1. A variable line \( \frac{x}{a} + \frac{y}{b} = 1 \) is such that \( a + b = 4 \). The locus of the midpoint of the portion of the line intercepted between the axes is
   1) \( x + y = 4 \)  
   2) \( x + y = 8 \)  
   3) \( x + y = 1 \)  
   4) \( x + y = 2 \)

2. The point \((5, -7)\) lies outside the circle
   1) \( x^2 + y^2 - 8x = 0 \)  
   2) \( x^2 + y^2 - 5x + 7y = 0 \)  
   3) \( x^2 + y^2 - 5x + 7y - 1 = 0 \)  
   4) \( x^2 + y^2 - 8x + 7y - 2 = 0 \)

3. If the circles \( x^2 + y^2 = 9 \) and \( x^2 + y^2 + 2\alpha x + 2y + 1 = 0 \) touch each other internally, then \( \alpha = \)
   1) \( \pm \frac{4}{3} \)  
   2) \( 1 \)  
   3) \( \frac{4}{3} \)  
   4) \( -\frac{4}{3} \)

4. The locus of the midpoints of the line joining the focus and any point on the parabola \( y^2 = 4ax \) is a parabola with the equation of directrix as
   1) \( x + a = 0 \)  
   2) \( 2x + a = 0 \)  
   3) \( x = 0 \).  
   4) \( x = \frac{a}{2} \)

5. The tangents drawn at the extremities of a focal chord of the parabola \( y^2 = 16x \)
   1) intersect on \( x = 0 \)  
   2) intersect on the line \( x + 4 = 0 \)  
   3) intersect at an angle of \( 60^0 \)  
   4) intersect at an angle of \( 45^0 \)

(Space for Rough Work)
6. On the set \( Z \), of all integers \(*\) is defined by \( a * b = a + b - 5 \). If \( 2 * (x * 3) = 5 \) then \( x = \)

1) 0  
2) 3  
3) 5  
4) 10

7. Which of the following is false?
1) Addition is commutative in \( N \).
2) Multiplication is associative in \( N \).
3) If \( a * b = a^b \) for all \( a, b \in N \) then \(*\) is commutative in \( N \).
4) Addition is associative in \( N \).

8. If \( \vec{a} \cdot \vec{i} = \vec{a} \cdot (\vec{i} + \vec{j}) = \vec{a} \cdot (\vec{i} + \vec{j} + \vec{k}) = 1 \) then \( \vec{a} = \)

1) \( \vec{i} + \vec{j} \)  
2) \( \vec{i} - \vec{k} \)  
3) \( \vec{i} \)  
4) \( \vec{i} + \vec{j} - \vec{k} \)

9. If \( \vec{a} \) and \( \vec{b} \) are unit vectors and \( |\vec{a} + \vec{b}| = 1 \) then \( |\vec{a} - \vec{b}| \) is equal to

1) \( \sqrt{2} \)  
2) 1  
3) \( \sqrt{5} \)  
4) \( \sqrt{3} \)

10. The projection of \( \vec{a} = 3\hat{i} - \hat{j} + 5\hat{k} \) on \( \vec{b} = 2\hat{i} + 3\hat{j} + \hat{k} \) is

1) \( \frac{8}{\sqrt{35}} \)  
2) \( \frac{8}{\sqrt{39}} \)  
3) \( \frac{8}{\sqrt{14}} \)  
4) \( \sqrt{14} \)

(Space for Rough Work)
11. If \( f : \mathbb{R} \to \mathbb{R} \) is defined by \( f(x) = x^3 \) then \( f^{-1}(8) = \)

1) \( \{2\} \)  
2) \( \{2, \ 2w, \ 2w^2\} \)  
3) \( \{2, \ -2\} \)  
4) \( \{2, \ 2\} \)

12. \( R \) is a relation on \( \mathbb{N} \) given by \( R = \{(x, y) \mid 4x + 3y = 20\} \). Which of the following belongs to \( R \)?

1) \((-4, \ 12)\)  
2) \((5, \ 0)\)  
3) \((3, \ 4)\)  
4) \((2, \ 4)\)

13. If \( \log_{10}7 = 0.8451 \) then the position of the first significant figure of \( 7^{-20} \) is

1) 16  
2) 17  
3) 20  
4) 15

14. \( \frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots \) upto \( n \) terms =

1) \( \frac{n}{4n+6} \)  
2) \( \frac{1}{6n+4} \)  
3) \( \frac{n}{6n+4} \)  
4) \( \frac{n}{3n+7} \)

15. The ten's digit in \( 1!+4!+7!+10!+12!+13!+15!+16!+17! \) is divisible by

1) 4  
2) 3!  
3) 5  
4) 7

(Space for Rough Work)
16. The equation \( \frac{x^2}{2-\lambda} - \frac{y^2}{\lambda-5} - 1 = 0 \) represents an ellipse if

1) \( \lambda > 5 \)
2) \( \lambda < 2 \)
3) \( 2 < \lambda < 5 \)
4) \( 2 > \lambda > 5 \)

17. The equation to the normal to the hyperbola \( \frac{x^2}{16} - \frac{y^2}{9} = 1 \) at \((-4, 0)\) is

1) \( 2x - 3y = 1 \)
2) \( x = 0 \)
3) \( x = 1 \)
4) \( y = 0 \)

18. The converse of the contrapositive of the conditional \( p \rightarrow \neg q \) is

1) \( p \rightarrow q \)
2) \( \neg p \rightarrow \neg q \)
3) \( \neg q \rightarrow p \)
4) \( \neg p \rightarrow q \)

19. The perimeter of a certain sector of a circle is equal to the length of the arc of the semicircle. Then the angle at the centre of the sector in radians is

1) \( \pi - 2 \)
2) \( \pi + 2 \)
3) \( \frac{\pi}{3} \)
4) \( \frac{2\pi}{3} \)

20. The value of \( \tan \frac{67 \frac{1}{2}}{2} + \cot \frac{67 \frac{1}{2}}{2} \) is

1) \( \sqrt{2} \)
2) \( 3\sqrt{2} \)
3) \( 2\sqrt{2} \)
4) \( 2 - \sqrt{2} \)

(Space for Rough Work)
21. If $e_1$ and $e_2$ are the eccentricities of a hyperbola $3x^2 - 3y^2 = 25$ and its conjugate, then

1) $e_1^2 + e_2^2 = 2$
2) $e_1^2 + e_2^2 = 4$
3) $e_1 + e_2 = 4$
4) $e_1 + e_2 = \sqrt{2}$

22. If $p$ and $q$ are prime numbers satisfying the condition $p^2 - 2q^2 = 1$, then the value of $p^2 + 2q^2$ is

1) 5
2) 15
3) 16
4) 17

23. If $A(\text{adj} \ A) = 5I$ where $I$ is the identity matrix of order 3, then $|\text{adj} \ A|$ is equal to

1) 125
2) 25
3) 5
4) 10

24. The number of solutions for the equation $\sin 2x + \cos 4x = 2$ is

1) 0
2) 1
3) 2
4) Infinite

25. $\int e^x \cdot x^5 \, dx$ is

1) $e^x \left[ x^5 + 5x^4 + 20x^3 + 60x^2 + 120x + 120 \right] + C$
2) $e^x \left[ x^5 - 5x^4 - 20x^3 - 60x^2 - 120x - 120 \right] + C$
3) $e^x \left[ x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120 \right] + C$
4) $e^x \left[ x^5 + 5x^4 + 20x^3 - 60x^2 - 120x + 120 \right] + C$

(Space for Rough Work)
26. If \( f(x) \) is an even function and \( f'(x) \) exists, then \( f'(e) + f'(-e) \) is
   1) \( > 0 \)            2) \( 0 \)
   3) \( \geq 0 \)            4) \( < 0 \)

27. If \( \alpha \) is a complex number satisfying the equation \( \alpha^2 + \alpha + 1 = 0 \) then \( \alpha^{31} \) is equal to
   1) \( \alpha \)            2) \( \alpha^2 \)
   3) \( 1 \)            4) \( i \)

28. The derivative of \( \sin(x^3) \) w.r.t. \( \cos(x^3) \) is
   1) \(-\tan(x^3)\)           2) \(\tan(x^3)\)
   3) \(-\cot(x^3)\)           4) \(\cot(x^3)\)

29. A unit vector perpendicular to both the vectors \( \hat{i} + \hat{j} \) and \( \hat{j} + \hat{k} \) is
   1) \( \frac{-\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}} \)
   2) \( \frac{\hat{i} + \hat{j} - \hat{k}}{3} \)
   3) \( \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}} \)
   4) \( \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}} \)

30. If \( A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \) and \( B = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \) then
   1) \( A = -B \)
   2) \( A = B \)
   3) \( B = 0 \)
   4) \( B = A^2 \)

(Space for Rough Work)
31. The locus of a point which moves such that the sum of its distances from two fixed points is a constant is
   1) a circle
   2) a parabola
   3) an ellipse
   4) a hyperbola

32. The centroid of the triangle $ABC$ where $A = (2, 3)$, $B = (8, 10)$ and $C = (5, 5)$ is
   1) $(5, 6)$
   2) $(6, 5)$
   3) $(6, 6)$
   4) $(15, 18)$

33. If $3x^2 + xy - y^2 - 3x + 6y + K = 0$ represents a pair of lines, then $K =$
   1) 0
   2) 9
   3) 1
   4) $-9$

34. The equation of the smallest circle passing through the points $(2, 2)$ and $(3, 3)$ is
   1) $x^2 + y^2 + 5x + 5y + 12 = 0$
   2) $x^2 + y^2 - 5x - 5y + 12 = 0$
   3) $x^2 + y^2 + 5x - 5y + 12 = 0$
   4) $x^2 + y^2 - 5x + 5y - 12 = 0$

35. The characteristic roots of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$ are
   1) 1, 3, 6
   2) 1, 2, 4
   3) 4, 5, 6
   4) 2, 4, 6

(Space for Rough Work)
36. If \( A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \), then \( A^{-1} = \)

1) \( \frac{-1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} \)

2) \( \frac{1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} \)

3) \( \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix} \)

4) \( \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \)

37. The set \( \{-1, 0, 1\} \) is not a multiplicative group because of the failure of

1) Closure law

2) Associative law

3) Identity law

4) Inverse law

38. The angle of elevation of the top of a TV tower from three points \( A, B \) and \( C \) in a straight line through the foot of the tower are \( \alpha, 2\alpha \) and \( 3\alpha \) respectively. If \( AB = a \), the height of the tower is

1) \( a \tan \alpha \)

2) \( a \sin \alpha \)

3) \( a \sin 2\alpha \)

4) \( a \sin 3\alpha \)

39. The angles \( A, B \) and \( C \) of a triangle \( ABC \) are in A.P. If \( b : c = \sqrt{3} : \sqrt{2} \), then the angle \( A \) is

1) \( 30^0 \)

2) \( 15^0 \)

3) \( 75^0 \)

4) \( 45^0 \)

40. \( \sin \left( 2 \sin^{-1} \frac{\sqrt{63}}{\sqrt{65}} \right) = \)

1) \( \frac{2\sqrt{126}}{65} \)

2) \( \frac{4\sqrt{65}}{65} \)

3) \( \frac{8\sqrt{63}}{65} \)

4) \( \frac{\sqrt{63}}{65} \)

(Space for Rough Work)
41. The general solution of $|\sin x| = \cos x$ is (when $n \in \mathbb{Z}$) given by

\begin{align*}
1) \quad n \pi + \frac{\pi}{4} \\
2) \quad 2n \pi + \frac{\pi}{4} \\
3) \quad n \pi \pm \frac{\pi}{4} \\
4) \quad n \pi - \frac{\pi}{4}
\end{align*}

42. The real root of the equation $x^3 - 6x + 9 = 0$ is

\begin{align*}
1) \quad -6 \\
2) \quad -9 \\
3) \quad 6 \\
4) \quad -3
\end{align*}

43. The digit in the unit’s place of $5^{834}$ is

\begin{align*}
1) \quad 0 \\
2) \quad 1 \\
3) \quad 3 \\
4) \quad 5
\end{align*}

44. The remainder when $3^{100} \times 2^{50}$ is divided by 5 is

\begin{align*}
1) \quad 1 \\
2) \quad 2 \\
3) \quad 3 \\
4) \quad 4
\end{align*}

45. $\int_{\sqrt{1 - \sin^4 x}} dx = \frac{1}{2} \sin^{-1}(\sin^2 x) + C$

\begin{align*}
1) \quad \frac{1}{2} \sin^{-1}(\sin^2 x) + C \\
2) \quad \frac{1}{2} \cos^{-1}(\sin^2 x) + C \\
3) \quad \tan^{-1}(\sin^2 x) + C \\
4) \quad \tan^{-1}(2 \sin x) + C
\end{align*}
46. The value of \[ \int_{-2}^{2} (ax^3 + bx + c) \, dx \] depends on the

1) value of \( b \) 
2) value of \( c \)
3) value of \( a \) 
4) values of \( a \) and \( b \)

47. The area of the region bounded by \( y = 2x - x^2 \) and the \( x \)-axis is

1) \( \frac{8}{3} \) sq. units 
2) \( \frac{4}{3} \) sq. units 
3) \( \frac{7}{3} \) sq. units 
4) \( \frac{2}{3} \) sq. units

48. The differential equation \( y \frac{dy}{dx} + x = c \) represents

1) a family of hyperbolas 
2) a family of circles whose centres are on the \( y \)-axis 
3) a family of parabolas 
4) a family of circles whose centres are on the \( x \)-axis

49. If \( f(x^5) = 5x^3 \), then \( f'(x) = \)

1) \( \frac{3}{\sqrt[5]{x^2}} \) 
2) \( \frac{3}{\sqrt{x}} \)
3) \( \frac{3}{x} \) 
4) \( \frac{3}{\sqrt[5]{x}} \)

50. \( f(x) = 2a - x \) in \(-a < x < a\)

\( = 3x - 2a \) in \( a \leq x \).

Then which of the following is true?

1) \( f(x) \) is discontinuous at \( x = a \) 
2) \( f(x) \) is not differentiable at \( x \)
3) \( f(x) \) is differentiable at all \( x \geq a \) 
4) \( f(x) \) is continuous at all \( x < a \)

(Space for Rough Work)
51. The maximum area of a rectangle that can be inscribed in a circle of radius 2 units is (in square units)
   1) 4
   2) 8π
   3) 8
   4) 5

52. If $Z$ is a complex number such that $Z = -\overline{Z}$, then
   1) $Z$ is purely real
   2) $Z$ is purely imaginary
   3) $Z$ is any complex number
   4) Real part of $Z$ is the same as its imaginary part

53. The value of $\sum_{K=1}^{6} \left[ \sin \frac{2K \pi}{7} - i \cos \frac{2K \pi}{7} \right]$ is
   1) $i$
   2) 0
   3) $-i$
   4) $-1$

54. $\lim_{x \to \infty} x \sin \left( \frac{2}{x} \right)$ is equal to
   1) $\infty$
   2) 0
   3) 2
   4) $\frac{1}{2}$

55. A stone is thrown vertically upwards and the height x ft. reached by the stone in t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in
   1) 2 seconds
   2) 2.5 seconds
   3) 3 seconds
   4) 1.5 seconds

(Space for Rough Work)
56. The maximum value of \( \frac{\log x}{x} \) in \((2, \infty)\) is

1) 1  \hspace{1cm} 2) \frac{2}{e}

3) \( e \)  \hspace{1cm} 4) \frac{1}{e}

57. If \( f(x) = be^{ax} + ae^{bx} \), then \( f''(0) = \)

1) 0  \hspace{1cm} 2) 2ab

3) \( ab(a+b) \)  \hspace{1cm} 4) \( ab \)

58. If \( \sqrt{\frac{1+\cos A}{1-\cos A}} = \frac{x}{y} \), then the value of \( \tan A = \)

1) \( \frac{x^2+y^2}{x^2-y^2} \)  \hspace{1cm} 2) \( \frac{2xy}{x^2+y^2} \)

3) \( \frac{2xy}{x^2-y^2} \)  \hspace{1cm} 4) \( \frac{2xy}{y^2-x^2} \)

59. \( \int \frac{\sec x}{\sec x + \tan x} \, dx = \)

1) \( \tan x - \sec x + C \)  \hspace{1cm} 2) \( \log (1+\sin x) + C \)

3) \( \sec x + \tan x + C \)  \hspace{1cm} 4) \( \log \sin x + \log \cos x + C \)

60. If \( \int f(x) \, dx = g(x) \), then \( \int f(x) \cdot g(x) \, dx = \)

1) \( \frac{1}{2} f^2(x) \)  \hspace{1cm} 2) \( \frac{1}{2} g^2(x) \)

3) \( \frac{1}{2} [g'(x)]^2 \)  \hspace{1cm} 4) \( f'(x) \cdot g(x) \)

(Space for Rough Work)