r_2 = 3$

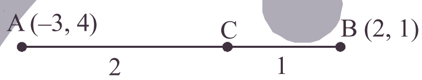
$C_1C_2 < r_1 - r_2$

54. If C is a point on the line segment joining A (–3, 4) and B (2, 1) such that AC = 2BC, then the coordinate of C is

(A) $\left(\frac{1}{3}, 2\right)$ (B) $\left(2, \frac{1}{3}\right)$ (C) (2, 7) (D) (7, 2)

Ans : (A)

Hints :



$C\left(\frac{4-3}{3}, \frac{2+4}{3}\right)$

$C\left(\frac{1}{3}, 2\right)$

55. If a, b, c are real, then both the roots of the equation $(x-b)(x-c) + (x-c)(x-a) + (x-a)(x-b) = 0$ are always

(A) positive (B) negative (C) real (D) imaginary

Ans : (C)

Hints : $3x^2 - 2x(a+b+c) + ab + bc + ca = 0$

$D = 4(a+b+c)^2 - 4.3(ab+bc+ca)$

$= 4(a^2 + b^2 + c^2 - ab - bc - ca)$

$= 2[(a-b)^2 + (b-c)^2 + (c-a)^2]$

$= [(a-b)^2 + (b-c)^2 + (c-a)^2]$

≥ 0

56. The sum of the infinite series $1 + \frac{1}{2!} + \frac{1.3}{4!} + \frac{1.3.5}{6!} + ...$ is

(A) e (B) e^2 (C) $\sqrt{e}$ (D) $\frac{1}{e}$

Ans : (C)

Hints : $T_n = \frac{1.3.5....(2n-1)}{\lfloor 2n}$

$= \dfrac{\lfloor 2n}{\lfloor 2n(2.4...2n)}$

$= \dfrac{\lfloor 2n}{2^n \lfloor n \lfloor 2n}$

$= \dfrac{x^n}{\lfloor n}$ $\qquad \dfrac{1}{2} = x$

$\therefore \dfrac{x}{\lfloor 1} + \dfrac{x^2}{\lfloor 2} + ... = e^x - 1$

$\exp = 1 + e^x - 1 = e^x = e^{1/2}$

57. The point (–4, 5) is the vertex of a square and one of its diagonals is $7x - y + 8 = 0$. The equation of the other diagonal is

(A) $7x - y + 23 = 0$ (B) $7y + x = 30$ (C) $7y + x = 31$ (D) $x - 7y = 30$

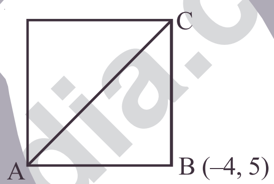
Ans : (C)

Hints : $x + 7y = k$ ......(1)

$-4 + 35 = k$

$31 = k$

$x + 7y - 31 = 0$



58. The domain of definition of the function $f(x) = \sqrt{1 + \log_e(1 - x)}$ is

(A) $-\infty < x \le 0$ (B) $-\infty < x \le \dfrac{e-1}{e}$ (C) $-\infty < x \le 1$ (D) $x \ge 1 - e$

Ans : (B)

Hints : $1 - x > 0 \Rightarrow x < 1$

$1 + \log_e(1 - x) \ge 0$

$\log_e(1 - x) \ge -1 \Rightarrow 1 - x \ge e^{-1}$

$x \le 1 - \dfrac{1}{e}$

$x \le \dfrac{e-1}{e}$

59. For what value of m, $\dfrac{a^{m+1} + b^{m+1}}{a^m + b^m}$ is the arithmetic mean of 'a' and 'b'?

(A) 1 (B) 0 (C) 2 (D) None

Ans : (B)

Hints : $\dfrac{a^{m+1} + b^{m+1}}{a^m + b^m} = \dfrac{a+b}{2}$

$m = 0$ Satisfy.

60. The value of the limit $\lim_{x \to 1} \frac{\sin(e^{x-1} - 1)}{\log x}$ is

(A) 0 (B) e (C) $\frac{1}{e}$ (D) 1

Ans : (D)

Hints : $\underset{h \to 0}{\text{Lt}} \frac{\sin(e^h - 1)}{\log(1+h)}$ Put $x = 1 + h$

$= \underset{h \to 0}{\text{Lt}} \frac{\sin(e^h - 1)}{(e^h - 1)} \cdot \frac{(e^h - 1)}{\log(1+h)}$

$= \underset{h \to 0}{\text{Lt}} \frac{\sin(e^h - 1)}{(e^h - 1)} \cdot \frac{(e^h - 1)}{h} \cdot \frac{h}{\log(1+h)}$

$= 1.\,1.\,1$

$= 1$

61. Let $f(x) = \frac{\sqrt{x+3}}{x+1}$ then the value of $\underset{x \to -3-0}{\text{Lt}} f(x)$ is

(A) 0 (B) does not exist (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Ans : (B)

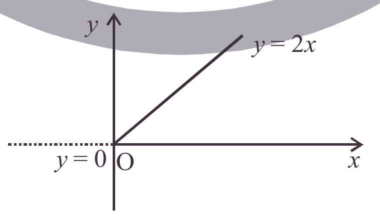
Hints : Because on left hand side of 3 function is not defined.

62. $f(x) = x + |x|$ is continuous for

(A) $x \in (-\infty, \infty)$ (B) $x \in (-\infty, \infty) - \{0\}$ (C) only $x > 0$ (D) no value of x

Ans : (A)

Hints : $f(x) = \begin{cases} 2x\,; & x \geq 0 \\ 0\,; & x < 0 \end{cases}$



63. $\tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right]$ is equal to

(A) $\frac{2a}{b}$ (B) $\frac{2b}{a}$ (C) $\frac{a}{b}$ (D) $\frac{b}{a}$

Ans : (B)

Hints : Let $\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right) = \theta$, then $\cos 2\theta = \frac{a}{b}$

$$\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right]+\tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right]$$

$$=\tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)=2\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)=\frac{2}{\cos 2\theta}=\frac{2}{a/b}=\frac{2b}{a}$$

64. If $i=\sqrt{-1}$ and n is a positive integer, then $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is euqal to

(A) 1 (B) i (C) i^n (D) 0

Ans : (D)

Hints : $i^n(1+i+i^2+i^3)=i^n(1+i-1-i)=0$

65. $\int \frac{dx}{x(x+1)}$ equals

(A) $\ln\left|\frac{x+1}{x}\right|+c$ (B) $\ln\left|\frac{x}{x+1}\right|+c$ (C) $\ln\left|\frac{x-1}{x}\right|+c$ (D) $\ln\left|\frac{x-1}{x+1}\right|+c$

where c is an arbitrary constant.

Ans : (B)

Hints : $\int\frac{dx}{x(x+1)}=\int\left(\frac{1}{x}-\frac{1}{x+1}\right)dx=\int\frac{dx}{x}-\int\frac{dx}{x+1}=\ln|x|-\ln|x+1|+C=\ln\left|\frac{x}{x+1}\right|+C$

66. If a, b, c are in G.P. ($a>1, b>1, c>1$), then for any real number x (with $x>0, x\neq 1$), $\log_a x, \log_b x, \log_c x$ are in

(A) G.P. (B) A.P. (C) H.P. (D) G.P. but not in H.P.

Ans : (C)

Hints : a, b, c are in G.P.

$\Rightarrow \log_x a, \log_x b, \log_x c$ are in A.P.

$\Rightarrow \frac{1}{\log_x a}, \frac{1}{\log_x b}, \frac{1}{\log_x c}$ are in H.P.

$\Rightarrow \log_a x, \log_b x, \log_c x$ are in H.P.

67. A line through the point A (2, 0) which makes an angle of 30º with the positive direction of x-axis is rotated about A in clockwise direction through an angle 15º. Then the equation of the straight line in the new position is

(A) $(2-\sqrt{3})x+y-4+2\sqrt{3}=0$ (B) $(2-\sqrt{3})x-y-4+2\sqrt{3}=0$

(C) $(2-\sqrt{3})x-y+4+2\sqrt{3}=0$ (D) $(2-\sqrt{3})x+y+4+2\sqrt{3}=0$

Ans : (B)

Hints : Equation of line in new position :

$y-0=\tan 15^\circ(x-2)$

$\Rightarrow y=\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)(x-2)$

$\Rightarrow y=\frac{(\sqrt{3}-1)^2}{2}(x-2)$

$\Rightarrow 2y = (4-2\sqrt{3})(x-2)$

$\Rightarrow y = (2-\sqrt{3})(x-2)$

$\Rightarrow (2-\sqrt{3})x - y - 4 + 2\sqrt{3} = 0$

68. The equation $\sqrt{3}\sin x + \cos x = 4$ has

(A) only one solution (B) two solutions (C) infinitely many solutions (D) no solution

Ans : (D)

Hints : $\sqrt{3}\sin x + \cos x = 2\sin\left(x+\frac{\pi}{6}\right) \le 2$. Therefore

$\sqrt{3}\sin x + \cos x = 4$ cannot have a solution

69. The slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx} = 3x^2$ and it passes through (–1 , 1). The equation of the curve is

(A) $y = x^3 + 2$ (B) $y = -x^3 - 2$ (C) $y = 3x^3 + 4$ (D) $y = -x^3 + 2$

Ans : (A)

Hints : $\frac{dy}{dx} = 3x^2 \Rightarrow \int dy = \int 3x^2 dx \Rightarrow y = x^3 + C$

Curve passes through (–1, 1). Hence $1 = -1 + C \Rightarrow C = 2$

$\therefore y = x^3 + 2$

70. The modulus of $\frac{1-i}{3+i} + \frac{4i}{5}$ is

(A) $\sqrt{5}$ unit (B) $\frac{\sqrt{11}}{5}$ unit (C) $\frac{\sqrt{5}}{5}$ unit (D) $\frac{\sqrt{12}}{5}$ unit

Ans : (C)

Hints : $\frac{1-i}{3+i} + \frac{4i}{5} = \frac{5-5i+4i(3+i)}{5(3+i)} = \frac{5-5i+12i-4}{5(3+i)} = \frac{1+7i}{5(3+i)} = \frac{(1+7i)(3-i)}{5(9+1)}$

$= \frac{3+21i-i+7}{5\times 10} = \frac{10+20i}{5\times 10} = \frac{1+2i}{5}$

$\therefore$ Modulus $= \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = \sqrt{\frac{1}{25}+\frac{4}{25}} = \sqrt{\frac{1}{5}} = \frac{\sqrt{5}}{5}$ unit

71. The equation of the tangent to the conic $x^2 - y^2 - 8x + 2y + 11 = 0$ at (2, 1) is

(A) $x + 2 = 0$ (B) $2x + 1 = 0$ (C) $x + y + 1 = 0$ (D) $x - 2 = 0$

Ans : (D)

Hints : Equation of tangent at (x_1, y_1) is

$xx_1 - yy_1 - 4(x + x_1) + (y + y_1) + 11 = 0$

$x_1 = 2; y = 1$

$\therefore$ Equation of tangent is

$2x - y - 4(x+2) + (y+1) + 11 = 0$

or $-2x - 8 + 12 = 0$

or $-2x+4=0$
or $2x=4$
or $x=2$
or $x-2=0$

72. A and B are two independent events such that $P(A \cup B') = 0.8$ and $P(A) = 0.3$. The P(B) is

(A) $\frac{2}{7}$ (B) $\frac{2}{3}$ (C) $\frac{3}{8}$ (D) $\frac{1}{8}$

Ans : (A)

Hints : Let $P(B)=x$

$P(A \cup B') = P(A) + P(B') - P(A \cap B') = 0.3 + (1-x) - 0.3(1-x)$

or $0.8 = 1 - x + 0.3x$
or $1 - 0.7x = 0.8$
or $0.7x = 0.2$
or $x = \frac{2}{7}$

73. The total number of tangents through the point (3, 5) that can be drawn to the ellipses $3x^2 + 5y^2 = 32$ and $25x^2 + 9y^2 = 450$ is

(A) 0 (B) 2 (C) 3 (D) 4

Ans : (C)

Hints : (3, 5) lies outside the ellipse $3x^2+5y^2=32$ and on the ellipse $25x^2+9y^2=450$. Therefore there will be 2 tangents for the first ellipse and one tangent for the second ellipse.

74. The value of $\lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+........\frac{n}{n^2+n^2}\right]$ is

(A) $\frac{\pi}{4}$ (B) log 2 (C) zero (D)1

Ans : (A)

Hints : $\lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+...+\frac{n}{n^2+n^2}\right]$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{n}{n^2+r^2} = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{1+\left(\frac{r}{n}\right)^2} = \int_0^1 \frac{dx}{1+x^2} = \left[\tan^{-1}x\right]_0^1 = \frac{\pi}{4}$$

75. A particle is moving in a straight line. At time t, the distance between the particle from its starting point is given by $x = t - 6t^2 + t^3$. Its acceleration will be zero at

(A) $t=1$ unit time (B) $t=2$ unit time (C) $t=3$ unit time (D) $t=4$ unit time

Ans : (B)

Hints : $x = t - 6t^2 + t^3$

$$\frac{dx}{dt} = 1 - 12t + 3t^2$$

$$\frac{d^2x}{dt^2} = -12 + 6t$$

Acceleration $= \frac{d^2x}{dt^2}$

$\therefore$ Acceleration $= 0 \Rightarrow 6t - 12 = 0 \Rightarrow t = 2$

76. Three numbers are chosen at random from 1 to 20. The probability that they are consecutive is

(A) $\frac{1}{190}$ (B) $\frac{1}{120}$ (C) $\frac{3}{190}$ (D) $\frac{5}{190}$

Ans : (C)

Hints : Total number of cases ; $^{20}C_3 = \frac{20\times19\times18}{2\times3} = 20 \times 19 \times 3 = 1140$

Total number of favourable cases = 18

$\therefore$ Required probability $= \frac{18}{1140} = \frac{3}{190}$

77. The co-ordinates of the foot of the perpendicular from (0, 0) upon the line $x + y = 2$ are

(A) (2, –1) (B) (–2, 1) (C) (1, 1) (D) (1, 2)

Ans : (C)

Hints : Let P be the foot of the perpendicular. P lies on a line perpendicular to $x + y = 2$.

$\therefore$ Equation of the line on which P lies is of the form : $x - y + k = 0$

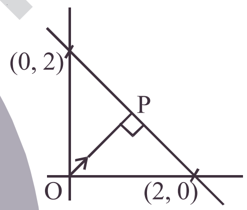
But this line passes through (0, 0).

$\therefore k = 0$

Hence, co-ordinates of P may be obtained by solving $x + y = 2$ and $y = x$

$\therefore x = 1, y = 1$

Hence, P $\equiv$ (1, 1)



78. If A is a square matrix then,

(A) $A + A^T$ is symmetric (B) AA^T is skew - symmetric (C) $A^T + A$ is skew-symmetric (D) A^TA is skew symmetric

Ans : (A)

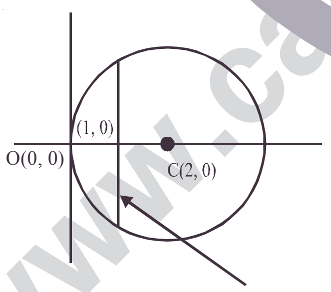
Hints : $(A + A^T)^T = A^T + (A^T)^T = A^T + A = A + A^T$

79. The equation of the chord of the circle $x^2 + y^2 - 4x = 0$ whose mid point is (1, 0) is

(A) $y = 2$ (B) $y = 1$ (C) $x = 2$ (D) $x = 1$

Ans : (D)

Hints :



Equation : $x = 1$

80. If $A^2 - A + I = 0$, then the inverse of the matrix A is

(A) $A - I$ (B) $I - A$ (C) $A + I$ (D) A

Ans : (B)

Hints : $A^2 - A + I = 0 \Rightarrow A^2 = A - I \Rightarrow A^2.A^{-1} = A.A^{-1} - A^{-1} \Rightarrow A = I - A^{-1} \Rightarrow A^{-1} = I - A$

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MATHEMATICS

SECTION-II

1. A train moving with constant acceleration takes t seconds to pass a certain fixed point and the front and back end of the train pass the fixed point with velocities u and v respectively. Show that the length of the trai is $\frac{1}{2}(u+v)t$.

A. $v = u + at$ $\qquad a = \frac{v-u}{t}$

$v^2 = u^2 + 2aS$

$\frac{v^2-u^2}{2a} = S \Rightarrow S = \frac{(v+u)(v-u)}{2a} = \frac{at(v+u)}{2a} = \frac{u+v}{2}t$

2. Show that

$$\frac{\sin\theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta} = \frac{1}{2}(\tan 27\theta - \tan\theta)$$

A. $T_1 = \frac{2\sin\theta}{2\cos 3\theta}.\frac{\cos\theta}{\cos\theta} = \frac{\sin 2\theta}{2.\cos 3\theta.\cos\theta}$

$= \frac{1}{2}.\frac{\sin(3\theta-\theta)}{\cos 3\theta.\cos\theta}$

$T_1 = \frac{1}{2}(\tan 3\theta - \tan\theta)$

$T_2 = \frac{1}{2}(\tan 9\theta - \tan 3\theta)$

$T_3 = \frac{1}{2}(\tan 27\theta - \tan 9\theta)$

$T_1 + T_2 + T_3 = \frac{1}{2}(\tan 27\theta - \tan\theta)$

3. If x = sin t, y = sin 2t, prove that

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$$

A. $y = \sin(2\sin^{-1}x)$

$\frac{dy}{dx} = \cos(2\sin^{-1}x).\frac{2}{\sqrt{1-x^2}}$

$\sqrt{1-x^2}\frac{dy}{dx} = 2\cos(2\sin^{-1}x)$

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = 4.\cos^2(2\sin^{-1}x) = 4[1-\sin^2(2\sin^{-1}x)]$$

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = 4[1-y^2]$$

Again differentiate

$$(1-x^2)2.\frac{dy}{dx}.\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2(-2x) = -8y\frac{dy}{dx}$$

Divide by $2\frac{dy}{dx}$

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=0$$

4. Show that, for a positive integer n, the coefficient of x^k $(0 \le K \le n)$ in the expansion of $1+(1+x)+(1+x)^2+........+(1+x)^n$ is ${}^{n+1}C_{n-k}$.

A. $S = \frac{1-(1+x)^{n+1}}{1-(1+x)} = \frac{(1+x)^{n+1}-1}{x}$

Coefficient of x^k in $\frac{(1+x)^{n+1}}{x}-\frac{1}{x}$ = Coefficient of x^{k+1} in $(1+x)^{n+1} = {}^{n+1}C_{k+1} = {}^{n+1}C_{n-k}$

5. If m, n be integers, then find the value of $\int_{-\pi}^{\pi}(\text{cosmx}-\text{sinnx})^2\text{dx}$

A. $I = \int_{-\pi}^{\pi}(\cos^2 mx+\sin^2 nx-2\sin nx.\cos mx)dx$

$$= \int_{-\pi}^{\pi}\cos^2 mx.dx+\int_{-\pi}^{\pi}\sin^2 nx.dx-2\int_{-\pi}^{\pi}\sin nx.\cos mx.dx$$

$$= 2\int_0^{\pi}\cos^2 mx.dx+2\int_0^{\pi}\sin^2 nx.dx-0 \qquad \text{(Odd .....)}$$

$$= 2\int_0^{\pi}(1+\cos 2mx)dx+\int_0^{\pi}(1-\cos 2nx)dx$$

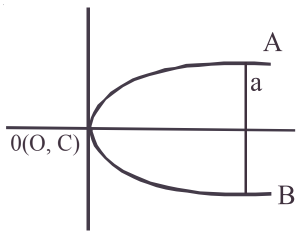
$$= \pi+\frac{1}{2m}(\sin 2mx)_0^{\pi}+\pi-\frac{1}{2n}(\sin 2nx)_0^{\pi}$$

$$= \pi+\pi+\frac{1}{2m}(0-0)-\frac{1}{2n}(0-0)$$

$$= 2\pi$$

6. Find the angle subtended by the double ordinate of length 2a of the parabola $y^2 = ax$ at its vertex.

A. $y^2 = ax,\ a^2 = ax,\ a = x$ [put y = a]



A (a, a), B(a, –a)

Slope OA $= \frac{a}{a} = 1$

Slope of OB $= \frac{-a}{a} = -1$

Ans. $= \frac{\pi}{2}$

7. If f is differentiable at x = a, find the value of

$$\lim_{x\to a} \frac{x^2 f(a) - a^2 f(x)}{x - a}.$$

A. $\lim_{x\to a} \frac{x^2 f(a) - a^2 f(x)}{x - a}$, $\frac{0}{0}$ form by LH

$= \lim_{x\to a} \frac{2x\,f(a) - a^2 f^1(x)}{1}$

$= 2af(a) - a^2 f^1(a)$

8. Find the values of 'a' for which the expression $x^2 - (3a - 1)x + 2a^2 + 2a - 11$ is always positve.

A. $x^2 - (3a - 1)x + 2a^2 + 2a - 11 > 0$

$D < 0$

$(3a - 1)^2 - 4(2a^2 + 2a - 11) < 0$

$9a^2 - 6a + 1 - 8a^2 - 8a + 44 < 0$

$a^2 - 14a + 45 < 0$

$(a - 9)(a - 5) < 0$

$5 < a < 9$

9. Find the sum of the first n terms of the series 0.2 + 0.22 + 0.222 + ...

A. $S = \frac{2}{9}[0.9 + 0.99 + 0.999 + ...]$

$= \frac{2}{9}[(1 - 0.1) + (1 - 0.01) + (1 - 0.001).......]$

$= \frac{2}{9}[n - (0.1 + 0.01.......... + n\ terms)]$

$= \frac{2}{9}n - \frac{2}{9}\frac{(0.1)[1-(0.1)^n]}{[1-(0.1)]}$

$\frac{2}{9}\text{n} - \frac{2}{9}\frac{(0.1)}{(0.9)}[1-(0.1)^\text{n}]$

$\frac{2}{9}\text{n} - \frac{2}{81} + \frac{2}{81}(0.1)^\text{n}$

10. The equation to the pairs of opposite sides of a parallelogram are $x^2 - 5x + 6 = 0$ and $y^2 - 6y + 5$. Find the equations of its diagonals.

A. $x = 2$ ......(i)

$x = 3$ ...... (ii)

$y = 1$ .... (iii)

$y = 5$ ..... (iv)

A (2, 1), B (3, 1), C (3, 5), D(2, 5)

Equation of AC

$\frac{x-2}{3-2} = \frac{y-1}{5-1}, \ x - 2 = \frac{y-1}{4}$

$4x - 8 = y - 1, \ 4x - y - 7 = 0$

Equation of BD $\frac{x-3}{2-3} = \frac{y-1}{5-1}$

$\frac{x-3}{-1} = \frac{y-1}{4}, \ -4x + 12 = y - 1$

$4x + y - 13 = 0$

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