COMMON ENTRANCE TEST - 2007

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<td>60</td>
<td>80 MINUTES</td>
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MENTION YOUR CET NUMBER

QUESTION BOOKLET DETAILS

VERSION CODE   SERIAL NUMBER
A - 1           553761

IMPORTANT INSTRUCTIONS TO CANDIDATES

(Candidates are advised to read the following instructions carefully, before answering on OMR answer sheet.)

1. Ensure that you have entered your Name and Register Number of 2nd PUC Annual Examination / 12th Std. in the space provided on the OMR answer sheet.
2. Ensure that CET No. has been entered and shaded the respective circles on the OMR answer sheet.
3. ENSURE THAT THE TIMING, MARKS PRINTED ON THE OMR ANSWER SHEET ARE NOT DAMAGED / MUTILATED / SPOILED.
4. This Question Booklet is issued to you by the invigilator after the 2nd Bell. i.e., after 02.00 p.m.
5. Enter the Serial Number of this question booklet on the OMR answer sheet.
6. Carefully enter the Version Code of this question booklet on the OMR answer sheet and SHADE the respective circles completely.
7. As answer sheets are designed to suit the Optical Mark Reader (OMR) system, please take special care while filling and shading the CET NO. & Version Code of this question booklet.
8. DO NOT FORGET TO SIGN AT THE BOTTOM PORTION OF OMR ANSWER SHEET IN THE SPACE PROVIDED.
9. Until the 3rd Bell is rung at 02.10 p.m.:
   • Do not remove the seal present on the right hand side of this question booklet.
   • Do not look inside this question booklet.
   • Do not start answering on the OMR answer sheet.
10. After the 3rd Bell is rung at 02.10 p.m., remove the seal present on the right hand side of this question booklet and start answering on the OMR answer sheet.
11. This question booklet contains 60 questions and each question will have four different options / choices.
12. During the subsequent 70 minutes:
   • Read each question carefully.
   • Determine the correct answer from out of the four available options / choices given under each question.
   • Completely darken / shade the relevant circle with a BLUE OR BLACK INK BALLPOINT PEN against the question number on the OMR answer sheet.

CORRECT METHOD OF SHADING THE CIRCLE ON THE OMR SHEET IS AS SHOWN BELOW:

1 2 3 4

13. Please note that even a minute unintended ink dot on the OMR sheet will also be recognised and recorded by the scanner. Therefore, avoid multiple markings of any kind on the OMR answer sheet.
14. Use the space provided on each page of the question booklet for Rough work AND do not use the OMR answer sheet for the same.
15. After the last bell is rung at 03.20 p.m., stop writing on the OMR answer sheet and affix your LEFT HAND THUMB IMPRESSION on the OMR answer sheet as per the instructions.
16. Hand over the OMR ANSWER SHEET to the room invigilator as it is.
17. After separating and retaining the top sheet (CET Cell Copy), the invigilator will return the bottom sheet replica (Candidate's copy) to you to carry home for self-evaluation.
18. Preserve the replica of the OMR answer sheet for a minimum period of One year.

Turn Over
MATHEMATICS

1. \(7^{2 \log_7 5}\) is equal to
   
   1) 5  
   2) \(\log_7 35\)  
   3) \(\log_7 25\)  
   4) 25

2. In the group \((G \oplus_{15})\), where \(G = \{3, 6, 9, 12\}\), \(\oplus_{15}\) is multiplication modulo 15, the identity element is
   
   1) 6  
   2) 3  
   3) 9  
   4) 12

3. A group \((G \ast)\) has 10 elements. The minimum number of elements of \(G\), which are their own inverses is
   
   1) 1  
   2) 2  
   3) 0  
   4) 9

4. If \(\vec{a}\) and \(\vec{b}\) are vectors such that \(|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|\), then the angle between \(\vec{a}\) and \(\vec{b}\) is
   
   1) 60°  
   2) 120°  
   3) 30°  
   4) 90°

5. \(\frac{3x^2 + 1}{x^2 - 6x + 8}\) is equal to
   
   1) \(\frac{49}{2(x-4)} - \frac{13}{2(x-2)}\)  
   2) \(3 + \frac{49}{2(x-4)} - \frac{13}{2(x-2)}\)  
   3) \(\frac{49}{2(x-4)} + \frac{13}{2(x-2)}\)  
   4) \(-\frac{49}{2(x-4)} + \frac{13}{2(x-2)}\)

(Space for Rough Work)
6. If \( \vec{a} = 2 \hat{i} + 3 \hat{j} - \hat{k} \), \( \vec{b} = \hat{i} + 2 \hat{j} - 5 \hat{k} \), \( \vec{c} = 3 \hat{i} + 5 \hat{j} - \hat{k} \), then a vector perpendicular to \( \vec{a} \) and in the plane containing \( \vec{b} \) and \( \vec{c} \) is

1) \( 17 \hat{i} + 21 \hat{j} - 123 \hat{k} \) 
2) \( -17 \hat{i} + 21 \hat{j} - 97 \hat{k} \) 
3) \( -17 \hat{i} - 21 \hat{j} - 97 \hat{k} \) 
4) \( -17 \hat{i} - 21 \hat{j} + 97 \hat{k} \)

7. \( \overrightarrow{OA} \) and \( \overrightarrow{BO} \) are two vectors of magnitudes 5 and 6 respectively. If \( \| \overrightarrow{BOA} \| = 60^\circ \), then \( \overrightarrow{OA} \cdot \overrightarrow{OB} \) is equal to

1) 15 
2) 0 
3) 15\sqrt{3} 
4) -15

8. A vector perpendicular to the plane containing the points \( A(1, -1, 2) \), \( B(2, 0, -1) \), \( C(0, 2, 1) \) is

1) \( 8 \hat{i} + 4 \hat{j} + 4 \hat{k} \) 
2) \( 4 \hat{i} + 8 \hat{j} - 4 \hat{k} \) 
3) \( \hat{i} + \hat{j} - \hat{k} \) 
4) \( 3 \hat{i} + \hat{j} + 2 \hat{k} \)

9. \( \frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \cdots \frac{1}{(3n-1)(3n+2)} = \)

1) \( \frac{n}{6n+3} \) 
2) \( \frac{n}{6n-4} \) 
3) \( \frac{n+1}{6n+4} \) 
4) \( \frac{n}{6n+4} \)

10. The ninth term of the expansion \( \left( 3x - \frac{1}{2x} \right)^8 \) is

1) \( \frac{-1}{512x^9} \) 
2) \( \frac{1}{512x^9} \) 
3) \( \frac{1}{256 \cdot x^8} \) 
4) \( \frac{-1}{256 \cdot x^8} \)

(Space for Rough Work)
11. If \( A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \) and \( B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} \) and \( B \) is the inverse of \( A \), then the value of \( \alpha \) is
   1) 0  2) 2  3) 4  4) 5

12. If \( A = \begin{bmatrix} 0 & x & 16 \\ x & 5 & 7 \\ 0 & 9 & x \end{bmatrix} \) is singular, then the possible values of \( x \) are
   1) 0, 1, -1  2) 0, +12, -12  3) 0, 5, -5  4) 0, 4, -4

13. If \( A = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \), then \( A \cdot \text{adj}(A) \) is equal to
   1) \( \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \)  2) \( \begin{bmatrix} 5 & 1 & 1 \\ 1 & 1 & 5 \\ 1 & 1 & 5 \end{bmatrix} \)
   3) \( \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \)  4) \( \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} \)

14. If \( f : R \rightarrow R \) is defined by \( f(x) = |x| \), then,
   1) \( f^{-1}(x) = \frac{1}{|x|} \)  2) \( f^{-1}(x) = -x \)
   3) \( f^{-1}(x) = \frac{1}{x} \)  4) The function \( f^{-1}(x) \) does not exist.

15. The value of \( \begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix} \) is
   1) \((x-p)(x-q)(x+p+q)\)  2) \(x(x-p)(x-q)\)
   3) \(pq(x-p)(x-q)\)  4) \((p-q)(x-q)(x-p)\)
16. The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y - 24 = 0$ is,
   1) 4  
   2) 3  
   3) 1  
   4) 2

17. If $3x + y + k = 0$ is a tangent to the circle $x^2 + y^2 = 10$, the values of $k$ are,
   1) $\pm 5$  
   2) $\pm 7$  
   3) $\pm 9$  
   4) $\pm 10$

18. The negation of the proposition “If 2 is prime, then 3 is odd” is
   1) 2 is prime and 3 is not odd  
   2) If 2 is not prime then 3 is not odd  
   3) If 2 is not prime then 3 is odd  
   4) 2 is not prime and 3 is odd.

19. The equation to two circles which touch the Y-axis at (0, 3) and make an intercept of 8 units on X-axis are
   1) $x^2 + y^2 + 6x - 10y + 9 = 0$  
   2) $x^2 + y^2 + 10x - 6y + 9 = 0$  
   3) $x^2 + y^2 + 10x + 6y + 9 = 0$  
   4) $x^2 + y^2 - 8x + 10y + 9 = 0$

20. The orthocentre of the triangle with vertices $A \left(0, 0\right)$, $B \left(5/2, 3/4\right)$, $C \left(-5, 3/2\right)$ is
   1) $\left(-5/2, 3/4\right)$  
   2) $\left(5/2, 3/4\right)$  
   3) $\left(0, 0\right)$  
   4) $\left(-5, 3/2\right)$

(Space for Rough Work)
21. \(x^2 + y^2 - 6x - 6y + 4 = 0\), \(x^2 + y^2 - 2x - 4y + 3 = 0\), \(x^2 + y^2 + 2kx + 2y + 1 = 0\) If the Radical centre of the above three circles exists, then which of the following cannot be the value of \(k\)?

1) 1
2) 2
3) 4
4) 5

22. If the circles \(x^2 + y^2 - 2x - 2y - 7 = 0\) and \(x^2 + y^2 + 4x + 2y + k = 0\) cut orthogonally, then the length of the common chord of the circles is

1) 2
2) \(\sqrt{12/13}\)
3) 8
4) 5

23. The co-ordinates of the foot of the perpendicular drawn from the point \((3, 4)\) on the line \(2x + y - 7 = 0\) is

1) \((1, 5)\)
2) \(\left(\frac{9}{5}, \frac{17}{5}\right)\)
3) \((1, -5)\)
4) \((-5, 1)\)

24. The area enclosed by the pair of lines \(xy = 0\), the line \(x - 4 = 0\) and \(y + 5 = 0\) is

1) 10 sq. units.
2) 20 sq. units
3) 0 sq. units.
4) \(\frac{5}{4}\) sq. units.

25. If the area of the auxiliary circle of the ellipse \(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\)(\(a > b\)) is twice the area of the ellipse, then the eccentricity of the ellipse is

1) \(\frac{\sqrt{3}}{2}\)
2) \(\frac{1}{\sqrt{2}}\)
3) \(\frac{1}{2}\)
4) \(\frac{1}{\sqrt{3}}\)

(Space for Rough Work)
26. A graph $G$ has 'm' vertices of odd degree and 'n' vertices of even degree. Then which of the following statements is necessarily true?
   
   1) $m + n$ is an even number  
   2) $m + n$ is an odd number  
   3) $m + 1$ is an odd number  
   4) $n + 1$ is an even number

27. If $P$ is any point on the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$, and $S$ and $S'$ are the foci, then $PS + PS' =$
   
   1) 8  
   2) 4  
   3) 12  
   4) 10

28. The value of $\sin \left[ 2\cos^{-1} \frac{\sqrt{5}}{3} \right]$ is
   
   1) $\frac{2\sqrt{5}}{3}$  
   2) $\frac{\sqrt{5}}{3}$  
   3) $\frac{2\sqrt{5}}{9}$  
   4) $\frac{4\sqrt{5}}{9}$

29. If $\frac{x^2}{36} - \frac{y^2}{k^2} = 1$ is a hyperbola, then which of the following statements can be true?
   
   1) $(3, 1)$ lies on the hyperbola  
   2) $(-3, 1)$ lies on the hyperbola  
   3) $(5, 2)$ lies on the hyperbola  
   4) $(10, 4)$ lies on the hyperbola

30. The focus of the parabola is
   
   1) $\left( \frac{1}{3}, \frac{-3}{2} \right)$  
   2) $\left( \frac{-1}{3}, \frac{3}{2} \right)$  
   3) $\left( \frac{1}{3}, \frac{-1}{2} \right)$  
   4) $\left( \frac{1}{3}, \frac{3}{2} \right)$

(Space for Rough Work)
31. The solution of $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$ is

1) $\frac{1}{\sqrt{3}}$  
2) $\frac{-1}{\sqrt{3}}$  
3) $\sqrt{3}$  
4) $-\sqrt{3}$

32. $\sin^2 17.5^0 + \sin^2 72.5^0$ is equal to

1) $\tan^2 45^0$  
2) $\cos^2 90^0$  
3) $\sin^2 45^0$  
4) $\cos^2 30^0$

33. The conjugate of the complex number $\frac{(1+i)^2}{1-i}$ is

1) $1+i$  
2) $1-i$  
3) $-1-i$  
4) $-1+i$

34. $ABC$ is a triangle with $\angle A = 30^0$, $BC = 10$ cm.

The area of the circum-circle of the triangle is

1) $5$ sq. cms.  
2) $100\pi$ sq. cms.  
3) $\frac{100\pi}{3}$ sq. cms.  
4) $25$ sq. cms.

35. If $\sin 3\theta = \sin \theta$, how many solutions exist such that $-2\pi < \theta < 2\pi$?

1) 9  
2) 8  
3) 7  
4) 5

(Space for Rough Work)
36. The imaginary part of $i^4$ is
   1) 1  
   2) 0  
   3) $-1$  
   4) 2

37. The amplitude of $(1+i)^5$ is
   1) $\frac{-3\pi}{4}$  
   2) $\frac{3\pi}{4}$  
   3) $\frac{5\pi}{4}$  
   4) $-\frac{5\pi}{4}$

38. $ABC$ is a triangle. $G$ is the centroid. $D$ is the mid point of $BC$. If $A = (2, 3)$ and $G = (7, 5)$,
then the point $D$ is
   1) $\left(\frac{19}{2}, 6\right)$  
   2) $\left(\frac{9}{2}, 4\right)$  
   3) $\left(8, \frac{13}{2}\right)$  
   4) $\left(\frac{11}{2}, \frac{11}{2}\right)$

39. $\lim_{x \to 1} \frac{Tan(x^2 - 1)}{x - 1}$ is equal to
   1) $\frac{1}{2}$  
   2) 2  
   3) $\frac{-1}{2}$  
   4) $-2$

40. If $y = 2^{Log x}$, then $\frac{dy}{dx}$ is
   1) $2^{Log x} \cdot Log 2$  
   2) $\frac{2^{Log x}}{Log 2}$  
   3) $\frac{2^{Log x} \cdot Log 2}{x}$  
   4) $\frac{2^{Log x}}{x}$

(Space for Rough Work)
41. If \( \sec^{-1}\left(\frac{1+x}{1-y}\right) = a \), then \( \frac{dy}{dx} \) is

1) \( \frac{y+1}{x-1} \) 
2) \( \frac{y-1}{x+1} \) 
3) \( \frac{x-1}{y+1} \) 
4) \( \frac{x-1}{y-1} \)

42. If \( y = \cos^2\frac{3x}{2} - \sin^2\frac{3x}{2} \), then \( \frac{d^2y}{dx^2} \) is

1) \( 9y \) 
2) \( -3\sqrt{1-y^2} \) 
3) \( 3\sqrt{1-y^2} \) 
4) \( -9y \)

43. If the function \( f(x) = \begin{cases} \frac{1-\cos x}{x^2} & \text{for } x \neq 0 \\ k & \text{for } x = 0 \end{cases} \) is continuous at \( x = 0 \), then the value of \( k \) is

1) \( 0 \) 
2) \( 1 \) 
3) \( -1 \) 
4) \( \frac{1}{2} \)

44. If \( 1, w, w^2 \) are the cube roots of unity then \( (1+w)(1+w^2)(1+w^4)(1+w^8) \) is equal to

1) \( 0 \) 
2) \( 1 \) 
3) \( w \) 
4) \( w^2 \)

45. If \( x^y = y^x \) then \( \frac{dy}{dx} \) is

1) \( \frac{x}{y} \) 
2) \( \frac{-y}{x} \) 
3) \( \frac{1+\log x}{1+\log y} \) 
4) \( 1+\log\left(\frac{x}{y}\right) \)

(Space for Rough Work)
46. The point on the curve $y^2 = x$, the tangent at which makes an angle $45^0$ with X-axis is

1) $(\frac{1}{2}, \frac{1}{4})$  
2) $(\frac{1}{4}, \frac{1}{2})$  
3) $(\frac{1}{2}, \frac{1}{2})$  
4) $(\frac{1}{2}, -\frac{1}{2})$

47. The length of the subtangent to the curve $x^2 y^2 = a^4$ at $(-a, a)$ is

1) $2a$  
2) $\frac{a}{2}$  
3) $\frac{a}{3}$  
4) $a$

48. The number of positive divisors of 252 is

1) 5  
2) 9  
3) 10  
4) 18

49. The remainder obtained when $5^{124}$ is divided by 124 is

1) 0  
2) 5  
3) 1  
4) 2

50. Which of the following is not a group with respect to the given operation?

1) The set of odd integers under addition.
2) The set of even integers under addition.
3) $\{1, -1\}$ under multiplication.
4) $\{0\}$ under addition.
51. The range in which \( y = -x^2 + 6x - 3 \) is increasing is

1) \( x > 3 \)  
2) \( x < 3 \)  
3) \( 5 < x < 6 \)  
4) \( 7 < x < 8 \)

52. The value of the integral \( \int_{0}^{\pi/2} (\sin^{100}x - \cos^{100}x) \, dx \) is

1) \( \frac{100!}{(100)^{100}} \)  
2) \( \frac{1}{100} \)  
3) 0  
4) \( \frac{\pi}{100} \)

53. \( OA \) and \( OB \) are two roads enclosing an angle of 120°. \( X \) and \( Y \) start from 'O' at the same time. \( X \) travels along \( OA \) with a speed of 4 km/hour and \( Y \) travels along \( OB \) with a speed of 3 km/hour. The rate at which the shortest distance between \( X \) and \( Y \) is increasing after 1 hour is

1) 37 km/hour  
2) \( \sqrt{37} \) km/hour  
3) \( \sqrt{13} \) km/hour  
4) 13 km/hour

54. If \( \int_{0}^{3} x \cdot f(3x) \, dx = \int_{0}^{3} t \cdot f(t) \, dt \), then the value of \( k \) is

1) 3  
2) 9  
3) \( \frac{1}{3} \)  
4) \( \frac{1}{9} \)

55. The value of \( \int \frac{1}{1 + \cos 8x} \, dx \) is

1) \( \frac{\tan 8x}{8} + C \)  
2) \( \frac{\tan 2x}{8} + C \)  
3) \( \frac{\tan 4x}{8} + C \)  
4) \( \frac{\tan 4x}{4} + C \)

(Space for Rough Work)
56. The value of \( \int e^x \left( x^5 + 5x^4 + 1 \right) \, dx \) is

1) \( e^x \cdot x^5 + e^x + C \)  
2) \( e^x \cdot x^5 \)  
3) \( 5x^4 e^x \)  
4) \( e^{x+1} \cdot x^5 + C \)

57. The value of \( \int \frac{x^2 + 1}{x^2 - 1} \, dx \) is

1) \( \log \left( \frac{x+1}{x-1} \right) + C \)  
2) \( \log \left( \frac{x-1}{x+1} \right) + C \)  
3) \( \log \left( x^2 - 1 \right) + C \)  
4) \( x + \log \left( \frac{x-1}{x+1} \right) + C \)

58. The area bounded by the curve \( x = 4 - y^2 \) and the Y-axis is

1) 32 sq. units  
2) 16 sq. units  
3) \( \frac{16}{3} \) sq. units  
4) \( \frac{32}{3} \) sq. units

59. The differential equation of the family of straight lines whose slope is equal to y-intercept is

1) \( (x+1) \frac{dy}{dx} + y = 0 \)  
2) \( (x+1) \frac{dy}{dx} - y = 0 \)  
3) \( \frac{dy}{dx} = \frac{x+1}{y+1} \)  
4) \( \frac{dy}{dx} = \frac{x-1}{y-1} \)

60. The order and degree of the differential equation \( \left[ 1 + \left( \frac{dy}{dx} \right)^5 \right]^{\frac{1}{3}} = \frac{d^2 y}{dx^2} \) are respectively

1) 2, 1  
2) 1, 5  
3) 2, 3  
4) 2, 5

(Space for Rough Work)