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FIITJEE SOLUTIONS TO JEE(ADVANCED)-2014

CODE	5	PAPER-1	P1-14-5 1208625

Note: Front and back cover pages are reproduced from actual paper back to back here.

Time: 3 Hours **Maximum Marks: 180**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General

1. This booklet is your Question Paper. Do not break the seal of this booklet before being instructed to do so by the invigilators.
2. The question paper CODE is printed on the left hand top corner of this sheet and on the back cover page of this booklet.
3. Blank space and blank pages are provided in the question paper for your rough work. No additional sheets will be provided for rough work.
4. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers and electronic gadget of any kind are NOT allowed inside the examination hall.
5. Write your name and roll number in the space provided on the back cover of this booklet.
6. Answers to the questions and personal details are to be filled on an Optical Response Sheet, which is provided separately. The ORS is a doublet of two sheets – upper and lower, having identical layout. The upper sheet is a machine-gradable Objective Response Sheet (ORS) which will be collected by the invigilator at the end of the examination. The upper sheet is designed in such a way that darkening the bubble with a ball point pen will leave an identical impression at the corresponding place on the lower sheet. You will be allowed to take away the lower sheet at the end of the examination (**see Figure-1** on the back cover page for the correct way of darkening the bubbles for valid answers).
7. **Use a black ball point pen only to darken the bubbles on the upper original sheet**. Apply sufficient pressure so that the impression is created on the lower sheet. **See Figure -1** on the back cover page for appropriate way of darkening the bubbles for valid answers.
8. DO NOT TAMPER WITH / MUT!LATE THE ORS SO THIS BOOKLET.
9. On breaking the seal of the booklet check that it contains **28** pages and all the 60 questions and corresponding answer choices are legible. Read carefully the instruction printed at the beginning of each section.

B. Filling the right part of the ORS

10. The ORS also has a CODE printed on its left and right parts.
11. Verify that the CODE printed on the ORS (on both the left and right parts) is the same as that on the this booklet and put your signature in the Box designated as R4.
12. **IF THE CODES DO NOT MATCH, ASK FOR A CHANGE OF THE BOOKLET / ORS AS APPLICABLE.**
13. Write your Name, Roll No. and the name of centre and sign with pen in the boxes provided on the upper sheet of ORS**. Do not write any of this anywhere else**. Darken the appropriate bubble UNDER each digit of your Roll No. in such way that the impression is created on the bottom sheet. (**see example in Figure 2** on the back cover)

C. Question Paper Format

The question paper consists of three parts (Physics, Chemistry and Mathematics). Each part consists of two sections.

14. **Section 1** contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE THAN ONE are correct**.
15. **Section 2** contains **10 questions**. The answer to each of the questions is a single-digit integer, ranging from 0 to 9 (both inclusive)

Please read the last page of this booklet for rest of the instructions.

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PAPER-1 [Code – 5]
JEE(ADVANCED) 2014

PART-I: PHYSICS

SECTION – 1: (One or More than One Options Correct Type)

This section contains 10 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE THAN ONE are correct.

1. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005)m , the gas in the tube is (**Useful information:** $\sqrt{167RT} = 640\ J^{1/2}mole^{-1/2}$; $\sqrt{140RT} = 590\ J^{1/2}mole^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)

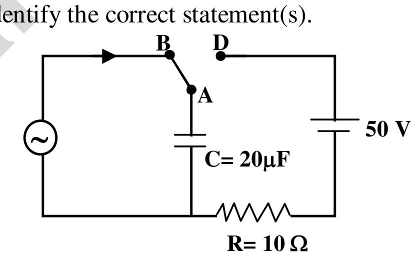
(A) Neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$
(B) Nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$
(C) Oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$
(D) Argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

Sol. **D**

$$\ell = \frac{1}{4\nu}\sqrt{\frac{\gamma RT}{M}}$$

Calculations for $\frac{1}{4\nu}\sqrt{\frac{\gamma RT}{M}}$ for gases mentioned in options A, B, C and D, work out to be 0.459 m, 0.363 m 0.340 m & 0.348 m respectively. As $\ell = (0.350 \pm 0.005)$m ; Hence correct option is D.

2. At time t = 0, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1$A and $\omega = 500$ rad/s starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If C = 20 μF, R = 10 Ω and the battery is ideal with emf of 50 V, identify the correct statement(s).



(A) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3}C.
(B) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.
(C) Immediately after A is connected to D, the current in R is 10 A.
(D) $Q = 2 \times 10^{-3}$C.

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Sol. **C, D**

As current leads voltage by $\pi/2$ in the given circuit initially, then ac voltage can be represent as

$V = V_0 \sin \omega t$

$\therefore \quad q = CV_0 \sin \omega t = Q \sin \omega t$

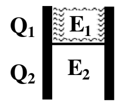
where, $Q = 2 \times 10^{-3}$ C

- At $t = 7\pi/6\omega$; $I = -\frac{\sqrt{3}}{2}I_0$ and hence current is anticlockwise.
- Current 'i' immediately after $t = \frac{7\pi}{6\omega}$ is

  $i = \frac{V_c + 50}{R} = 10$ A
- Charge flow $= Q_{final} - Q_{(7\pi/6\omega)} = 2 \times 10^{-6}$C

Hence C & D are correct options.

3. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects.



(A) $\frac{E_1}{E_2} = 1$ (B) $\frac{E_1}{E_2} = \frac{1}{K}$

(C) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (D) $\frac{C}{C_1} = \frac{2+K}{K}$

Sol. **A, D**

As $E = V/d$

$E_1/E_2 = 1$ (both parts have common potential difference)

Assume C_0 be the capacitance without dielectric for whole capacitor.

$$k\frac{C_0}{3} + \frac{2C_0}{3} = C$$

$$\frac{C}{C_1} = \frac{2+k}{k}$$

$$\frac{Q_1}{Q_2} = \frac{k}{2}.$$

4. One end of a taut string of length 3m along the x axis is fixed at x = 0. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is (are)

(A) $y(t) = A \sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$ (B) $y(t) = A \sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(C) $y(t) = A \sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$ (D) $y(t) = A \sin\frac{5\pi x}{2}\cos 250\pi t$

Sol. **A, C, D**

Taking $y(t) = A\, f(x)\, g(t)$ & Applying the conditions:

1; here x = 3m is antinode & x = 0 is node

2; possible frequencies are odd multiple of fundamental frequency.

where, $\nu_{fundamental} = \frac{v}{4\ell} = \frac{25}{3}$Hz

The correct options are A, C, D.

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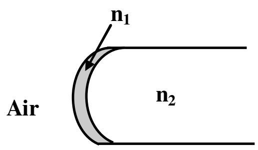
5. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then



(A) $|f_1| = 3R$ (B) $|f_1| = 2.8\ R$
(C) $|f_2| = 2R$ (D) $|f_2| = 1.4\ R$

Sol. **A, C**
For air to glass

$$\frac{1.5}{f_1} = \frac{1.4-1}{R} + \frac{1.5-1.4}{R}$$

$\therefore$ $f_1 = 3R$
For glass to air.

$$\frac{1}{f_2} = \frac{1.4-1.5}{-R} + \frac{1-1.4}{-R}$$

$\therefore$ $f_2 = 2R$

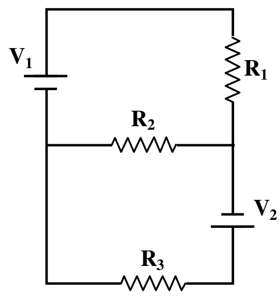
6. Heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter 2d. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40K?
(A) 4 if wires are in parallel (B) 2 if wires are in series
(C) 1 if wires are in series (D) 0.5 if wires are in parallel.

Sol. **B, D**

$$H = \frac{V^2}{R}4 = \frac{V^2}{R/2}t_1 = \frac{V^2}{R/8}t_2$$

$t_1 = 2$ min.
$t_2 = 0.5$ min.

7. Two ideal batteries of emf V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if



(A) $V_1 = V_2$ and $R_1 = R_2 = R_3$
(B) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$
(C) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$
(D) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

Sol. **A, B, D**

$$V_1 = \frac{R_1(V_1+V_2)}{R_1+R_3} \Rightarrow V_1R_3 = V_2R_1$$

$$V_2 = \frac{R_3(V_1+V_2)}{R_1+R_3} \Rightarrow V_2R_1 = V_2R_3$$

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8. Let $E_1(r), E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then

(A) $Q = 4\sigma\pi r_0^2$
(B) $r_0 = \dfrac{\lambda}{2\pi\sigma}$
(C) $E_1(r_0/2) = 2E_2(r_0/2)$
(D) $E_2(r_0/2) = 4E_3(r_0/2)$

Sol. **C**

$$\frac{Q}{4\pi\varepsilon_0 r_0^2} = \frac{\lambda}{2\pi\varepsilon_0 r_0} = \frac{\sigma}{2\varepsilon_0}$$

$$E_1\left(\frac{r_0}{2}\right) = \frac{Q}{\pi\varepsilon_0 r_0^2}\ ,\ E_2\left(\frac{r_0}{2}\right) = \frac{\lambda}{\pi\varepsilon_0 r_0}\ ,\ E_3\left(\frac{r_0}{2}\right) = \frac{\sigma}{2\varepsilon_0}$$

$$\therefore\quad E_1\left(\frac{r_0}{2}\right) = 2E_2\left(\frac{r_0}{2}\right)$$

9. A light source, which emits two wavelengths λ_1 = 400 nm and λ_2 = 600 nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then

(A) $\beta_2 > \beta_1$
(B) $m_1 > m_2$
(C) From the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1
(D) The angular separation of fringes for λ_1 is greater than λ_2

Sol. **A, B, C**

$$\beta = \frac{D\lambda}{d}$$

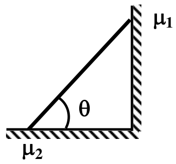
$\because \lambda_2 > \lambda_1 \Rightarrow \beta_2 > \beta_1$

Also $m_1\beta_1 = m_2\beta_2 \Rightarrow m_1 > m_2$

Also $3\left(\dfrac{D}{d}\right)(600\ \text{nm}) = (2\times5-1)\left(\dfrac{D}{2d}\right)400\,\text{nm}$

Angular width $\theta = \dfrac{\lambda}{d}$

10. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then



(A) $\mu_1 = 0\ \ \mu_2 \neq 0$ and $N_2 \tan\theta = \dfrac{mg}{2}$
(B) $\mu_1 \neq 0\ \ \mu_2 = 0$ and $N_1 \tan\theta = \dfrac{mg}{2}$
(C) $\mu_1 \neq 0\ \ \mu_2 \neq 0$ and $N_2 = \dfrac{mg}{1+\mu_1\mu_2}$
(D) $\mu_1 = 0\ \ \mu_2 \neq 0$ and $N_1 \tan\theta = \dfrac{mg}{2}$

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Sol. **C, D**

Condition of translational equilibrium

$N_1 = \mu_2 N_2$

$N_2 + \mu_1 N_1 = Mg$

Solving $N_2 = \dfrac{mg}{1+\mu_1\mu_2}$

$N_1 = \dfrac{\mu_2 mg}{1+\mu_1\mu_2}$

Applying torque equation about corner (left) point on the floor

$mg\dfrac{\ell}{2}\cos\theta = N_1\ell\sin\theta + \mu_1 N_1 \ell\cos\theta$

Solving $\tan\theta = \dfrac{1-\mu_1\mu_2}{2\mu_2}$

SECTION -2: (One Integer Value Correct Type)

This section contains 10 questions. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

11. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{-7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is

Sol. **4**

$Y = \dfrac{FL}{\ell A}$ since the experiment measures only change in the length of wire

$\therefore \dfrac{\Delta Y}{Y}\times 100 = \dfrac{\Delta\ell}{\ell}\times 100$

From the observation ℓ_1 = MSR + 20 (LC)

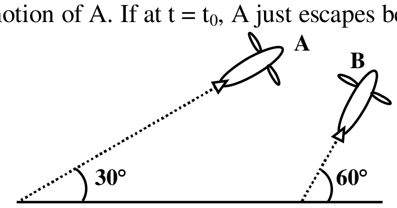
ℓ_2 = MSR + 45 (LC)

⇒ change in lengths = 25(LC)

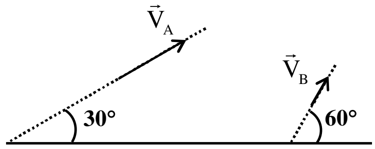
and the maximum permissible error in elongation is one LC

$\therefore \dfrac{\Delta Y}{Y}\times 100 = \dfrac{(LC)}{25(LC)}\times 100 = 4$

12. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in the figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time t = 0 s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is



Sol. **5**



The relative velocity of B with respect to A is perpendicular to line of motion of A.

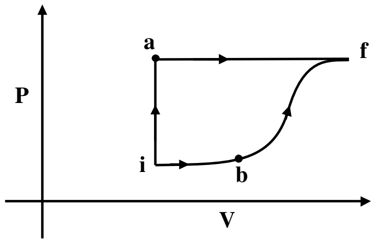
$\therefore \quad V_B \cos 30° = V_A$

$\Rightarrow \quad V_B = 200$ m/s

And time t_0 = (Relative distance) / (Relative velocity)

$= \dfrac{500}{V_B \sin 30°} = 5\,\text{sec}$

13. A thermodynamic system is taken from an initial state *i* with internal energy U_i = 100 J to the final state *f* along two different paths *iaf* and *ibf*, as schematically shown in the figure. The work done by the system along the paths *af*, *ib* and *bf* are W_{af} = 200 J, W_{ib} = 50 J and W_{bf} = 100 J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is U_b = 200 J and Q_{iaf} = 500 J, the ratio $\mathbf{Q_{bf} / Q_{ib}}$ is



Sol. **2**

U_b = 200 J, $U_i = 100$ J

Process *iaf*

Process	W(in Joule)	ΔU(in Joule)	Q(in Joule)
ia		0	
af		200	
Net	300	200	500

$\Rightarrow \quad U_f$ = 400 Joule

Process *ibf*

Process	W(in Joule)	ΔU(in Joule)	Q(in Joule)
ib	100	50	150
bf	200	100	300
Net	300	150	450

$\Rightarrow \quad \dfrac{Q_{bf}}{Q_{ib}} = \dfrac{300}{150} = 2$

14. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\dfrac{X_0}{X_1} = 3$, the value of $\dfrac{R_1}{R_2}$ is

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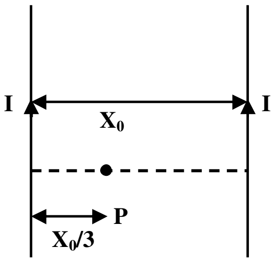
Sol. **3**

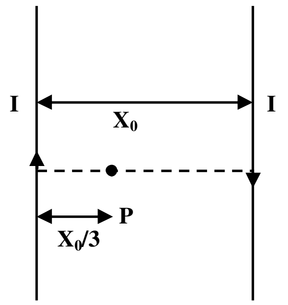
Case – I



$$B_1 = \frac{1}{2}\left(\frac{\mu_0}{2\pi}\right)\left(\frac{3I}{x_0}\right)$$

$$R_1 = \frac{mv}{qB_1}$$

Case – II



$$R_2 = \frac{mv}{qB_2}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{1/3}{1/9} = 3$$

15. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is

Sol. **3**

$d \propto \rho^x S^y F^z$

$\Rightarrow \; [L] \equiv [ML^{-3}]^x [MT^{-3}]^y \; [T^{-1}]^z$

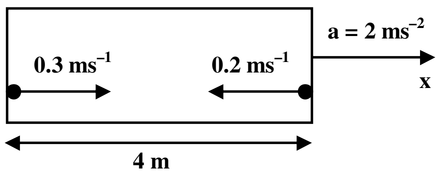
$\Rightarrow \; x + y = 0, -3x = 1, -3y - z = 0$

$\Rightarrow \; x = \frac{-1}{3}, \; y = \frac{1}{3}, \; z = -1$

$\Rightarrow \; y = \frac{1}{n}$

$\Rightarrow \; n = 3$

16. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along + x direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in + x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in −x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is



Sol. **2**

Maximum displacement of the left ball from the left wall of the chamber is 2.25 cm, so the right ball has to travel almost the whole length of the chamber (4m) to hit the left ball. So the time taken by the right ball is 1.9 sec (approximately 2 sec)

17. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0 – 30 V. If connected to a $\frac{2n}{249}\Omega$ resistance, it becomes an ammeter of range 0 – 1.5 A. The value of n is

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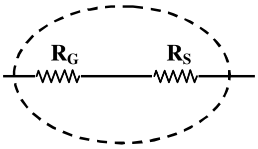
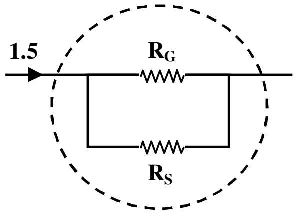
Sol. **5**





$i = \dfrac{V}{R}$

$0.006 = \dfrac{30}{4990 + R}$

R = 10

$i_{RG} = 0.006$

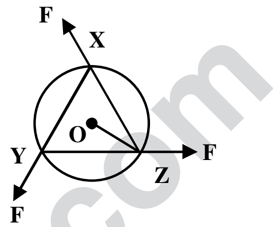
$i_{RS} = 1.494$

Since R_G and R_S are in parallel, $i_G R_G = i_S R_S$

$0.006\,R = 1.494\left(\dfrac{2n}{249}\right)$

$\therefore\ n = 5$

18. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is



Sol. **2**

$\tau = I\alpha$

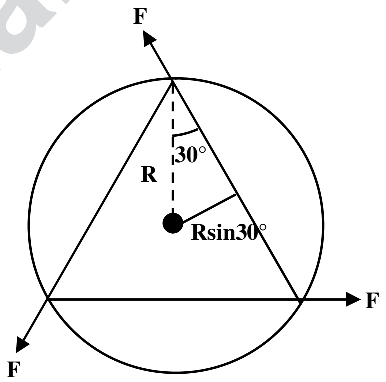
$3\ FR\sin 30° = I\alpha$

$I = \dfrac{MR^2}{2}$

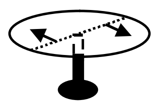
$\alpha = 2$

$\omega = \omega_0 + \alpha t$

$\omega = 2$ rad/s



19. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is



Sol. **4**

Since net torque about centre of rotation is zero, so we can apply conservation of angular momentum of the system about center of disc

$L_i = L_f$

$0 = I\omega + 2mv\,(r/2)$; comparing magnitude

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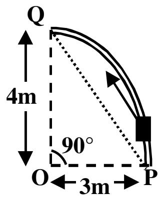
$$\therefore \left(\frac{0.45\times0.5\times0.5}{2}\right)\omega = 0.05\times9\times\frac{0.5}{2}\times2$$

$\therefore\ \omega = 4$

20. Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4 m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n×10) Joules. The value of n is (take acceleration due to gravity = 10 ms^{-2})



Sol. **5**

Using work energy theorem

$W_{mg} + W_F = \Delta KE$

$-mgh + Fd = \Delta KE$

$-1\times10\times4 + 18(5) = \Delta KE$

$\Delta KE = 50$

$\therefore\ n = 5$

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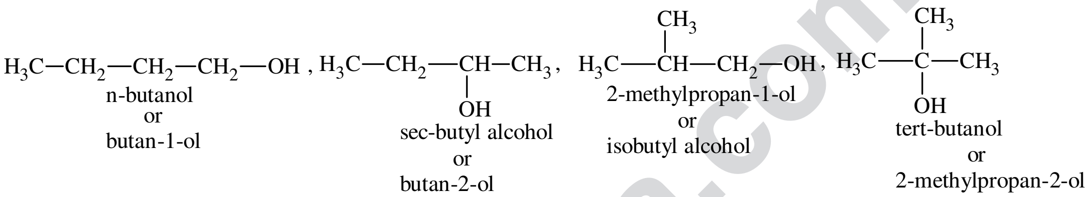
PART-II: CHEMISTRY

SECTION – 1 (One or More Than One Options Correct Type)

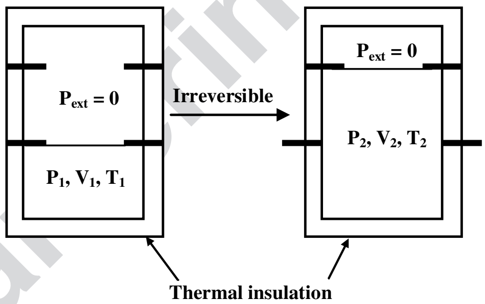
This section contains 10 multiple choice type questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE THAN ONE are correct.

21. The correct combination of names for isomeric alcohols with molecular formula $C_4H_{10}O$ is/are
(A) tert-butanol and 2-methylpropan-2-ol
(B) tert-butanol and 1, 1-dimethylethan-1-ol
(C) n-butanol and butan-1-ol
(D) isobutyl alcohol and 2-methylpropan-1-ol

Sol. **A, C, D**
Isomeric alcohols of $C_4H_{10}O$ are



22. An ideal gas in a thermally insulated vessel at internal pressure = P_1, volume = V_1 and absolute temperature = T_1 expands irreversibly against zero external pressure, as shown in the diagram. The final internal pressure, volume and absolute temperature of the gas are P_2, V_2 and T_2, respectively. For this expansion,



(A) $q = 0$
(B) $T_2 = T_1$
(C) $P_2V_2 = P_1V_1$
(D) $P_2V_2^{\gamma} = P_1V_1^{\gamma}$

Sol. **A, B, C**
Since container is thermally insulated. So, q = 0, and it is a case of free expansion therefore W = 0 and $\Delta E = 0$
So, $T_1 = T_2$
Also, $P_1V_1 = P_2V_2$

23. Hydrogen bonding plays a central role in the following phenomena:
(A) Ice floats in water.
(B) Higher Lewis basicity of primary amines than tertiary amines in aqueous solutions.
(C) Formic acid is more acidic than acetic acid.
(D) Dimerisation of acetic acid in benzene.

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Sol. **A, B, D**

(A) Ice has cage-like structure in which each water molecule is surrounded by four other water molecules tetrahedrally through hydrogen boding, due to this density of ice is less than water and it floats in water.

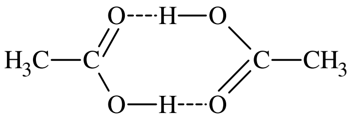
(B) $R-NH_2 + H-OH \rightleftharpoons \underset{(I)}{R-\overset{\oplus}{N}H_3} + OH^-$

$(R)_3N + H-OH \rightleftharpoons \underset{(II)}{(R)_3-\overset{\oplus}{N}H} + OH^-$

The cation (I) more stabilized through hydrogen boding than cation (II). So, R–NH_2 is better base than $(R)_3N$ in aqueous solution.

(C) HCOOH is stronger acid than CH_3COOH due to inductive effect and not due to hydrogen bonding.

(D) Acetic acid dimerizes in benzene through intermolecular hydrogen bonding.



24. In a galvanic cell, the salt bridge
(A) does not participate chemically in the cell reaction.
(B) stops the diffusion of ions from one electrode to another.
(C) is necessary for the occurrence of the cell reaction.
(D) ensures mixing of the two electrolytic solutions.

Sol. **A**

25. For the reaction:

$I^- + ClO_3^- + H_2SO_4 \longrightarrow Cl^- + HSO_4^- + I_2$

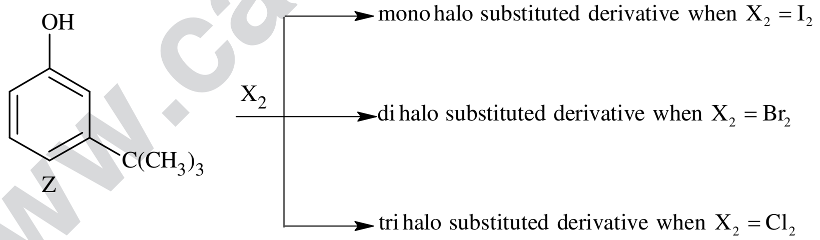
The correct statement(s) in the balanced equation is/are:
(A) Stoichiometric coefficient of HSO_4^- is 6.
(B) Iodide is oxidized.
(C) Sulphur is reduced.
(D) H_2O is one of the products.

Sol. **A, B, D**

The balanced equation is,

$ClO_3^- + 6I^- + 6H_2SO_4 \longrightarrow 3I_2 + Cl^- + 6HSO_4^- + 3H_2O$

26. The reactivity of compound Z with different halogens under appropriate conditions is given below:



The observed pattern of electrophilic substitution can be explained by
(A) the steric effect of the halogen
(B) the steric effect of the tert-butyl group
(C) the electronic effect of the phenolic group
(D) the electronic effect of the tert-butyl group

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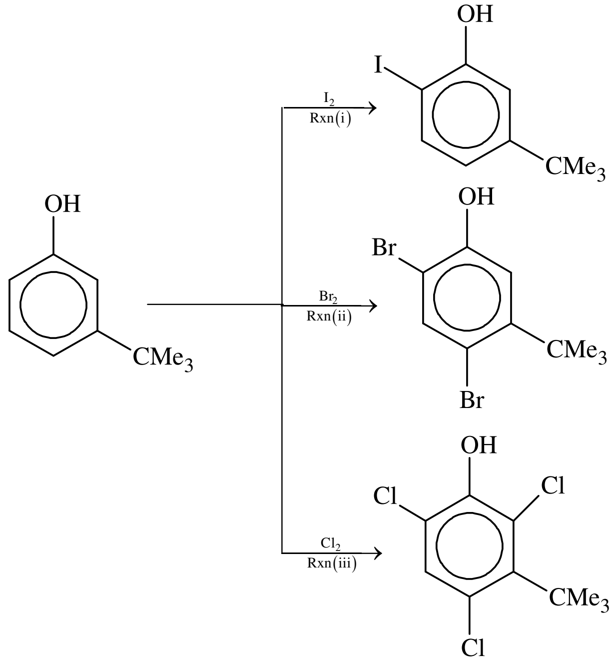
Sol. **A, B, C**

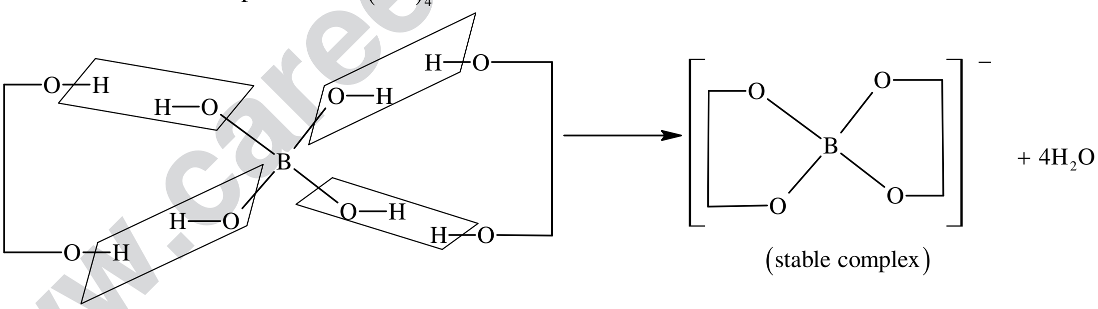


27. The correct statement(s) for orthoboric acid is/are
(A) It behaves as a weak acid in water due to self ionization.
(B) Acidity of its aqueous solution increases upon addition of ethylene glycol.
(C) It has a three dimensional structure due to hydrogen bonding.
(D) It is a weak electrolyte in water.

Sol. **B, D**
(A) H_3BO_3 is a weak monobasic Lewis acid.

$$H_3BO_3 + H-OH \rightleftharpoons B(OH)_4^- + H^+ \quad \ldots \text{(i)}$$

(B) Equilibrium (i) is shifted in forward direction by the addition of syn-diols like ethylene glycol which forms a stable complex with $B(OH)_4^-$.



(C) It has a planar sheet like structure due to hydrogen bonding.

(D) H_3BO_3 is a weak electrolyte in water.

28. Upon heating with Cu_2S, the reagent(s) that give copper metal is/are
(A) $CuFeS_2$
(B) CuO
(C) Cu_2O
(D) $CuSO_4$

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Sol. **B, C, D**

(A) $2CuFeS_2 + O_2 \xrightarrow{\Delta} Cu_2S + 2FeS + SO_2$

(B) $4CuO \xrightarrow{1100^{\circ}C} 2Cu_2O + O_2$

$2Cu_2O + Cu_2S \xrightarrow{\Delta} 6Cu + SO_2$

(C) $Cu_2S + 2Cu_2O \xrightarrow{\Delta} 6Cu + SO_2$

(D) $CuSO_4 \xrightarrow{720^{\circ}C} CuO + SO_2 + \frac{1}{2}O_2$

$4CuO \xrightarrow{1100^{\circ}C} 2Cu_2O + O_2$

$2Cu_2O + Cu_2S \xrightarrow{\Delta} 6Cu + SO_2$

29. The pair(s) of reagents that yield paramagnetic species is/are

(A) Na and excess of NH_3
(B) K and excess of O_2
(C) Cu and dilute HNO_3
(D) O_2 and 2- ethylanthraquinol

Sol. **A, B, C**

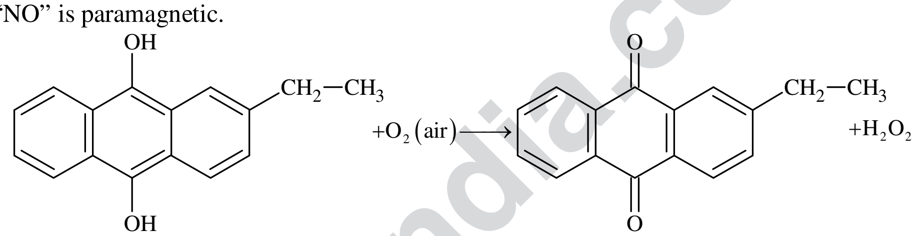
(A) sodium (Na) when dissolved in excess liquid ammonia, forms a blue coloured paramagnetic solution.

(B) $K + O_2 \longrightarrow \underset{\text{(potassium superoxide)}}{KO_2}$ and KO_2 is paramagnetic.

(C) $3Cu + \underset{\text{(dilute)}}{8HNO_3} \longrightarrow 3Cu(NO_3)_2 + 2NO + 4H_2O$

Where "NO" is paramagnetic.

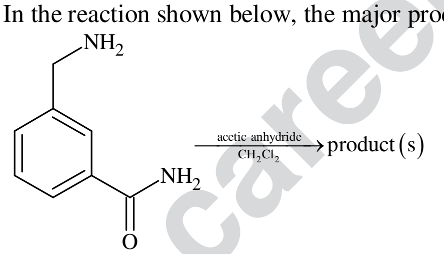
(D)



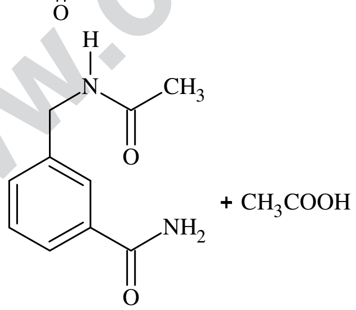
(2 – ethyl anthraquinol)

Where "H_2O_2" is diamagnetic.

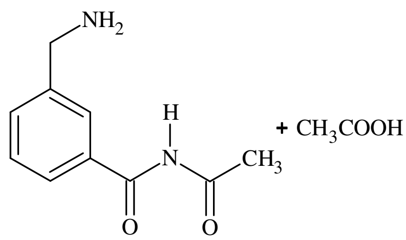
30. In the reaction shown below, the major product(s) formed is/are



(A)



(B)



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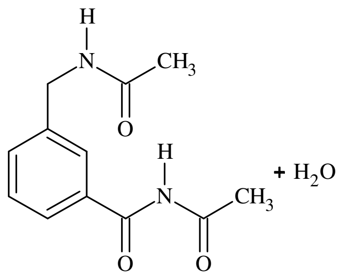
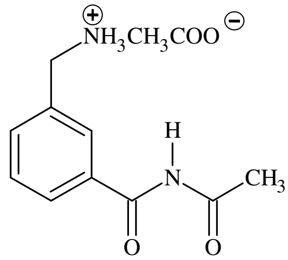
(C)

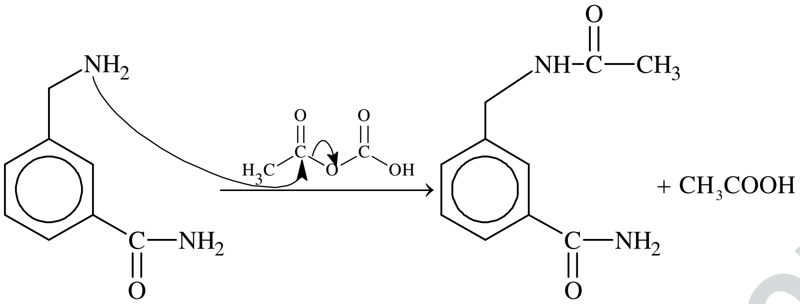


(D)



Sol. **A**

Only amines undergo acetylation and not acid amides.



SECTION – 2: (Only Integer Value Correct Type)

This section contains 10 questions. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

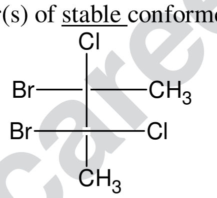
31. Among PbS, CuS, HgS, MnS, Ag_2S, NiS, CoS, Bi_2S_3 and SnS_2, the total number of BLACK coloured sulphides is

Sol. **6**

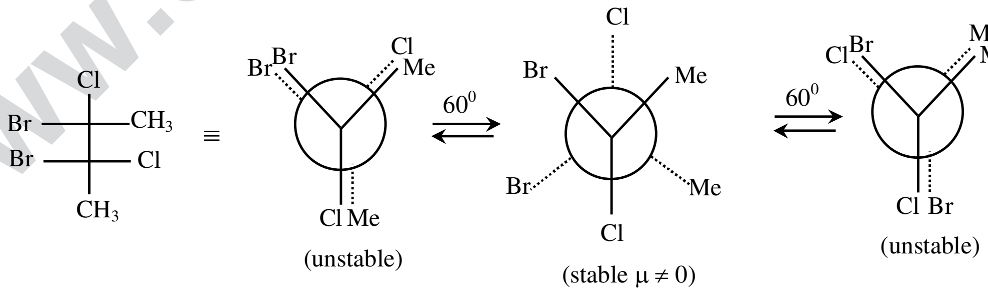
Black coloured sulphides $\{PbS, CuS, HgS, Ag_2S, NiS, CoS\}$

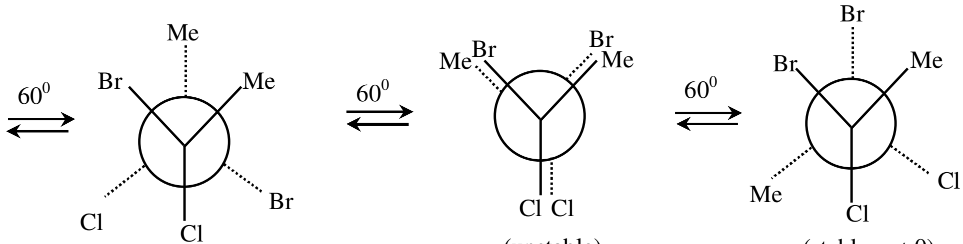
*** Bi_2S_3 in its crystalline form is dark brown but Bi_2S_3 precipitate obtained is black in colour.**

32. The total number(s) of stable conformers with non-zero dipole moment for the following compound is(are)



Sol. **3**





(stable $\mu \neq 0$) (unstable) (stable $\mu \neq 0$)

33. Consider the following list of reagents:
Acidified $K_2Cr_2O_7$, alkaline $KMnO_4, CuSO_4, H_2O_2, Cl_2, O_3, FeCl_3, HNO_3$ and $Na_2S_2O_3$.
The total number of reagents that can oxidise aqueous iodide to iodine is

Sol. **7**

$K_2Cr_2O_7, CuSO_4, H_2O_2, Cl_2, O_3, FeCl_3, HNO_3$

$K_2Cr_2O_7 + 7H_2SO_4 + 6KI \longrightarrow 4K_2SO_4 + Cr_2(SO_4)_3 + 3I_2 + 7H_2O$

$2CuSO_4 + 4KI \longrightarrow Cu_2I_2 + I_2 + 2K_2SO_4$

$H_2O_2 + 2KI \longrightarrow I_2 + 2KOH$

$Cl_2 + 2KI \longrightarrow 2KCl + I_2$

$O_3 + 2KI + H_2O \longrightarrow 2KOH + I_2 + O_2$

$2FeCl_3 + 2KI \longrightarrow 2FeCl_2 + I_2 + 2KCl$

$8HNO_3 + 6KI \longrightarrow 6KNO_3 + 2NO + 4H_2O + 3I_2$

$2KMnO_4 + KI + H_2O \longrightarrow KIO_3 + 2MnO_2 + 2KOH$

34. A list of species having the formula XZ_4 is given below.
$XeF_4, SF_4, SiF_4, BF_4^-, BrF_4^-, \left[Cu(NH_3)_4\right]^{2+}, \left[FeCl_4\right]^{2-}, \left[CoCl_4\right]^{2-}$ and $\left[PtCl_4\right]^{2-}$.
Defining shape on the basis of the location of X and Z atoms, the total number of species having a square planar shape is

Sol. **4**

$XeF_4 \rightarrow$ Square planar

$BrF_4^- \rightarrow$ Square planar

$\left[Cu(NH_3)_4\right]^{2+} \rightarrow$ Square planar

$\left[Pt\,Cl_4\right]^{2-} \rightarrow$ Square planar

$SF_4 \rightarrow$ See – saw

$SiF_4 \rightarrow$ Tetrahedral

$BF_4^- \rightarrow$ Tetrahedral

$\left[FeCl_4\right]^{2-} \rightarrow$ Tetrahedral

$\left[CoCl_4\right]^{2-} \rightarrow$ Tetrahedral

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35. Consider all possible isomeric ketones, including stereoisomers of MW = 100. All these isomers are independently reacted with $NaBH_4$ (NOTE: stereoisomers are also reacted separately). The total number of ketones that give a racemic product(s) is/are

Sol. **5**

(1) $CH_3-CH_2-\overset{*}{C}H(CH_3)-\overset{O}{\overset{\parallel}{C}}-CH_3$ Will not give a racemic mixture on reduction with $NaBH_4$

(2) $CH_3-CH_2-CH_2-CH_2-\overset{O}{\overset{\parallel}{C}}-CH_3$ Will give a racemic mixture on reduction with $NaBH_4$

(3) $CH_3-CH(CH_3)-CH_2-\overset{O}{\overset{\parallel}{C}}-CH_3$ Will give a racemic mixture on reduction with $NaBH_4$

(4) $H_3C-C(CH_3)_2-\overset{O}{\overset{\parallel}{C}}-CH_3$ Will give a racemic mixture on reduction with $NaBH_4$

(5) $CH_3-CH_2-\overset{O}{\overset{\parallel}{C}}-CH_2-CH_2-CH_3$ Will give a racemic mixture on reduction with $NaBH_4$

(6) $CH_3-CH_2-\overset{O}{\overset{\parallel}{C}}-CH(CH_3)-CH_3$ Will give a racemic mixture on reduction with $NaBH_4$

36. In an atom, the total number of electrons having quantum numbers $n = 4$, $|m_l| = 1$ and $m_s = -1/2$ is

Sol. **6**

$n = 4$

$\ell = 0,1,2,3$

$|m_\ell| = 1 \Rightarrow \pm 1$

$m_s = -\frac{1}{2}$

For $\ell = 0,\ m_\ell = 0$

$\ell = 1,\ m_\ell = -1,\ 0,\ +1$

$\ell = 2,\ m_\ell = -2, -1, 0, +1, +2$

$\ell = 3,\ m_\ell = -3, -2, -1, 0, +1, +2, +3$

So, six electrons can have $|m_\ell| = 1$ & $m_s = -\frac{1}{2}$

37. If the value of Avogadro number is 6.023×10^{23} mol^{-1} and the value of Boltzmann constant is $1.380 \times 10^{-23}\, JK^{-1}$, then the number of significant digits in the calculated value of the universal gas constant is

Sol. **4**

$k = \frac{R}{N_A}$

$R = kN_A$

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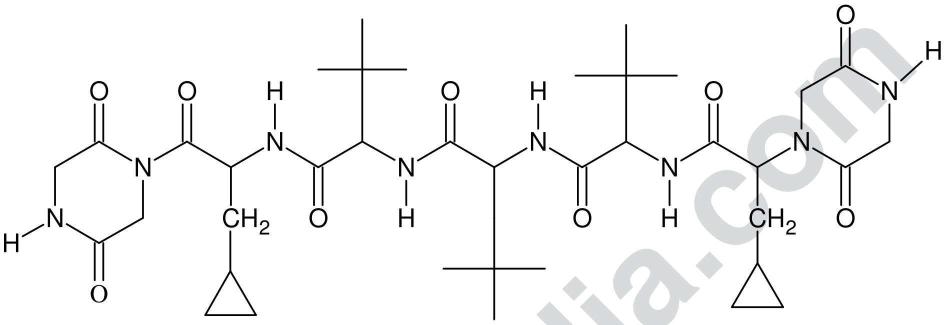
$= 1.380 \times 10^{-23} \times 6.023 \times 10^{23}$
$= 8.31174$
≈ 8.312

38. MX_2 dissociates in M^{2+} and X^- ions in an aqueous solution, with a degree of dissociation (α) of 0.5. The ratio of the observed depression of freezing point of the aqueous solution to the value of the depression of freezing point in the absence of ionic dissociation is

Sol. **2**

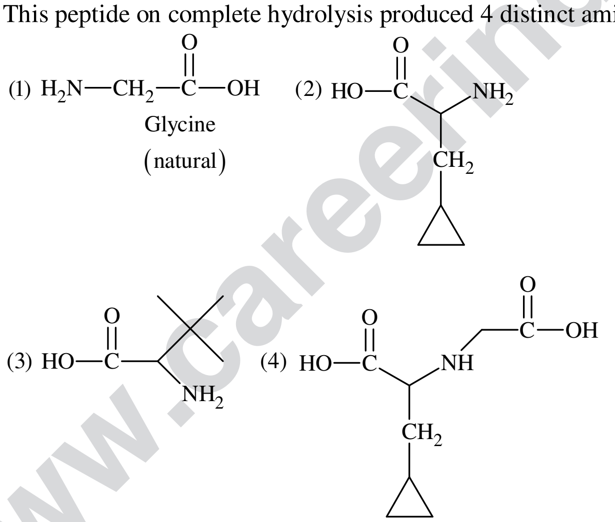
$MX_2 \rightleftharpoons M^{2+} + 2X^-$
$1-\alpha \qquad \alpha \qquad 2\alpha$
$i = 1 + 2\alpha \qquad \{\alpha = 0.5\}$
$i = 2$

39. The total number of *distinct naturally occurring amino acids* obtained by complete acidic hydrolysis of the peptide shown below is



Sol. **1**

This peptide on complete hydrolysis produced 4 distinct amino acids which are given below:



Only glycine is naturally occurring amino acid.

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40. A compound H_2X with molar weight of 80g is dissolved in a solvent having density of 0.4 gml^{-1}. Assuming no change in volume upon dissolution, the molality of a 3.2 molar solution is

Sol. **8**

Here, $V_{solution} \approx V_{solvent}$

Since, in 1ℓ solution , 3.2 moles of solute are present,

So, 1ℓ solution $\approx$ 1ℓ solvent (d = 0.4g/ml) $\approx$ 0.4 kg

So, molality $\left(m\right) = \frac{\text{moles of solute}}{\text{mass of solvent}\left(kg\right)} = \frac{3.2}{0.4} = 8$

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PART-III: MATHEMATICS

SECTION – 1 : (One or More than One Options Correct Type)

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE** or **MORE THAN ONE are correct.**

41. Let $f : (0, \infty) \to R$ be given by $f(x) = \int\limits_{1/x}^{x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$, then

(A) $f(x)$ is monotonically increasing on $[1, \infty)$
(B) $f(x)$ is monotonically decreasing on $(0, 1)$
(C) $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in (0, \infty)$
(D) $f(2^x)$ is an odd function of x on R

Sol. **A, C, D**

$$f'(x) = \frac{2e^{-\left(x+\frac{1}{x}\right)}}{x}$$

Which is increasing in $[1, \infty)$

Also, $f(x) + f\left(\frac{1}{x}\right) = 0$

$$g(x) = f(2^x) = \int\limits_{2^{-x}}^{2^x} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt$$

$$g(-x) = \int\limits_{2^x}^{2^{-x}} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt = -g(x)$$

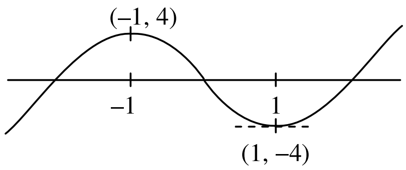
Hence, an odd function

42. Let $a \in R$ and let $f : R \to R$ be given by $f(x) = x^5 - 5x + a$, then

(A) $f(x)$ has three real roots if $a > 4$
(B) $f(x)$ has only one real roots if $a > 4$
(C) $f(x)$ has three real roots if $a < -4$
(D) $f(x)$ has three real roots if $-4 < a < 4$

Sol. **B, D**

Let $y = x^5 - 5x$



43. For every pair of continuous functions $f, g : [0, 1] \to R$ such that $\max\{f(x) : x \in [0, 1]\} = \max\{g(x) : x \in [0, 1]\}$, the correct statement(s) is(are)

(A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$
(B) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$
(C) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0, 1]$
(D) $(f(c))^2 = (g(c))^2$ for some $c \in [0, 1]$

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Sol. **A, D**

Let f (x) and g (x) achieve their maximum value at x_1 and x_2 respectively

$h(x) = f(x) - g(x)$

$h(x_1) = f(x_1) - g(x_1) \geq 0$

$h(x_2) = f(x_2) - g(x_2) \leq 0$

$\Rightarrow h(c) = 0$ where $c \in [0, 1] \Rightarrow f(c) = g(c)$.

44. A circle S passes through the point (0, 1) and is orthogonal to the circles $(x-1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then

(A) radius of S is 8
(B) radius of S is 7
(C) centre of S is (–7, 1)
(D) centre of S is (–8, 1)

Sol. **B, C**

Given circles

$x^2 + y^2 - 2x - 15 = 0$

$x^2 + y^2 - 1 = 0$

Radical axis $x + 7 = 0$ … (1)

Centre of circle lies on (1)

Let the centre be $(-7, k)$

Let equation be $x^2 + y^2 + 14x - 2ky + c = 0$

Orthogonallity gives

$-14 = c - 15 \Rightarrow c = 1$ … (2)

$(0, 1) \to 1 - 2k + 1 = 0 \Rightarrow k = 1$

Hence radius $= \sqrt{7^2 + k^2 - c} = \sqrt{49+1-1} = 7$

Alternate solution

Given circles $x^2 + y^2 - 2x - 15 = 0$

$x^2 + y^2 - 1 = 0$

Let equation of circle $x^2 + y^2 + 2gx + 2fy + c = 0$

Circle passes through (0, 1)

$\Rightarrow 1 + 2f + c = 0$

Applying condition of orthogonality

$-2g = c - 15, 0 = c - 1$

$\Rightarrow c = 1, g = 7, f = -1$

$r = \sqrt{49+1-1} = 7$; centre (–7, 1)

45. Let $\vec{x}$, $\vec{y}$ *and* $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a non-zero vector perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$ and $\vec{b}$ is a non-zero vector perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$, then

(A) $\vec{b} = (\vec{b}\cdot\vec{z})(\vec{z}-\vec{x})$
(B) $\vec{a} = (\vec{a}\cdot\vec{y})(\vec{y}-\vec{z})$
(C) $\vec{a}\cdot\vec{b} = -(\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})$
(D) $\vec{a} = (\vec{a}\cdot\vec{y})(\vec{z}-\vec{y})$

Sol. **A, B, C**

$\vec{a}$ is in direction of $\vec{x}\times(\vec{y}\times\vec{z})$

i.e. $(\vec{x}\cdot\vec{z})\vec{y} - (\vec{x}\cdot\vec{y})\vec{z}$

$\Rightarrow \vec{a} = \lambda_1\left[2\times\frac{1}{2}(\vec{y}-\vec{z})\right]$

$\vec{a} = \lambda_1(\vec{y}-\vec{z})$ … (1)

Now $\vec{a}\cdot\vec{y} = \lambda_1(\vec{y}\cdot\vec{y} - \vec{y}\cdot\vec{z})$

$= \lambda_1 (2 - 1) \Rightarrow \lambda_1 = \vec{a} \cdot \vec{y}$ … (2)

From (1) and (2), $\vec{a} = \vec{a} \cdot \vec{y}\left(\vec{y} - \vec{z}\right)$

Similarly, $\vec{b} = \left(\vec{b} \cdot \vec{z}\right)\left(\vec{z} - \vec{x}\right)$

Now, $\vec{a} \cdot \vec{b} = \left(\vec{a} \cdot \vec{y}\right)\left(\vec{b} \cdot \vec{z}\right)\left[\left(\vec{y} - \vec{z}\right) \cdot \left(\vec{z} - \vec{x}\right)\right]$

$= \left(\vec{a} \cdot \vec{y}\right)\left(\vec{b} \cdot \vec{z}\right)[1 - 1 - 2 + 1]$

$= -\left(\vec{a} \cdot \vec{y}\right)\left(\vec{b} \cdot \vec{z}\right)$.

46. From a point P(λ, λ, λ), perpendiculars PQ and PR are drawn respectively on the lines $y = x$, $z = 1$ and $y = -x$, $z = -1$. If P is such that ∠QPR is a right angle, then the possible value(s) of λ is(are)

(A) $\sqrt{2}$ (B) 1

(C) −1 (D) $-\sqrt{2}$

Sol. **C**

Line 1: $\dfrac{x}{1} = \dfrac{y}{1} = \dfrac{z-1}{0} = r$, Q (r, r, 1)

Line 2: $\dfrac{x}{1} = \dfrac{y}{-1} = \dfrac{z+1}{0} = k$, R (k, − k, − 1)

$\overrightarrow{PQ} = (\lambda - r)\,\hat{i} + (\lambda - r)\,\hat{j} + (\lambda - 1)\,\hat{k}$

and $\lambda - r + \lambda - r = 0$ as $\overrightarrow{PQ}$ is ⊥ to L_1

$\Rightarrow 2\lambda = 2r \Rightarrow \lambda = r$

$\overrightarrow{PR} = (\lambda - k)\hat{i} + (\lambda + k)\hat{j} + (\lambda + 1)\hat{k}$

and $\lambda - k - \lambda - k = 0$ as $\overrightarrow{PR}$ is ⊥ to L_2

$\Rightarrow k = 0$

so PQ ⊥ PR

$(\lambda - r)(\lambda - k) + (\lambda - r)(\lambda + k) + (\lambda - 1)(\lambda + 1) = 0$

$\Rightarrow \lambda = 1, -1$

For λ = 1 as points P and Q coincide

$\Rightarrow \lambda = -1$.

47. Let M be a 2 × 2 symmetric matrix with integer entries. Then M is invertible if

(A) the first column of M is the transpose of the second row of M

(B) the second row of M is the transpose of the first column of M

(C) M is a diagonal matrix with non-zero entries in the main diagonal

(D) the product of entries in the main diagonal of M is not the square of an integer

Sol. **C, D**

Let $M = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$ (where a, b, c ∈ I)

then Det M = $ab - c^2$

if a = b = c, Det(M) = 0

if c = 0, a, b ≠ 0, Det(M) ≠ 0

if ab ≠ square of integer, Det(M) ≠ 0

48. Let M and N be two 3 × 3 matrices such that MN = NM. Further, if $M \neq N^2$ and $M^2 = N^4$, then

(A) determinant of $(M^2 + MN^2)$ is 0

(B) there is a 3 × 3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix

(C) determinant of $(M^2 + MN^2) \geq 1$

(D) for a 3 × 3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix

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Sol. **A, B**

$M^2 = N^4 \Rightarrow M^2 - N^4 = O \Rightarrow (M - N^2)(M + N^2) = O$

As M, N commute.

Also, $M \neq N^2$, Det $((M - N^2)(M + N^2)) = 0$

As $M - N^2$ is not null $\Rightarrow$ Det $(M + N^2) = 0$

Also Det $(M^2 + MN^2) = $ (Det M) (Det $(M + N^2)) = 0$

$\Rightarrow$ There exist non-null U such that $(M^2 + MN^2)\,U = O$

49. Let $f : [a, b] \to [1, \infty)$ be a continuous function and let $g : R \to R$ be defined as

$$g(x) = \begin{cases} 0 & \text{if} \quad x < a, \\ \int_a^x f(t)\,dt & \text{if} \quad a \le x \le b, \\ \int_a^b f(t)\,dt & \text{if} \quad x > b \end{cases}$$

Then

(A) g(x) is continuous but not differentiable at *a*

(B) g(x) is differentiable on R

(C) g(x) is continuous but not differentiable at *b*

(D) g(x) is continuous and differentiable at either *a* or *b* but not both

Sol. **A, C**

Since $f(x) \ge 1 \ \forall\ x \in [a, b]$

for g(x)

LHD at x = a is zero

and RHD at (x = a) = $\lim\limits_{x \to a^+} \dfrac{\int_a^x f(t)\,dt - 0}{x - a} = \lim\limits_{x \to a^+} f(x) \ge 1$

Hence g(x) is not differentiable at x = a

Similarly LHD at x = b is greater than 1

g(x) is not differentiable at x = b

50. Let $f : \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right) \to R$ be given by $f(x) = (\log(\sec x + \tan x))^3$. Then

(A) $f(x)$ is an odd function
(B) $f(x)$ is a one-one function
(C) $f(x)$ is an onto function
(D) $f(x)$ is an even function

Sol. **A, B, C**

$f(x) = (\log(\sec x + \tan x))^3 \quad \forall\ x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$f(-x) = -f(x)$, hence f (x) is odd function

Let $g(x) = \sec x + \tan x \quad \forall\ x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$\Rightarrow g'(x) = \sec x(\sec x + \tan x) > 0\ \forall\ x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$\Rightarrow$ g (x) is one-one function

Hence $(\log_e(g(x)))^3$ is one-one function.

and $g(x) \in (0, \infty)\ \forall\ x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$\Rightarrow \log(g(x)) \in R$. Hence f (x) is an onto function.

SECTION – 2 : (One Integer Value Correct Type)

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This section contains 10 questions. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

51. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is _________

Sol. **7**

When n_5 takes value from 10 to 6 the carry forward moves from 0 to 4 which can be arranged in

$${}^4C_0 + \frac{{}^4C_1}{4} + \frac{{}^4C_2}{3} + \frac{{}^4C_3}{2} + \frac{{}^4C_4}{1} = 7$$

Alternate solution

Possible solutions are

1, 2, 3, 4, 10
1, 2, 3, 5, 9
1, 2, 3, 6, 8
1, 2, 4, 5, 8
1, 2, 4, 6, 7
1, 3, 4, 5, 7
2, 3, 4, 5, 6

Hence 7 solutions are there.

52. Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is _________

Sol. **5**

Number of red lines = ${}^nC_2 - n$
Number of blue lines = n
Hence, ${}^nC_2 - n = n$
${}^nC_2 = 2n$

$$\frac{n(n-1)}{2} = 2n$$

$n - 1 = 4 \Rightarrow n = 5$.

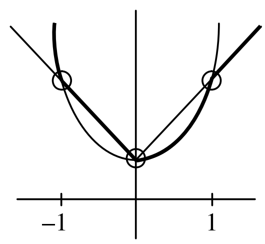
53. Let $f : R \to R$ and $g : R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h : R \to R$ by

$$h(x) = \begin{cases} \max \quad \{f(x), g(x)\} & if \quad x \leq 0 \\ \min \quad \{f(x), g(x)\} & if \quad x > 0 \end{cases}.$$

Then number of points at which h(x) is not differentiable is __________

Sol. **3**

$$h(x) = \begin{cases} x^2+1 & , \quad x \in (-\infty, -1] \\ -x+1 & , \quad x \in [-1, 0] \\ x^2+1 & , \quad x \in [0, 1] \\ x+1 & , \quad x \in [1, \infty) \end{cases}$$



Hence, not differentiable at x = –1, 0, 1

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54. Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is $b + 2$, then the value of $\frac{a^2+a-14}{a+1}$ is ________

Sol. **4**

$\frac{b}{a} = \frac{c}{b} = \text{(integer)}$

$b^2 = ac \Rightarrow c = \frac{b^2}{a}$

$\frac{a+b+c}{3} = b+2$

$a + b + c = 3b + 6 \Rightarrow a - 2b + c = 6$

$a - 2b + \frac{b^2}{a} = 6 \Rightarrow 1 - \frac{2b}{a} + \frac{b^2}{a^2} = \frac{6}{a}$

$\left(\frac{b}{a} - 1\right)^2 = \frac{6}{a} \Rightarrow a = 6$ only

55. Let $\vec{a}$, $\vec{b}$, *and* $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c} = p\vec{a}+q\vec{b}+r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is ______

Sol. **4**

$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$\vec{a}\times\vec{b}+\vec{b}\times\vec{c} = p\vec{a}+q\vec{b}+r\vec{c}$

$\vec{a}\cdot(\vec{b}\times\vec{c}) = p + q(\vec{a}\cdot\vec{b}) + r(\vec{a}\cdot\vec{c})$

And $[\vec{a}\ \ \vec{b}\ \ \vec{c}] = \frac{1}{\sqrt{2}}$

$p + \frac{q}{2} + \frac{r}{2} = [\vec{a}\ \vec{b}\ \vec{c}]$ ….. (1)

$\frac{p}{2} + q + \frac{r}{2} = 0$ ….. (2)

$\frac{p}{2} + \frac{q}{2} + r = [\vec{a}\ \vec{b}\ \vec{c}]$ ….. (3)

$\Rightarrow p = r = -q$

$\frac{p^2+2q^2+r^2}{q^2} = 4$

56. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point (1, 3) is ______

Sol. **8**

$2\,(y - x^5)\left(\frac{dy}{dx} - 5x^4\right)$

$= 1\,(1 + x^2)^2 + (x)\,(2\,(1 + x^2)\,(2x))$

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Now put x = 1, y = 3 and $\frac{dy}{dx} = m$.

$2(3-1)(m-5) = 1(4) + (1)(4)(2)$

$m - 5 = \frac{12}{4}$

$m = 5 + 3 = 8$

$\frac{dy}{dx} = m = 8$.

57. The value of $\int_0^1 4x^3 \left\{\frac{d^2}{dx^2}(1-x^2)^5\right\} dx$ is ___________

Sol. **2**

$$\int_0^1 \underset{\mathrm{I}}{4x^3} \frac{d^2}{dx^2}\underset{\mathrm{II}}{(1-x^2)^5}\,dx$$

$$= \left[4x^3 \frac{d}{dx}(1-x^2)^5\right]_0^1 - \int_0^1 12x^2 \frac{d}{dx}(1-x^2)^5\,dx$$

$$= \left[4x^3 \times 5(1-x^2)^4(-2x)\right]_0^1 - 12\left[\left[x^2(1-x^2)^5\right]_0^1 - \int_0^1 2x(1-x^2)^5\,dx\right]$$

$$= 0 - 0 - 12[0-0] + 12\int_0^1 2x(1-x^2)^5\,dx$$

$$= 12 \times \left[-\frac{(1-x^2)^6}{6}\right]_0^1$$

$$= 12\left[0 + \frac{1}{6}\right] = 2$$

58. The largest value of the non-negative integer a for which $\lim_{x\to1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$ is ______

Sol. **2**

$$\lim_{x\to1}\left(\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right)^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$

$$\lim_{x\to1}\left(\frac{\frac{\sin(x-1)}{(x-1)} - a}{\frac{\sin(x-1)}{(x-1)} + 1}\right)^{(1+\sqrt{x})} = \frac{1}{4} \Rightarrow \left(\frac{1-a}{2}\right)^2 = \frac{1}{4}$$

$\Rightarrow a = 0, a = 2$

$\Rightarrow a = 2$

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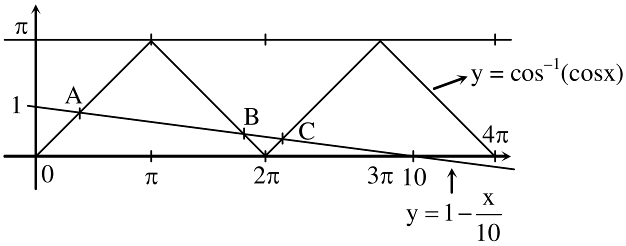
59. Let $f : [0, 4\pi] \to [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation $f(x) = \frac{10-x}{10}$ is ___________

Sol. **3**

$f : [0, 4\pi] \to [0, \pi]$, $f(x) = \cos^{-1}(\cos x)$

$\Rightarrow$ point A, B, C satisfy $f(x) = \frac{10-x}{10}$

Hence, 3 points



60. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is ________

Sol. **6**

$2 \le d_1(p) + d_2(p) \le 4$

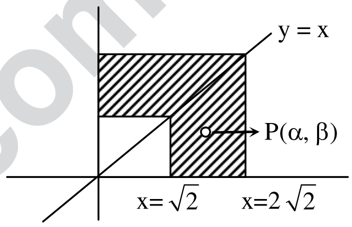
For $P(\alpha, \beta)$, $\alpha > \beta$

$\Rightarrow 2\sqrt{2} \le 2\alpha \le 4\sqrt{2}$

$\sqrt{2} \le \alpha \le 2\sqrt{2}$

$\Rightarrow$ Area of region $= \left(\left(2\sqrt{2}\right)^2 - \left(\sqrt{2}\right)^2\right)$

$= 8 - 2 = 6$ sq. units



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CODE 5

D. Marking Scheme

16. For each questions in **Section1,** you will be awarded **3 marks** if you darken all the bubbles(s) corresponding to the correct answer(s) and **zero mark** if no bubbles are darkened. **No negative** marks will be awarded for incorrect answers in this section.

17. For each question in **Section 2**, you will be awarded **3 marks** if you darken only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. **No negative** marks will be awarded from incorrect answer in this section.

Appropriate way of darkening the bubble for your answer to be evaluated:

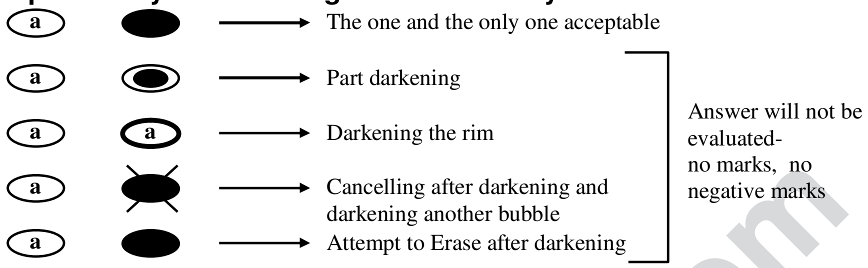

Figure-1 : Correct way of bubbling for a valid answer and a few examples of invalid answers.
Any other form of partial marking such as ticking or crossing the bubble will be considered invalid.

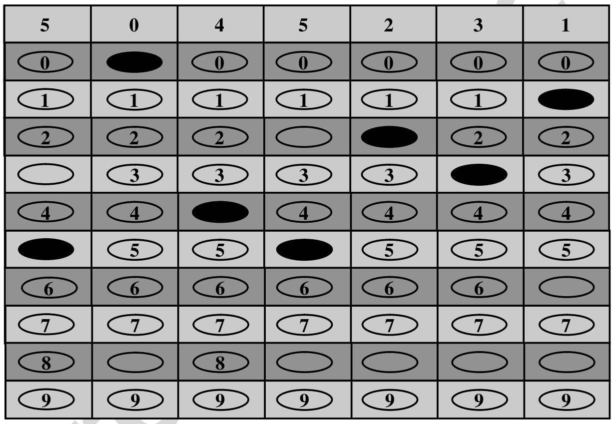

Figure-2 : Correct Way of Bubbling your Roll Number on the ORS. (Example Roll Number : 5045231)

Name of the Candidate	Roll Number
I have read all instructions and shall abide by them. …………………………………………… Signature of the Candidate	I have verified all the information filled by the candidate. …………………………………………… Signature of the invigilator