COMMON ENTRANCE TEST - 2010

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DOs:
1. Check whether the CET No. has been entered and shaded in the respective circles on the OMR answer sheet.
2. This Question Booklet is issued to you by the Invigilator after the 2nd Bell, i.e., after 02.30 p.m.
3. The Serial Number of this question booklet should be entered on the OMR answer sheet.
4. The Version Code of this question booklet should be entered on the OMR answer sheet and the respective circles should also be shaded completely.
5. Compulsorily sign at the bottom portion of the OMR answer sheet in the space provided.

DON'Ts:
1. **THE TIMING AND MARKS PRINTED ON THE OMR ANSWER SHEET SHOULD NOT BE DAMAGED/MUTILATED/SPALOVED.**
2. Until the 3rd Bell is rung at 02.40 p.m.:
   - Do not remove the seal/staple present on the right hand side of this question booklet.
   - Do not look inside this question booklet.
   - Do not start answering on the OMR answer sheet.

**IMPORTANT INSTRUCTIONS TO CANDIDATES**

1. This question booklet contains 60 questions and each question will have four different options / choices.
2. After the 3rd Bell is rung at 02.40 p.m., remove the seal/staple present on the right hand side of this question booklet and start answering on the OMR answer sheet.
3. During the subsequent 70 minutes:
   - Read each question carefully.
   - Choose the correct answer from out of the four available options / choices given under each question.
   - Completely darken/shade the relevant circle with a BLUE OR BLACK INK BALLPOINT PEN against the question number on the OMR answer sheet.

**CORRECT METHOD OF SHADING THE CIRCLE ON THE OMR SHEET IS AS SHOWN BELOW:**

1 2 3 4

4. Please note that even a minute unintended ink dot on the OMR sheet will also be recognized and recorded by the scanner. Therefore, avoid multiple markings of any kind on the OMR answer sheet.
5. Use the space provided on each page of the question booklet for Rough Work. Do not use the OMR answer sheet for the same.
6. After the last bell is rung at 03.50 p.m., stop writing on the OMR answer sheet and affix your LEFT HAND THUMB IMPRESSION on the OMR answer sheet as per the instructions.
7. Hand over the OMR ANSWER SHEET to the room Invigilator as it is.
8. After separating and retaining the top sheet (KEA Copy), the Invigilator will return the bottom sheet replica (Candidate's copy) to you to carry home for self-evaluation.
9. Preserve the replica of the OMR answer sheet for a minimum period of ONE year.

SR - 17  

Turn Over
MATHEMATICS

1. The chord of the circle \( x^2 + y^2 - 4x = 0 \) which is bisected at \((1, 0)\) is perpendicular to the line ..................
   1) \( x = 1 \)  
   2) \( y = 1 \)  
   3) \( y = x \)  
   4) \( x + y = 0 \)

2. In \( \triangle ABC \), if \( a = 2 \), \( B = \tan^{-1} \frac{1}{2} \) and \( C = \tan^{-1} \frac{1}{3} \), then \((A, b) = \)
   1) \( \left( \frac{3\pi}{4}, \frac{2\sqrt{2}}{\sqrt{5}} \right) \)  
   2) \( \left( \frac{\pi}{4}, \frac{2}{\sqrt{5}} \right) \)  
   3) \( \left( \frac{3\pi}{4}, \frac{2}{\sqrt{5}} \right) \)  
   4) \( \left( \frac{\pi}{4}, \frac{2\sqrt{2}}{\sqrt{5}} \right) \)

3. The straight line \( 2x + 3y - k = 0 \), \( k > 0 \) cuts the \( X \)- and \( Y \)-axes at \( A \) and \( B \). The area of \( \triangle OAB \), where \( O \) is the origin, is 12 sq. units. The equation of the circle having \( AB \) as diameter is ..................
   1) \( x^2 + y^2 - 6x + 4y = 0 \)  
   2) \( x^2 + y^2 - 4x - 6y = 0 \)  
   3) \( x^2 + y^2 - 6x - 4y = 0 \)  
   4) \( x^2 + y^2 + 4x - 6y = 0 \)

4. Let \( P(x, y) \) be the midpoint of the line joining \((1, 0)\) to a point on the curve
   \[
   y^2 = \begin{vmatrix}
   x + 1 & x + 2 \\
   x + 3 & x + 5
   \end{vmatrix}
   \]. The locus of \( P \) is symmetrical about ..................
   1) \( x = 1 \)  
   2) \( y = 1 \)  
   3) \( Y \)-axis  
   4) \( X \)-axis

5. The function \( f(x) = |x - 2| + x \) is ..................
   1) continuous at \( x = 2 \) but not at \( x = 0 \).  
   2) continuous at both \( x = 2 \) and \( x = 0 \).  
   3) differentiable at both \( x = 2 \) and \( x = 0 \).  
   4) differentiable at \( x = 2 \) but not at \( x = 0 \).

(Space for Rough Work)
6. If \(a, -a, \ b\) are the roots of \(x^3 - 5x^2 - x + 5 = 0\), then \(b\) is a root of

\[
\begin{align*}
1) \quad & x^2 - 3x - 10 = 0 \\
2) \quad & x^2 + 5x - 30 = 0 \\
3) \quad & x^2 + 3x - 20 = 0 \\
4) \quad & x^2 - 5x + 10 = 0
\end{align*}
\]

7. In the binomial expansion of \((1+x)^{15}\), the coefficients of \(x^7\) and \(x^{13}\) are equal. Then \(r\) is

\[
\begin{align*}
1) \quad & 4 \\
2) \quad & 6 \\
3) \quad & 8 \\
4) \quad & 7
\end{align*}
\]

8. The \(n^{th}\) term of the series \(1 + 3 + 7 + 13 + 21 + ...\) is 9901. The value of \(n\) is

\[
\begin{align*}
1) \quad & 900 \\
2) \quad & 99 \\
3) \quad & 100 \\
4) \quad & 90
\end{align*}
\]

9. If \(\frac{1}{(3-5x)(2+3x)} = A \frac{1}{3-5x} + B \frac{1}{2+3x}\), then \(A : B\) is

\[
\begin{align*}
1) \quad & 3 : 5 \\
2) \quad & 3 : 2 \\
3) \quad & 2 : 3 \\
4) \quad & 5 : 3
\end{align*}
\]

10. Which of the following is NOT true?

\[
\begin{align*}
1) \quad & p \rightarrow (q \land r) \equiv (p \rightarrow q) \land (p \rightarrow r). \\
2) \quad & \neg (p \leftrightarrow q) \equiv (p \land \neg q) \lor (\neg p \land q). \\
3) \quad & (p \land \neg q) \leftrightarrow (p \rightarrow q) \text{ is a tautology.} \\
4) \quad & \{(p \rightarrow q) \land (q \rightarrow r)\} \rightarrow (p \rightarrow r) \text{ is a tautology.}
\end{align*}
\]

(Space for Rough Work)
11. Let \( R \) be an equivalence relation defined on a set containing 6 elements. The minimum number of ordered pairs that \( R \) should contain is ........................................
   1) 64
   2) 36
   3) 12
   4) 6

12. The line joining \( A(2, -7) \) and \( B(6, 5) \) is divided into 4 equal parts by the points \( P, Q \) and \( R \) such that \( AQ = RP = QB \). The midpoint of \( PR \) is ..........................
   1) (4, -1)  
   2) (8, -2)  
   3) (4, 12)  
   4) (-8, 1)

13. Let \( P = (-1, 0) \), \( Q = (0, 0) \) and \( R = (3, 3\sqrt{3}) \) be three points. The equation of the bisector of the angle \( PQR \) is ..........................
   1) \( x + \sqrt{3}y = 0 \)  
   2) \( \sqrt{3}x + y = 0 \)  
   3) \( x - \sqrt{3}y = 0 \)  
   4) \( \sqrt{3}x - y = 0 \)

14. If \( m \) is the slope of one of the lines represented by \( ax^2 + 2hxy + by^2 = 0 \), then
   \[ (h + bm)^2 = \] ..........................
   1) \( h^2 + ab \)  
   2) \( h^2 - ab \)  
   3) \( (a+b)^2 \)  
   4) \( (a-b)^2 \)

15. \( \cot 12^\circ \cot 102^\circ + \cot 102^\circ \cot 66^\circ + \cot 66^\circ \cot 12^\circ = \) ..........................
   1) -1  
   2) 2  
   3) -2  
   4) 1

(Space for Rough Work)
16. \((\sin \theta + \cos \theta)(\tan \theta + \cot \theta) = \) 
   
   1) \(\sec \theta + \csc \theta\) 
   2) \(\sec \theta \cdot \csc \theta\) 
   3) \(\sin \theta \cdot \cos \theta\) 
   4) 1

17. The sides of a triangle are \(6 + \sqrt{12}, \sqrt{48}\) and \(\sqrt{24}\). The tangent of the smallest angle of the triangle is 
   
   1) \(\sqrt{3}\) 
   2) 1 
   3) \(\frac{1}{\sqrt{3}}\) 
   4) \(\sqrt{2} - 1\)

18. A simple graph contains 24 edges. Degree of each vertex is 3. The number of vertices is 
   
   1) 21 
   2) 16 
   3) 8 
   4) 12

19. \(\lim_{n \to \infty} \left\{ n \sin \frac{2\pi}{3n} \cdot \cos \frac{2\pi}{3n} \right\} = \) 
   
   1) 1 
   2) \(\frac{\pi}{3}\) 
   3) \(\frac{\pi}{6}\) 
   4) \(\frac{2\pi}{3}\)

20. The function \(f(x) = [x]\), where \([x]\) denotes the greatest integer not greater than \(x\), is...
   
   1) continuous for all real values of \(x\). 
   2) continuous only at rational values of \(x\). 
   3) continuous for all nonintegral values of \(x\). 
   4) continuous only at positive integral values of \(x\).

(Space for Rough Work)
21. If \( a > b > 0 \), \( \sec^{-1}\left(\frac{a+b}{a-b}\right) = 2 \sin^{-1} x \), then \( x = \) ________________

1) \( \frac{a}{\sqrt{a+b}} \)  
2) \( \frac{a}{\sqrt{a+b}} \)  
3) \( \frac{b}{\sqrt{a+b}} \)  
4) \( \frac{b}{\sqrt{a+b}} \)

22. If \( x \neq n\pi, x \neq (2n+1)\frac{\pi}{2}, n \in \mathbb{Z} \), then \( \frac{\sin^{-1}(\cos x) + \cos^{-1}(\sin x)}{\tan^{-1}(\cot x) + \cot^{-1}(\tan x)} = \) ________________

1) \( \frac{\pi}{4} \)  
2) \( \frac{\pi}{3} \)  
3) \( \frac{\pi}{2} \)  
4) \( \frac{\pi}{6} \)

23. The general solution of \( 1 + \sin^2 x = 3 \sin x \cdot \cos x \), \( \tan x \neq \frac{1}{2} \) is ________________

1) \( n\pi - \frac{\pi}{4}, n \in \mathbb{Z} \)  
2) \( n\pi + \frac{\pi}{4}, n \in \mathbb{Z} \)  
3) \( 2n\pi + \frac{\pi}{4}, n \in \mathbb{Z} \)  
4) \( 2n\pi - \frac{\pi}{4}, n \in \mathbb{Z} \)

24. The least positive integer \( n \), for which \( \frac{(1+i)^n}{(1-i)^{n-2}} \) is positive, is ________________

1) 1  
2) 2  
3) 3  
4) 4

25. If \( x + iy = \left(-1 + i\sqrt{3}\right)^{2010} \), then \( x = \) ________________

1) 1  
2) -1  
3) \(-2^{2010}\)  
4) \(2^{2010}\)

(Space for Rough Work)
26. The greatest value of $x$ satisfying $21 \equiv 385 \pmod{x}$ and $587 \equiv 167 \pmod{x}$ is ..........
   1) 28  
   2) 56  
   3) 156  
   4) 32

27. The number $\left(49^{3} - 4\right) \left(49^{3} - 49\right)$ is divisible by .................
   1) 6!  
   2) 5!  
   3) 7!  
   4) 9!

28. The least positive integer $x$ satisfying $2^{2010} \equiv 3x \pmod{5}$ is ..................
   1) 1  
   2) 2  
   3) 3  
   4) 4

29. If $A$ and $B$ are two square matrices of the same order such that $AB = B$ and $BA = A$, then $A^2 + B^2$ is always equal to .................
   1) $2AB$  
   2) $2BA$  
   3) $I$  
   4) $A + B$

30. If $A$ is a $3 \times 3$ nonsingular matrix and if $|A| = 3$, then $\left|(2A)^{-1}\right| =$ ..................
   1) $\frac{1}{3}$  
   2) $\frac{1}{24}$  
   3) 24  
   4) 3

(Space for Rough Work)
31. If \( A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \), then \( A^2 + xA + yI = 0 \) for \((x, y) = \ldots\).

1) \((4, -1)\)  
2) \((1, 3)\)  
3) \((-4, 1)\)  
4) \((-1, 3)\)

32. The constant term of the polynomial \( \begin{vmatrix} x+3 & x & x+2 \\ x & x+1 & x-1 \\ x+2 & 2x & 3x+1 \end{vmatrix} \) is \( \ldots\).

1) \(-1\)  
2) 1  
3) 0  
4) 2

33. If \( \vec{a}, \vec{b} \) and \( \vec{c} \) are nonzero coplanar vectors, then \( \begin{bmatrix} 2\vec{a} - \vec{b} & 3\vec{b} - \vec{c} & 4\vec{c} - \vec{a} \end{bmatrix} = \ldots\).

1) 27  
2) 9  
3) 25  
4) 0

34. A space vector makes the angles 150\(^\circ\) and 60\(^\circ\) with the positive direction of X- and Y-axes. The angle made by the vector with the positive direction of Z-axis is \( \ldots\).

1) 180\(^\circ\)  
2) 120\(^\circ\)  
3) 90\(^\circ\)  
4) 60\(^\circ\)

35. If \( \vec{a}, \vec{b} \) and \( \vec{c} \) are unit vectors, such that \( \vec{a} + \vec{b} + \vec{c} = \vec{0} \), then \( 3\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \ldots\).

1) \(-3\)  
2) 3  
3) \(-1\)  
4) 1

(Space for Rough Work)
36. If \( i, j, k \) are unit vectors along the positive direction of \( X, Y \) and \( Z \)-axes, then a FALSE statement in the following is .................................................................

\[
\begin{align*}
1) \quad \sum i \cdot (j \times k) &= 0 \\
2) \quad \sum i \cdot (j + k) &= 0 \\
3) \quad \sum i \times (j + k) &= \vec{0} \\
4) \quad \sum i \times (j \times k) &= \vec{0}
\end{align*}
\]

37. In \( P(X) \), the power set of a nonempty set \( X \), a binary operation \( \ast \) is defined by \( A \ast B = A \cup B \forall A, B \in P(X) \). Under \( \ast \), a TRUE statement is ............................................

1) commutative law is not satisfied.
2) associative law is not satisfied.
3) identity law is not satisfied.
4) inverse law is not satisfied.

38. The inverse of 2010 in the group \( Q^* \) of all positive rationals under the binary operation \( \ast \) defined by \( a \ast b = \frac{ab}{2010} \forall a, b \in Q^* \), is ............................................

\[
\begin{align*}
1) \quad 1 \\
2) \quad 2010 \\
3) \quad 2009 \\
4) \quad 2011
\end{align*}
\]

39. If the three functions \( f(x), g(x) \) and \( h(x) \) are such that \( h(x) = f(x) \cdot g(x) \) and \( f'(x) \cdot g'(x) = c \), where \( c \) is a constant, then \( f'(x) + \frac{g'(x)}{g(x)} + \frac{2c}{f(x) \cdot g(x)} \) is equal to ..............

\[
\begin{align*}
1) \quad \frac{h^*(x)}{h(x)} \\
2) \quad \frac{h(x)}{h'(x)} \\
3) \quad h'(x) \cdot h^*(x) \\
4) \quad \frac{h(x)}{h^*(x)}
\end{align*}
\]

40. The derivative of \( e^{ax} \cos bx \) with respect to \( x \) is \( re^{ax} \cos \left( bx + \tan^{-1} \frac{b}{a} \right) \). When \( a > 0, b > 0 \), the value of \( r \) is ..................

\[
\begin{align*}
1) \quad ab \\
2) \quad a + b \\
3) \quad \sqrt{a^2 + b^2} \\
4) \quad \frac{1}{\sqrt{ab}}
\end{align*}
\]

(Space for Rough Work)

SR - 17
41. If \( x = a \cos^3 \theta \) and \( y = a \sin^3 \theta \), then \( \frac{dy}{dx} = \ldots \)

1) \( -\frac{\sqrt{x}}{\sqrt{y}} \)  
2) \( -\frac{\sqrt{y}}{\sqrt{x}} \)  
3) \( \frac{\sqrt{y}}{\sqrt{x}} \)  
4) \( \frac{\sqrt{x}}{\sqrt{y}} \)

42. If \( y = \tan^{-1} \sqrt{x^2 - 1} \), then the ratio \( \frac{d^2y}{dx^2} : \frac{dy}{dx} = \ldots \)

1) \( \frac{1 + 2x^2}{x(x^2 + 1)} \)  
2) \( \frac{x(x^2 + 1)}{1 - 2x^2} \)  
3) \( \frac{x(x^2 - 1)}{1 + 2x^2} \)  
4) \( \frac{1 - 2x^2}{x(x^2 - 1)} \)

43. \( P \) is the point of contact of the tangent from the origin to the curve \( y = \log_e x \). The length of the perpendicular drawn from the origin to the normal at \( P \) is \ldots \)

1) \( 2\sqrt{e^2 + 1} \)  
2) \( \sqrt{e^2 + 1} \)  
3) \( \frac{1}{2e} \)  
4) \( \frac{1}{e} \)

44. For the curve \( 4x^3 = 5y^4 \), the ratio of the cube of the subtangent at a point on the curve to the square of the subnormal at the same point is \ldots \)

1) \( \left( \frac{4}{5} \right)^3 \)  
2) \( \left( \frac{5}{4} \right)^4 \)  
3) \( x \left( \frac{4}{5} \right)^3 \)  
4) \( y \left( \frac{5}{4} \right)^4 \)

45. The set of real values of \( x \) for which \( f(x) = \frac{x}{\log x} \) is increasing, is \ldots \)

1) \( \{ x : x < e \} \)  
2) \( \{ 1 \} \)  
3) \( \{ x : x \geq e \} \)  
4) empty

---

(Space for Rough Work)
46. A wire of length 20 cm is bent in the form of a sector of a circle. The maximum area that can be enclosed by the wire is ....................

1) 10 sq. cm
2) 30 sq. cm
3) 20 sq. cm
4) 25 sq. cm

47. Two circles centered at (2, 3) and (5, 6) intersect each other. If the radii are equal, the equation of the common chord is ....................

1) \(x + y - 8 = 0\)
2) \(x - y - 8 = 0\)
3) \(x + y + 1 = 0\)
4) \(x - y + 1 = 0\)

48. Equation of the circle centered at (4, 3) touching the circle \(x^2 + y^2 = 1\) externally, is ......

1) \(x^2 + y^2 + 8x - 6y + 9 = 0\)
2) \(x^2 + y^2 - 8x + 6y + 9 = 0\)
3) \(x^2 + y^2 - 8x - 6y + 9 = 0\)
4) \(x^2 + y^2 + 8x + 6y + 9 = 0\)

49. The points (1, 0), (0, 1), (0, 0) and (2k, 3k), \(k \neq 0\) are concyclic if \(k = \) ....................

1) \(\frac{5}{13}\)
2) \(\frac{5}{13}\)
3) \(\frac{1}{5}\)
4) \(-\frac{1}{5}\)

50. The locus of the point of intersection of the tangents drawn at the ends of a focal chord of the parabola \(x^2 = -8y\) is ....................

1) \(y = 2\)
2) \(y = -2\)
3) \(x = 2\)
4) \(x = -2\)

(Space for Rough Work)
51. The condition for the line $y = mx + c$ to be a normal to the parabola $y^2 = 4ax$ is ..........  
   1) $c = \frac{a}{m}$  
   2) $c = 2am + am^3$  
   3) $c = -2am - am^3$  
   4) $c = -\frac{a}{m}$  

52. The eccentric angle of the point $(2, \sqrt{3})$ lying on $\frac{x^2}{16} + \frac{y^2}{4} = 1$ is .................  
   1) $\frac{\pi}{3}$  
   2) $\frac{\pi}{6}$  
   3) $\frac{\pi}{4}$  
   4) $\frac{\pi}{2}$  

53. The distance of the focus of $x^2 - y^2 = 4$, from the directrix which is nearer to it, is .........  
   1) $2\sqrt{2}$  
   2) $\sqrt{2}$  
   3) $4\sqrt{2}$  
   4) $8\sqrt{2}$  

54. If $\int f(x) \sin x \cdot \cos x \, dx = \frac{1}{2(b^2 - a^2)} \log f(x) + c$, where $c$ is the constant of integration, then $f(x) =$ .....................  
   1) $\frac{2}{ab \sin 2x}$  
   2) $\frac{2}{(b^2 - a^2) \sin 2x}$  
   3) $\frac{2}{ab \cos 2x}$  
   4) $\frac{2}{(b^2 - a^2) \cos 2x}$  

55. If $\int \frac{\sqrt{x}}{x(x + 1)} \, dx = k \tan^{-1}m$, then $(k, m)$ is .....................  
   1) $(1, \sqrt{x})$  
   2) $(2, \sqrt{x})$  
   3) $(2, x)$  
   4) $(1, x)$  

(Space for Rough Work)
56. \[ \int_0^\frac{\pi}{2} \frac{\sin x + \cos x}{3 + \sin 2x} \, dx = \] .................................

1) \( \frac{1}{2} \log 3 \) 2) \( 2 \log 3 \)

3) \( \frac{1}{4} \log 3 \) 4) \( \log 3 \)

57. \( \int_0^1 x (1-x)^{\frac{3}{2}} \, dx = \) .................................

1) \( \frac{24}{35} \) 2) \( \frac{-8}{35} \)

3) \( \frac{-2}{35} \) 4) \( \frac{4}{35} \)

58. The area bounded by the curve \( y = \begin{cases} x^2, & x < 0 \\ x, & x \geq 0 \end{cases} \) and the line \( y = 4 \) is .................................

1) \( \frac{40}{3} \) 2) \( \frac{16}{3} \)

3) \( \frac{32}{3} \) 4) \( \frac{8}{3} \)

59. The order and degree of the differential equation \( y = \frac{dp}{dx} x + \sqrt{a^2 p^2 + b^2} \) where \( p = \frac{dy}{dx} \) (here \( a \) and \( b \) are arbitrary constants) respectively are .................................

1) 1, 2 2) 2, 1

3) 2, 2 4) 1, 1

60. The general solution of the differential equation \( 2x \frac{dy}{dx} - y = 3 \) is a family of .................................

1) straight lines 2) circles

3) hyperbolas 4) parabolas

(Space for Rough Work)