BITSAT-PILANI

ENGINEERING ENTRANCE

SOLVED PAPER 2005

Mathematics

1. The equation of a parabola which passes through the intersection of a straight line $x + y = 0$ and the circle $x^2 + y^2 + 4y = 0$ is :
   (a) $y^2 = 4x$ (b) $y^2 = x$
   (c) $y^2 = 2x$ (d) none of these

2. The point $(4, -3)$ with respect to the ellipse $4x^2 + 5y^2 = 1$ is :
   (a) lies on the curve
   (b) is inside the curve
   (c) is outside the curve
   (d) is focus of the curve

3. If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ then the angle between the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ is:
   (a) 60° (b) 90°
   (c) 45° (d) 55°

4. Let S be a set containing n elements and we select two subsets A and B of S at random, then the probability that $A \cup B = S$ and $A \cap B = \phi$, is :
   (a) 2^n (b) n^2
   (c) $1/n$ (d) $1/2^n$

5. $\begin{vmatrix} 1+\sin^2\theta & \sin^2\theta & \sin^2\theta \\ \cos^2\theta & 1+\cos^2\theta & \cos^2\theta \\ 4\sin 4\theta & 4\sin 4\theta & 1+4\sin 4\theta \end{vmatrix} = 0$,
   then $\sin 4\theta$ equals to :
   (a) 1/2 (b) 1
   (c) $-1/2$ (d) -1

6. The value of the constant α and β such that $\lim_{x \to \infty} \left(\frac{x^2+1}{x+1} - \alpha x - \beta \right) = 0$ are respectively :
   (a) (1, 1) (b) (−1, 1)
   (c) (1, −1) (d) (0, 1)

7. Let the homogeneous system of linear equations $px + y + z = 0$, $x + qy + z = 0$, and $x + y + rz = 0$, where $p, q, r \neq 1$, have a non-zero solution, then the value of $\frac{1}{1-p} + \frac{1}{1-q} + \frac{1}{1-r}$ is :
   (a) −1 (b) 0
   (c) 2 (d) 1

8. A point (α, β) lies on a circle $x^2 + y^2 = 1$, then locus of the point $(3\alpha + 2, \beta)$ is a/an :
   (a) straight line (b) ellipse
   (c) parabola (d) none of these

9. If θ is an acute angle and $\sin \frac{\theta}{2} = \sqrt{\frac{x-1}{2x}}$, then $\tan \theta$ is equal to :
   (a) $x^2 - 1$ (b) $\sqrt{x^2 - 1}$
   (c) $\sqrt{x^2 + 1}$ (d) $x^2 + 1$

10. The value of
    $$\left[\int_0^{\sin^2\theta} \sin^{-1}\sqrt{\phi}\, d\phi + \int_0^{\cos^2\theta} \cos^{-1}\sqrt{\phi}\, d\phi \right]$$
    is equal to :
    (a) π (b) $\pi/2$
    (c) $\pi/3$ (d) $\pi/4$

11. $\int_0^{2n\pi} \left\{ |\sin x| - \left| \frac{1}{2} \sin x \right| \right\} dx$ equals :
    (a) n
    (b) $2n$
    (c) $-2n$
    (d) none of the above

12. Range of the function $f(x) = \frac{x^2}{x^2+1}$ is :
    (a) (−1, 0) (b) (−1, 1)
    (c) [0, 1] (d) (1, 1)

13. If $\sin^{-1}(1-x) - 2\sin^{-1} x = \pi/2$, then x is equals :

(a) $\{0, -1/2\}$ (b) $\{1/2, 0\}$
(c) $\{0\}$ (d) $(-1, 0)$

14. $\sin A$, $\sin B$, $\cos A$ are in GP. Roots of $x^2 + 2x \cot B + 1 = 0$ are always :

(a) real (b) imaginary
(c) greater than 1 (d) equal

15. If $\int_{\log 2}^{x} \frac{du}{(e^u - 1)^{1/2}} = \frac{\pi}{6}$, then e^x is equal to :

(a) 1 (b) 2
(c) 4 (d) -1

16. Total number of books is $2n+1$. One is allowed to select a minimum of the one book and a maximum of n books. If total number of selections if 63, then value of n is :

(a) 3 (b) 6
(c) 2 (d) none of these

17. $x^2 = xy$ is a relation which is :

(a) symmetric (b) reflexive
(c) transitive (d) none of these

18. Let the determinant of a 3×3 matrix A be 6, then B is a matrix defined by $B = 5A^2$. Then determinant of B is :

(a) 180 (b) 100
(c) 80 (d) none of these

19. Let $f(x) = \begin{cases} 1 & \forall \quad x < 0 \\ 1 + \sin x & \forall \quad 0 \le x \le \pi/2 \end{cases}$

then what is the value of $f'(x)$ at $x = 0$?

(a) 1 (b) -1
(c) ∞ (d) does not exist

20. The length intercepted by the curve $y^2 = 4x$ on the line satisfying $dy/dx = 1$ and passing through point (0, 1), is given by:

(a) 1 (b) 2
(c) 0 (d) none of these

21. Area bounded by curve $y = x^2$ and $y = 2 - x^2$ is:

(a) 8/3 sq units (b) 3/8 sq units
(c) 3/2 sq units (d) none of these

22. $\lim_{\theta \to 0} \frac{4\theta(\tan\theta - 2\theta\tan\theta)}{(1 - \cos 2\theta)}$ is :

(a) $1/\sqrt{2}$ (b) 1/2
(c) 1 (d) 2

23. The largest value of $2x^3 - 3x^2 - 12x + 5$ for $-2 \le x \le 4$ occurs at x is equal to :

(a) -4 (b) 0
(c) 1 (d) 4

24. The number of solutions for the equations $|z-1| = |z-2| = |z-i|$ is :

(a) one solution (b) 3 solutions
(c) 2 solutions (d) no solution

25. Let A and B are two events and $P(A') = 0.3$ $P(B) = 0.4$, $P(A \cap B') = 0.5$ then $P(A \cup B')$ is:

(a) 0.5 (b) 0.8
(c) 1 (d) 0.1

26. $(10101101)_2 = (\ldots\ldots\ldots\ldots)_{10}$:

(a) 137 (b) 173
(c) 170 (d) none of these

27. Given function $f(x) = \left(\frac{e^{2x} - 1}{e^{2x} + 1}\right)$ is :

(a) increasing (b) decreasing
(c) even (d) none of these

28. The solution of $x^2 + y^2 - 2xy\frac{dy}{dx} = 0$ is :

(a) $x^2 - y^2 = cx$ (b) $x^2 + y^2 = cx$
(c) $2(x^2 - y^2) = cx$ (d) none of these

29. $f(x) = ax^2 + bx + c$ and $g(x) = px^2 + qx$ with $g(1) = f(1)$, $g(2) - f(2) = 1$, $g(3) - f(3) = 4$, then $g(4) - f(4)$ is :

(a) 0 (b) 5
(c) 6 (d) none of these

30. If the vectors $\alpha\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} + \beta\hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}$ $(\alpha, \beta, \gamma \neq 1)$ are coplanar, then the value of $\frac{1}{1-\alpha} + \frac{1}{1-\beta} + \frac{1}{1-\gamma}$ is :

(a) -1 (b) 0
(c) 1 (d) 1/2

31. The circumcentre of a triangle formed by the line $xy + 2x + 2y + 4 = 0$ and $x + y + 2 = 0$ is :

(a) $(-1, -1)$ (b) $(0, -1)$
(c) $(1, 1)$ (d) $(-1, 0)$

32. The number of common tangents to circle $x^2 + y^2 + 2x + 8y - 23 = 0$ and $x^2 + y^2 - 4x - 10y + 9 = 0$, is :

(a) 1 (b) 3
(c) 2 (d) none of these

33. If $\frac{x}{\alpha} + \frac{y}{\beta} = 1$ touches the circle $x^2 + y^2 = a^2$, then point $(1/\alpha, 1/\beta)$ lies on a/an:

(a) straight line (b) circle
(c) parabola (d) ellipse

34. The point of intersection of the line $\frac{x-1}{3}=\frac{y+2}{4}=\frac{z-3}{-2}$ and plane $2x-y+3z-1=0$ is :

(a) (10, –10, 3) (b) (10, 10, –3)
(c) (–10, 10, 3) (d) none of these

35. The tangents from a point $(2\sqrt{2}, 1)$ to the hyperbola $16x^2-25y^2=400$ include an angle equal to :

(a) $\pi/2$ (b) $\pi/4$
(c) π (d) $\pi/3$

36. Let α, β, γ and δ are four positive real number such that their product is unity, then the least value of $(1+\alpha)(1+\beta)(1+\gamma)(1+\delta)$ is:

(a) 6 (b) 16
(c) 0 (d) 32

37. Value of $\sum_{k=1}^{6}\left(\frac{2k\pi}{7}\right)-i\cos\left(\frac{2k\pi}{7}\right)$ is equal to:

(a) –1 (b) 1
(c) 0 (d) none of these

38. The degree of the differential equation

$$y(x)=1+\frac{dy}{dx}+\frac{1}{1\cdot 2}\left(\frac{dy}{dx}\right)^2+\frac{1}{1\cdot 2\cdot 3}\left(\frac{dy}{dx}\right)^3+\ldots$$

is:

(a) 2 (b) 3
(c) 1 (d) none of these

39. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points such that their abscissa x_1 and x_2 are the roots of the equation $x^2+2x-3=0$ while the ordinate y_1 and y_2 are the roots of the equation $y^2+4y-12=0$. The centre of the circle with PQ as diameter is :

(a) (–1, –2) (b) (1,2)
(c) (1, –2) (d) (–1, 2)

40. The equation of plane passing through a point $A(2, -1, 3)$ and parallel to the vectors $\vec{\mathbf{a}}=(3, 0, -1)$ and $\vec{\mathbf{b}}=(-3, 2, 2)$ is :

(a) $2x-3y+6z-25=0$
(b) $2x-3y+6z+25=0$
(c) $3x-2y+6z-25=0$
(d) $3x-2y+6z+25=0$

41. The equation of a straight line drawn through the focus of the parabola $y^2=-4x$ at an angle of 120° to the x-axis is :

(a) $y+\sqrt{3}(x-1)=0$
(b) $y-\sqrt{3}(x-1)=0$
(c) $y+\sqrt{3}(x+1)=0$
(d) $y-\sqrt{3}(x+1)=0$

42. Let $x=\alpha+\beta$, $y=\alpha\omega+\beta\omega^2$, $z=\alpha\omega^2+\beta\omega$, ω is an imaginary cube root of unity. Product of xyz is :

(a) $\alpha^2+\beta^2$ (b) $\alpha^2-\beta^2$
(c) $\alpha^3+\beta^3$ (d) $\alpha^3-\beta^3$

43. If $r=[2\phi+\cos^2(2\phi+\pi/4)]^{1/2}$, then what is the value of the derivative of $dr/d\phi$ at $\phi=\pi/4$?

(a) $2\left(\frac{1}{\pi+1}\right)^{1/2}$ (b) $2\left(\frac{2}{\pi+1}\right)^{2}$
(c) $\left(\frac{2}{\pi+1}\right)^{1/2}$ (d) $2\left(\frac{2}{\pi+1}\right)^{1/2}$

44. If a vector $\vec{\alpha}$ lie in the plane of $\vec{\beta}$ and $\vec{\gamma}$, then which is correct?

(a) $[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]=0$ (b) $[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]=1$
(c) $[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]=3$ (b) $[\vec{\beta}\,\vec{\gamma}\,\vec{\alpha}]=1$

45. If $\vec{\alpha}=2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-\hat{\mathbf{k}}$, $\vec{\beta}=-\hat{\mathbf{i}}+2\hat{\mathbf{j}}-4\hat{\mathbf{k}}$, $\vec{\gamma}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$, then what is the value of $(\vec{\alpha}\times\vec{\beta})\cdot(\vec{\alpha}\times\vec{\gamma})$?

(a) 47
(b) 74
(c) –74
(d) none of the above

Physics

46. If M is the mass of the earth and R its radius, the ratio of the gravitational acceleration and the gravitational constants is:

(a) $\frac{R^2}{M}$ (b) $\frac{M}{R^2}$
(c) MR^2 (d) $\frac{M}{R}$

47. A student unable to answer a question on Newton's laws of motion attempts to pull himself up by tugging on his hair. He will not succeed :

(a) as the force exerted is small
(b) the frictional force while gripping, is small

(c) Newton's law of inertia is not applicable to living beings
(d) as the force applied is internal to the system

48. Which one of the following is not a unit of Young's modulus?
(a) Nm^{-1}
(b) Nm^{-2}
(c) dyne cm^{-2}
(d) mega pascal

49. A piece of blue glass heated to a high temperature and a piece of red glass at room temperature, are taken inside a dimly lit room, then:
(a) the blue piece will look blue and red will look as usual
(b) red looks brighter red and blue looks ordinary blue
(c) blue shines like brighter red compared to the red piece
(d) both the pieces will look equally red

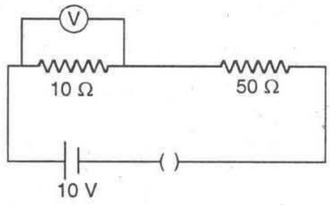
50. A 5.0 A current is setup in an external circuit by a 6.0 V storage battery for 6.0 min. The chemical energy of the battery is reduced by:
(a) 1.08×10^4 J
(b) 1.08×10^{-4} J
(c) 1.8×10^4 J
(d) 1.8×10^{-4} J

51. The current in a simple series circuit is 5.0 A. When an additional resistance of 2.0 Ω is inserted, the current drops to 4.0 A. The original resistance of the circuit in ohms was :
(a) 1.25 (b) 8
(c) 10 (d) 20

52. Two resistances are connected in two gaps of a metre bridge. The balance point is 20 cm from the zero end. A resistance of 15 Ω is connected in series with the smaller of the two. The null point shifts to 40 cm. The value of the smaller resistance in ohms is :
(a) 3 (b) 6
(c) 9 (d) 12

53. By using only two resistance coils-singly, in series or in parallel one should be able to obtain resistances of 3, 4, 12 and 16 Ω. The separate resistances of the coil are :
(a) 3 and 4 (b) 4 and 12
(c) 12 and 16 (d) 16 and 3

54. In the given circuit, the voltmeter records 5 V. The resistance of the voltmeter in ohms is :



(a) 200 (b) 100
(c) 10 (d) 50

55. The wavelength of the radiation emitted by a body depends upon :
(a) the nature of the surface
(b) the area of the surface
(c) the temperature of the surface
(d) all of the above factors

56. Which mirror is to be used to obtain a parallel beam of light from a small lamp?
(a) Plane mirror
(b) Convex mirror
(c) Concave mirror
(d) Any one of the above

57. Which of the following is a wrong statement?
(a) $D = 1/f$ where f is the focal length and D is called the refractive power of a lens.
(b) Power is expressed in a diopter when f is in metres
(c) Power is expressed in diopter and does not depend on the system of unit used to measure f
(d) D is positive for convergent lens and negative for divergent lens

58. An electric field of 1500 V/m and a magnetic field of 0.40 Wb/m^2 act on a moving electron. The minimum uniform speed along a straight line the electron could have is:
(a) 1.6×10^{15} m/s (b) 6×10^{-16} m/s
(c) 3.75×10^3 m/s (d) 3.75×10^2 m/s

59. In an ammeter 10% of main current is passing through the galvanometer. If the resistance of the galvanometer is G, then the shunt resistance, in ohms is:
(a) $9G$ (b) $\frac{G}{9}$
(c) $90G$ (d) $\frac{G}{90}$

60. Among the following properties describing diamagnetism identify the property that is wrongly stated:

(a) Diamagnetic material do not have permanent magnetic moment

(b) Diamagnetism is explained in terms of electromagnetic induction

(c) Diamagnetic materials have a small positive susceptibility

(d) The magnetic moment of individual electrons neutralize each other

61. The induction coil works on the principle of :

(a) self-induction

(b) mutual induction

(c) Ampere's rule

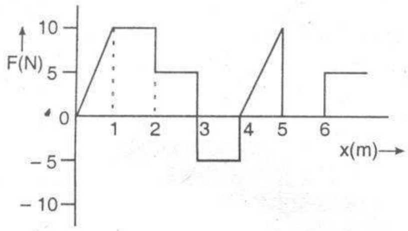
(d) Fleming's right hand rule

62. The square root of the product of inductance and capacitance has the dimension of :

(a) length (b) mass

(c) time (d) no dimension

63. The relationship between the force F and position x of a body is as shown in figure. The work done in displacing the body from $x = 1$ m to $x = 5$ m will be :



(a) 30 J (b) 15 J

(c) 25 J (d) 20 J

64. From the top of a tower of two stones, whose masses are in the ratio 1 : 2 are thrown on straight up with an initial speed u and the second straight down with the same speed u. Then neglecting air resistance:

(a) the heavier stone hits the ground with a higher speed

(b) the lighter stone hits the ground with a higher speed

(c) both the stones will have the same speed when they hit the ground

(d) the speed cannot be determined with the given data

65. Infrared radiation was discovered in 1800 by :

(a) William Wollaston

(b) William Herschel

(c) Wilhelm Roentgen

(d) Thomas Young

66. A particle on the trough of a wave at any instant will come to the mean position after a time : (T = time period)

(a) $T/2$ (b) $T/4$

(c) T (d) $2T$

67. The disc of a siren containing 60 holes rotates at a constant speed of 360 rpm. The emitted sound is in unison with a tuning fork of frequency :

(a) 10 Hz (b) 360 Hz

(c) 216 Hz (d) 60 Hz

68. The ratio of velocity of sound in hydrogen and oxygen at STP is :

(a) 16 : 1 (b) 8 : 1

(c) 4 : 1 (d) 2 : 1

69. In an experiment with sonometer a tuning fork of frequency 256 Hz resonates with a length of 25 cm and another tuning fork resonates with a length of 16 cm. Tension of the string remaining constant the frequency of the second tuning fork is :

(a) 163.84 Hz

(b) 400 Hz

(c) 320 Hz

(d) 204.8 Hz

70. The wave theory of light, in its original form, was first postulated by:

(a) Isaac Newton

(b) Christian Huygens

(c) Thomas Young

(d) Augustin Jean Fresnel

71. If a liquid does not wet glass, its angle of contact is :

(a) zero (b) acute

(c) obtuse (d) right angle

72. Electron of mass m and charge q is travelling with a speed v along a circular path of radius r at right angles to a uniform magnetic field of intensity B. If the speed of the electron is doubled and the magnetic field is halved the resulting path would have a radius:

(a) $2r$ (b) $4r$

(c) $r/4$ (d) $r/2$

73. Two coherent light beams of intensity I and $4I$ are superposed. The maximum and minimum possible intensities in the resulting beam are :

(a) $9I$ and I (b) $9I$ and $3I$
(c) $5I$ and I (d) $5I$ and $3I$

74. The electron in a hydrogen atom makes a transition form $n = n_1$ to $n = n_2$ state. The time period of the electron in the initial state (n_1) is eight times that in the final state(n_2). The possible values of n_1 and n_2 are :

(a) $n_1 = 8, n_2 = 1$
(b) $n_1 = 4, n_2 = 2$
(c) $n_1 = 2, n_2 = 4$
(d) $n_1 = 1, n_2 = 8$

75. If the forward voltage in a diode is increased, the width of the depletion region :

(a) increases
(b) decreases
(c) fluctuates
(d) no change

76. Two nucleons are at a separation of one Fermi. Protons have a charge of $+1.6 \times 10^{-19}$ C. The net nuclear force between them is F_1, if both are neutrons, F_2 if both are protons and F_3 if one is proton and the other is neutron. Then :

(a) $F_1 = F_2 > F_3$
(b) $F_1 = F_2 = F_3$
(c) $F_1 < F_2 < F_3$
(d) $F_1 > F_2 > F_3$

77. The potential to which a conductor is raised, depends on :

(a) the amount of charge
(b) geometry and size of the conductor
(c) both (a) and (b)
(d) only on (a)

78. The work done in carrying a charge q once round a circle of radius r with a charge Q at the centre is :

(a) $\frac{qQ}{4\pi\varepsilon_0 r}$

(b) $\frac{qQ}{4\pi\varepsilon_0^2 r^2}$

(c) $\frac{qQ}{4\pi\varepsilon_0 r^2}$

(d) none of the above

79. An air filled parallel plate condenser has a capacity of 2pF. The separation of the plates is doubled and the interspace between the plates is filled with wax. If the capacity is increased to 6 pF, the dielectric constant of wax is :

(a) 2 (b) 3
(c) 4 (d) 6

80. The energy that should be added to an electron to reduce its de-Broglie wavelength from 1 nm to 0.5 nm is :

(a) four times the initial energy
(b) equal to the initial energy
(c) twice the initial energy
(d) thrice the initial energy

81. Mean life of a radioactive sample is 100 s. Then its half-life (in minutes) is:

(a) 0.693 (b) 1
(c) 10^{-4} (d) 1.155

82. Consider two nuclei of the same radioactive nuclide. One of the nuclei was created in a supernova explosion 5 billion years ago. The probability of decay during the next time is :

(a) different for each nuclei
(b) nuclei created in explosion decays first
(c) nuclei created in the reactor decays first
(d) independent of the time of creation

83. Bohr's atom model assumes :

(a) the nucleus is of infinite mas and is at rest
(b) electrons in a quantized orbit will not radiate energy
(c) mass of electron remains constant
(d) all the above conditions

84. Identify the wrong statement in the following. Coulomb's law correctly described the electric force that :

(a) binds the electrons of an atom to its nucleus
(b) binds the protons and neutrons in the nucleus of an atom
(c) binds atoms together to form molecules
(d) binds atoms and molecules to form solids

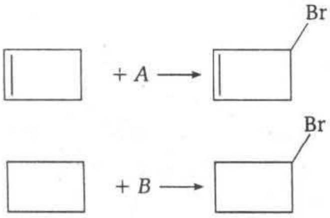
85. When unpolarised light beam is incident from air onto glass ($n = 1.5$) at the polarising angle:

(a) reflected beam is polarised 100 percent
(b) reflected and refracted beams are partially polarised
(c) the reason for (a) is that almost all the light is reflected
(d) all of the above

86. Which of the following silver salts is insoluble in water ?

(a) $AgClO_4$ (b) Ag_2SO_4
(c) AgF (d) $AgNO_3$

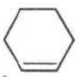
87. Suitable reagents *A* and *B* for the following reactions are:

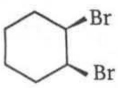


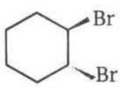
(a) Br, Br_2 (b) Br_2, NBS
(c) NBS, NBS (d) NBS, Br_2

88. KF combines with HF to form KHF_2. The compound contains the species:

(a) K^+ , F^- and H^+ (b) K^+ , F^- and HF
(c) K^+ and $[HF_2]^-$ (d) $[KHF]^+$ and F_2

89.  + $Br_2 \rightarrow A$, *A* will have configuration :

(a) 

(b) 

(c) both (a) and (b) (d) none of these

90. Among the following sets of quantum numbers. Which one is incorrect for 4*d* electron ?

(a) 4, 3, 2, $+\frac{1}{2}$ (b) 4, 2, 1, 0
(c) 4, 2, −2, $+\frac{1}{2}$ (d) 4, 2, 1, $-\frac{1}{2}$

91. Raffinose is :

(a) trisaccharide
(b) monosaccharide
(c) disaccharide
(d) none of the above

92. The molecular electronic configuration of Be_2 is :

(a) $\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2p^2$
(b) $KK\sigma 2s^2$
(c) $\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2$
(d) none of the above

93. Which of the following is deliquescent ?

(a) $ZnCl_2$ (b) Hg_2Cl_2
(c) $HgCl_2$ (d) $CdCl_2$

94. The velocity of electron in first orbit of H-atom as compared to the velocity of light is :

(a) $\frac{1}{10}$th (b) $\frac{1}{100}$th
(d) $\frac{1}{1000}$th (d) same

95.  $\xrightarrow[H_2O_2]{OsO_4}$ *A*, *A* is :

(a) *meso* diol
(b) racemic diol
(c) both (a) and (b)
(d) none of the above

96. In which of the following reactions is $K_p < K_c$?

(a) $I_2\ (g) \rightleftharpoons 2I(g)$
(b) $2BrCl(g) \rightleftharpoons Cl_2\ (g) + Br_2\ (g)$
(c) $CO(g) + 3H_2(g) \rightleftharpoons CH_4(g) + H_2O(g)$
(d) All of the above

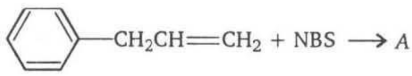
97. For the reaction (at 1240 K and 1 atm.)

$$CaCO_3(s) \longrightarrow CaO(s) + CO_2\ (g)$$

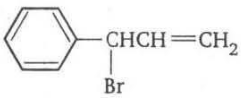
$\Delta H = 176$ kJ/mol; ΔE will be :

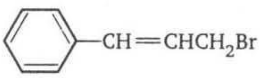
(a) 160 kJ (b) 165.6 kJ
(c) 186.4 kJ (d) 180 kJ

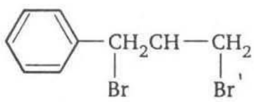
98. Following compound is treated with NBS

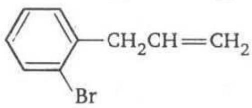


Compound formed *A* is :

(a) 

(b) 

(c) 

(d) 

99. The standard reduction potential of the reation, $H_2O + e^- \longrightarrow \frac{1}{2} H_2 + OH$ at 298 K is:

(a) $E° = \frac{RT}{F} \ln K_w$

(b) $E° = -\frac{RT}{F} \ln [P_{H_2}]^{1/2} [OH^-]$

(c) $E° = -\frac{RT}{F} \ln \frac{[P_{H_2}]^{1/2}}{[H^+]}$

(d) $E° = -\frac{RT}{F} \ln K_w$

100. Glycerol is oxidised by bismuth nitrate to produce:

(a) oxalic acid (b) mesooxalic acid
(c) glyceric acid (d) glyoxalic acid

101. Unit of frequency factor (A) is :

(a) mol/L
(b) mol/L × s
(c) depends upon order
(d) it does not have any unit

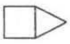
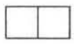
102. The change in pressure will not affect the equilibrium constant for :

(a) $N_2 + 3H_2 \rightleftharpoons 2NH_3$
(b) $PCl_5 \rightleftharpoons PCl_3 + Cl_2$
(c) $H_2 + I_2 \rightleftharpoons 2HI$
(d) all of the above

103. The volume strength of 1.5 N H_2O_2 solution is :

(a) 4.8 (b) 8.4
(c) 4.2 (d) 2.4

104. Bicyclo (1, 1, 0) butane is :

(a)  (b) 

(c)  (d) 

105. How many hydrogen bonds are present between pair of thymine and adenine in DNA?

(a) 1-hydrogen bond
(b) 2-hydrogen bonds
(c) 3-hydrogen bonds
(d) No bonds occur

106. Graham's law deals with the relation between :

(a) pressure and volume
(b) density and rate of diffusion
(c) rate of diffusion and volume
(d) rate of diffusion and viscosity

107. The rms speed of hydrogen is $\sqrt{7}$ times the rms speed of nitrogen. If T is the temperature of the gas, then:

(a) $T_{H_2} = T_{N_2}$ (b) $T_{H_2} > T_{N_2}$
(c) $T_{H_2} < T_{N_2}$ (d) $T_{H_2} = \sqrt{7} T_{N_2}$

108. In P_4O_{10}, the :

(a) second bond in P=O is formed by $p\pi$-$d\pi$ back bonding
(b) P=O bond is formed by $p\pi$-$p\pi$ bonding
(c) P=O bond is formed by $d\pi$-$d\pi$ bonding
(d) P=O bond is formed by $d\pi$-$d\pi$-3σ back bonding

109. Grignard reagent reacts with HCHO to produce:

(a) secondary alcohol
(b) anhydride
(c) and acid
(d) primary alcohol

110. Dacron is polymer of :

(a) glycol and formaldehyde
(b) glycol and phenol
(c) glycol and phthalic acid
(d) glycol and terephthalic acid

111. The product obtained by heating diethyl ether with HI is :

(a) C_2H_5I
(b) C_2H_5OH
(c) $C_2H_5OH + C_2H_5I$
(d) $C_2H_5—C_2H_5$

112. For the gaseous reaction involving the complete combustion of isobutane:

(a) $\Delta H = \Delta E$ (b) $\Delta H > \Delta E$
(c) $\Delta H < \Delta E$ (d) none of these

113. Natural rubber is a polymer of :

(a) styrene (b) isoprene
(c) ethylene (d) butadiene

114. Charles' law is represented mathematically as :

(a) $V_t = KV_0 t$ (b) $V_t = \frac{KV_0}{t}$

(c) $V_t = V_0 \left(1 + \frac{273}{t}\right)$ (d) $V_t = V_0 \left(1 + \frac{t}{273}\right)$

115. Cyanohydrin of which of the following forms lactic acid:

(a) HCHO (b) CH_3CHO
(c) CH_3CH_2CHO (d) CH_3COCH_3

116. Dinitrogen pentoxide (N_2O_5), a colourless solid, is prepared by :

(a) heating NH_4NO_2 with an excess of oxygen
(b) dehydrating HNO_3 with CaO
(c) dehydrating HNO_3 with P_4O_{10}
(d) heating a mixture of HNO_2 and $Ca(NO_3)_2$

117. Which gas has the highest partial pressure in atmosphere?
(a) CO_2 (b) H_2O
(b) O_2 (d) N_2

118. Acetone and acetaldehyde can be distinguished by :
(a) Molisch test (b)Tollen's test
(c) Schiff's test (d) Iodoform test

119. Incorrect statement for pyrophosphorus acid $H_4P_2O_5$ is :
(a) contains P in +5 oxidation state
(b) it is dibasic acid
(c) it is strongly reducing in nature
(d) it contains one P—O—P bond

120. Which of the following compounds is not an "interpseudohalogen"?
(a) Cl_2N_3 (b) BrCN
(c) ClCN (d) ICN

121. The oxidation number of oxygen in hydrogen peroxide is :
(a) +1 (b) −1
(c) +2 (d) −2

122. Isopropyl bromide on Wurtz reaction gives:
(a) hexane
(b) propane
(c) 2, 3-dimethyl butane
(d) *neo*-hexane

123. Tetraethyl lead is a:
(a) solvent (b) petroleum additive
(c) oxidising agent (d) fire extinguisher

124. Solvay process is used for the manufacture of :
(a) NaOH (b) Na_2CO_3
(c) NH_3 (d) NaCl

125. Milk of magnesia is used as:
(a) antichlor (b) antacid
(c) antiseptic (d) food preservative

English

Directions : *Choose the correct meanings of Phrases/Idioms, out of the four responses given under each.*

126. To meet one's Waterloo :
(a) To meet a strong adversary
(b) To met with humiliation
(c) To die fighting
(d) To meet one's final defeat

127. Through thick and thin :
(a) Big and small
(b) Large object
(c) Under all conditions
(d) Thin and fat

128. An axe to grind :
(a) Difficult job
(b) Hard labour
(c) Private ends to serve
(d) Punishment

129. His wit's end :
(a) Finished
(b) Confused
(c) Comedy
(d) Very intelligent

Directions : *Find out the part which contains an error in the following sentences. If there is no error, the answer is (d).*

130. It is not advisable (a) to take heavy luggages (b) while on journey these days. (c) No error. (d)

131. Mr. Bose (a) accompanied by his wife and children (b) were present there. (c) No error. (d)

132. You must pay respect to (a) those who has (b) respect for you. (c) No error. (d)

Directions : *Choose the word that is most nearly opposite in meaning to the word given in capital letters at the question place.*

133. DREARY :
(a) Drab (b) Dangerous
(c) Beautiful (d) Bright

134. GREGARIOUS:
(a) Antisocial
(b) Horrendous
(c) Similar
(d) Glorious

135. MISERLY:

(a) Charitable (b) Spendthrift
(c) Liberal (d) Generous

Directions : *Choose the word that is most nearly the same in meaning to the word given in capital letters at the question place.*

136. DILIGENT:

(a) Industrious (b) Energetic
(c) Modest (d) Inteligent

137. RENOUNCE:

(a) Reform (b) Revoke
(c) Retain (d) Resign

138. PROLIFIC:

(a) Plenty (b) Competent
(c) Predominant (d) Fertile

Directions : *In each of the following questions four parts of a sentence are given as P, Q, R and S. Put them in proper order to produce the correct sentence.*

139. P: I decided to call on him
Q : at the earliest opportunity
R : having heard of the palmist
S : before I came into town

(a) RSPQ (b) PQRS
(c) SQPR (d) QPRS

140. P : when a chemical substance
Q : the food poisoning occurred
R : in the food preparations
S : was mistaken for salt and used

(a) RQPS (b) SRQP
(c) QPSR (d) PSRQ

Reasoning

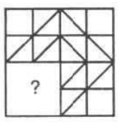
141. Victory is related to *Happiness* in the same way as *Failure* is related to :

(a) Defeat (b) Anger
(c) Frustration (d) Sadness

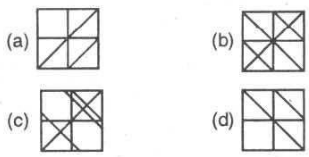
142. In the following question, four groups of letters are given. Three of them are alike in a certain way while one is different. Select the one which is different.

(a) RSXY (b) NOUV
(c) MNST (d) DEJK

143. Complete the pattern in fig (x) by selecting one of the figures from the four alternatives:



(X)

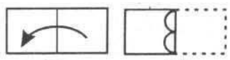


144. In the following question, a statement/ group of statements is given followed by some conclusions. Choose the conclusion which logically follows from the given statement.

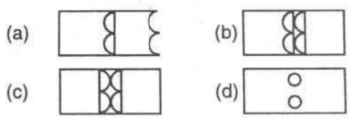
Statements :
1. Only students can participate in the race.
2. Some participants in the race are females.
3. All female participants in the race are invited for coaching.

Conclusions :
(a) All participants in the race are invited for coaching.
(b) All participants in the race are males.
(c) All students are invited for coaching.
(d) All participants in the race are students.

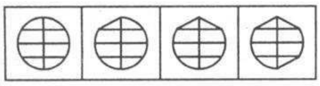
145. Consider the figures *X* and *Y* showing a rectangular sheet of paper folded in fig. *X* and punched in fig. *Y*. From amongst the answer figures *a, b, c* and *d*, select the figure, which will most closely resemble the unfolded position of fig. Y.

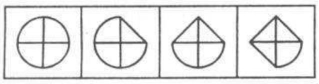


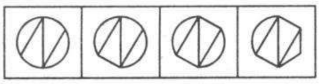
(X) (A)

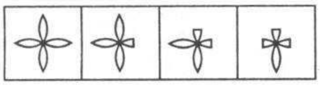


146. Which one of the given sets of figures follows the following rule.
Rule : "Sectors get converted to triangles one by one".

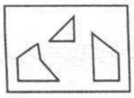
(a) 

(b) 

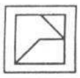
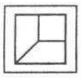
(c) 

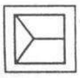
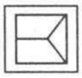
(d) 

Direction : *In the following question find out which of the figures (a), (b), (c) and (d) can be formed from the pieces given in (X).*

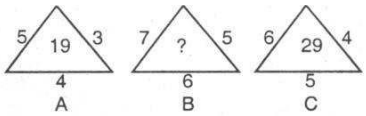
147. 

(X)

(a)  (b) 

(c)  (d) 

Direction : *Find the missing character from among the given alternatives.*

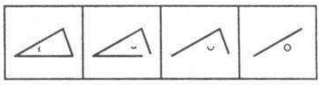
148. 

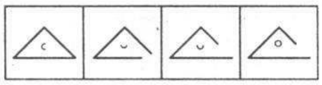
(a) 25 (b) 37
(c) 41 (d) 47

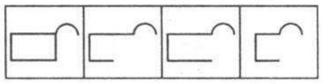
149. What terms will fill the blank spaces?
Z, X, V, T, R, (....), (....)
(a) O, K (b) N, M
(c) K, S (d) P, N

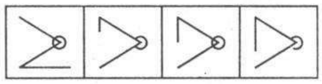
Direction : *In the following question, choose the set of figures which follows the given rule.*

150. Rule : *Closed figures become more and more open and open figures become more and more closed.*

(a) 

(b) 

(c) 

(d) 

ANSWERS

MATHEMATICS

1.	(c)	2.	(c)	3.	(b)	4.	(d)	5.	(c)	6.	(c)	7.	(d)	8.	(b)
9.	(b)	10.	(d)	11.	(b)	12.	(c)	13.	(c)	14.	(a)	15.	(c)	16.	(a)
17.	(c)	18.	(a)	19.	(d)	20.	(c)	21.	(a)	22.	(d)	23.	(d)	24.	(a)
25.	(b)	26.	(b)	27.	(a)	28.	(a)	29.	(d)	30.	(c)	31.	(a)	32.	(c)
33.	(b)	34.	(b)	35.	(a)	36.	(b)	37.	(d)	38.	(c)	39.	(a)	40.	(a)
41.	(c)	42.	(c)	43.	(d)	44.	(a)	45.	(c)						

PHYSICS

46.	(b)	47.	(d)	48.	(a)	49.	(c)	50.	(a)	51.	(b)	52.	(c)	53.	(d)
54.	(b)	55.	(c)	56.	(c)	57.	(c)	58.	(c)	59.	(b)	60.	(c)	61.	(b)
62.	(c)	63.	(b)	64.	(c)	65.	(b)	66.	(b)	67.	(b)	68.	(c)	69.	(b)
70.	(b)	71.	(c)	72.	(b)	73.	(a)	74.	(b)	75.	(b)	76.	(b)	77.	(c)
78.	(d)	79.	(d)	80.	(d)	81.	(d)	82.	(d)	83.	(d)	84.	(b)	85.	(a)

CHEMISTRY

86.	(b)	87.	(d)	88.	(c)	89.	(b)	90.	(b)	91.	(a)	92.	(c)	93.	(a)
94.	(b)	95.	(a)	96.	(a)	97.	(b)	98.	(b)	99.	(a)	100.	(b)	101.	(b)
102.	(d)	103.	(b)	104.	(c)	105.	(b)	106.	(b)	107.	(c)	108.	(a)	109.	(d)
110.	(d)	111.	(c)	112.	(b)	113.	(b)	114.	(d)	115.	(b)	116.	(c)	117.	(d)
118.	(b)	119.	(a)	120.	(a)	121.	(b)	122.	(c)	123.	(b)	124.	(b)	125.	(b)

ENGLISH

126.	(d)	127.	(c)	128.	(c)	129.	(b)	130.	(b)	131.	(c)	132.	(b)	133.	(d)
134.	(a)	135.	(d)	136.	(a)	137.	(d)	138.	(d)	139.	(a)	140.	(c)		

REASONING

141.	(c)	142.	(b)	143.	(d)	144.	(d)	145.	(d)	146.	(b)	147.	(a)	148.	(c)
149.	(d)	150.	(a)												

HINTS & SOLUTIONS

Mathematics

1. Given equation of line is

$$x + y = 0 \quad \text{...(i)}$$

and equation of circle is

$$x^2 + y^2 + 4y = 0 \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii),

$$x^2 + (-x)^2 + 4(-x) = 0$$

$\Rightarrow \quad 2x^2 - 4x = 0$

$\Rightarrow \quad 2x(x-2) = 0$

$\Rightarrow \quad x = 0, 2$ and $y = 0, -2$

Now taking option (c)

i.e., $\quad y^2 = 2x$

at poing $(0, 0) \Rightarrow 0 = 0$

and at point $(2, -2)$

$\Rightarrow \quad (-2)^2 = 2(2) \Rightarrow 4 = 4$

$\therefore$ option (c) is the correct answer.

2. Given equation of ellipse is $4x^2 + 5y^2 = 1$

or $\quad S \equiv 4x^2 + 5y^2 - 1 = 0 \quad \text{...(i)}$

At point $(4, -3)$

$$S \equiv 4(4)^2 + 5(-3)^2 - 1$$

$$\equiv 108 > 0$$

Therefore the given point lies outside the ellipse.

3. Given that

$$\vec{\mathbf{a}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}} \text{ and } \vec{\mathbf{b}} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

Now, $\vec{\mathbf{a}} + \vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}} + 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$$= 4\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$$

and $\vec{\mathbf{a}} - \vec{\mathbf{b}} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) - (3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$$= -2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$$

Let θ be the angle between $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ and $\vec{\mathbf{a}} - \vec{\mathbf{b}}$

$$\therefore \cos\theta = \frac{(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} - \vec{\mathbf{b}})}{|\vec{\mathbf{a}} + \vec{\mathbf{b}}|\,|\vec{\mathbf{a}} - \vec{\mathbf{b}}|}$$

$$= \frac{(4\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (-2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}})}{|4\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}|\,|-2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}|}$$

$$= \frac{-8 + 3 + 5}{\sqrt{16 + 1 + 1}\sqrt{4 + 9 + 25}} = 0$$

$\Rightarrow \quad \theta = 90°$

4. Let A and B be two subsets of S. There are following cases to make a subset of S, under the given condition *i.e.* $A \cup B = S$ and $A \cap B = \phi$

Case I : If set A has no element. The number of ways of selection of 0 element from set S is ${}^nC_0$.

Case II : If set A has one element. The number of ways of selection of one element from set S is ${}^nC_1$.

Case III : If set A has two elements. The number of ways of selection of two element from set S is ${}^nC_2$.

....

....

....

....

Case (*n*) : If set A has *n* elements. The number of ways of selection of *n* elements from set *S* is ${}^nC_n$.

$\therefore$ Total set of $A = {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n$

$= 2^n$

Total set of A and $B = 2^n \times 2^n = 2^{2n}$

$\therefore$ Required probability $= \dfrac{2^n}{2^{2n}} = \dfrac{1}{2^n}$

5. Let $A \equiv \begin{vmatrix} 1+\sin^2\theta & \sin^2\theta & \sin^2\theta \\ \cos^2\theta & 1+\cos^2\theta & \cos^2\theta \\ 4\sin 4\theta & 4\sin 4\theta & 1+4\sin 4\theta \end{vmatrix} = 0$

Applying $R_1 \to R_1 + R_2$

$$\Rightarrow \begin{vmatrix} 2 & 2 & 1 \\ \cos^2\theta & 1+\cos^2\theta & \cos^2\theta \\ 4\sin 4\theta & 4\sin 4\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - 2C_3$, $C_2 \to C_2 - 2C_3$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ -\cos^2\theta & 1-\cos^2\theta & \cos^2\theta \\ -2-4\sin 4\theta & -2-4\sin 4\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

$\Rightarrow$

$[\cos^2\theta(2+4\sin 4\theta) + (1-\cos^2\theta)(2+4\sin 4\theta)] = 0$

$\Rightarrow [2\cos^2\theta + 4\cos^2\theta\sin 4\theta + 2 + 4\sin 4\theta$

$- 2\cos^2\theta - 4\cos^2\theta\sin 4\theta] = 0$

$\Rightarrow \quad 2 + 4\sin 4\theta = 0$

$\Rightarrow \quad \sin 4\theta = -\dfrac{1}{2}$

6. Given that,

$$\lim_{x\to\infty}\left(\frac{x^2+1}{x+1} - \alpha x - \beta\right) = 0$$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{x^2+1-\alpha(x^2+x)-\beta(x+1)}{x+1}\right) = 0$$

Using L-Hospital's rule, we get

$$\lim_{x\to\infty}\left(\frac{2x-\alpha(2x+1)-\beta(1)}{1}\right) = 0$$

If this limit is zero, then the function

$2x - \alpha(2x+1) - \beta = 0$

or $\quad x(2-2\alpha) - (\alpha+\beta) = 0$

Equating the coefficient of x and constant terms, we get

$2 - 2\alpha = 0$ and $\alpha + \beta = 0$

$\Rightarrow \quad \alpha = 1, \ \beta = -1$

7. Given equations are

$px + y + z = 0, \ x + qy + z = 0, \ x + y + rz = 0$

Since the system have a non-zero solution, then

$$\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$

and $\quad C_3 \to C_3 - C_2$

$$\Rightarrow \begin{vmatrix} p & 1-p & 0 \\ 1 & q-1 & 1-q \\ 1 & 0 & r-1 \end{vmatrix} = 0$$

$$\Rightarrow (1-p)(1-q)(1-r)\begin{vmatrix} \frac{p}{1-p} & 1 & 0 \\ \frac{1}{1-q} & -1 & 1 \\ \frac{1}{1-r} & 0 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (1-p)(1-q)(1-r)\left[\frac{p}{1-p}(1) - 1\left(-\frac{1}{1-q} - \frac{1}{1-r}\right)\right] = 0$$

Since, $p, q, r \neq 1$

$$\therefore \quad \frac{p}{1-p} + \frac{1}{1-q} + \frac{1}{1-r} = 0$$

$$\Rightarrow \quad \frac{1}{1-p} - 1 + \frac{1}{1-q} + \frac{1}{1-r} = 0$$

$$\Rightarrow \quad \frac{1}{1-p} + \frac{1}{1-q} + \frac{1}{1-r} = 1$$

8. Given that (α, β) lies on the circle $x^2 + y^2 = 1$.

$\therefore \quad \alpha^2 + \beta^2 = 1$

or it can be rewritten as

$$\frac{1}{9}(9\alpha^2 + 4 + 12\alpha) + \beta^2 = 1 + \frac{1}{9}(4 + 12\alpha)$$

$$\Rightarrow \frac{1}{9}(3\alpha^2 + 2)^2 + \beta^2 = 1 + \frac{4}{9}(1 + 3\alpha + 1) - \frac{4}{9}$$

$$\Rightarrow \frac{1}{9}(3\alpha + 2)^2 + \beta^2 = \frac{5}{9} + \frac{4}{9}(3\alpha + 2)$$

The locus of $(3\alpha + 2, \beta)$ is

$$\frac{1}{9}x^2 + y^2 = \frac{5}{9} + \frac{4}{9}x$$

or $\qquad x^2 - 4x + 9y^2 - 5 = 0$

On comparing this equation with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$\Rightarrow a = 1, b = 9, h = 0, g = -2, f = 0, c = -5$

Now,

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$= 1 \times 9 \times (-5) + 2(0) - 1(0)^2 - 9(-2)^2 - 0$

$= -45 - 36 = -81 \neq 0$

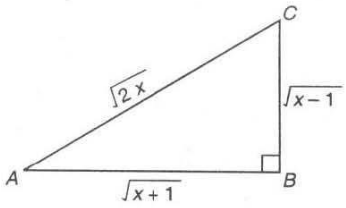
Now, $h^2 - ab = 0 - 9(1) = -9 < 0$

$\therefore \qquad \Delta \neq 0$ and $h^2 < ab$,

Hence, it is an ellipse.

9. Given that

$$\sin\frac{\theta}{2} = \sqrt{\frac{x-1}{2x}}$$



In ΔABC

$$\tan\frac{\theta}{2} = \sqrt{\frac{x-1}{x+1}}$$

$$\therefore \quad \tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$$

$$= \frac{2\sqrt{\frac{x-1}{x+1}}}{1-\frac{x-1}{x+1}} = \frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{2}{x+1}}$$

$$= \sqrt{x^2-1}$$

10. Let

$$f(\theta) = \left[\int_0^{\sin^2\theta} \sin^{-1}\sqrt{\phi}\, d\phi + \int_0^{\cos^2\theta} \cos^{-1}\sqrt{\phi}\, d\phi\right]$$

$$f'(\theta) = \frac{d}{d\theta}\sin^2\theta\left[\sin^{-1}\sqrt{\sin^2\theta}\right] + \frac{d}{d\theta}\cos^2\theta\left[\cos^{-1}\sqrt{\cos^2\theta}\right]$$

$= (2\sin\theta\cos\theta)\theta - (2\sin\theta\cos\theta)\theta$

$= 0$

$\therefore \qquad f(\theta) = \text{constant} = a$ (say)

$$\therefore \qquad f\left(\frac{\pi}{4}\right) = a$$

$$\Rightarrow \int_0^{1/2} \sin^{-1}\sqrt{\phi}\, d\phi + \int_0^{1/2} \cos^{-1}\sqrt{\phi}\, d\phi = a$$

$$\Rightarrow \int_0^{1/2} (\sin^{-1}\sqrt{\phi} + \cos^{-1}\sqrt{\phi})\, d\phi = a$$

$$\Rightarrow \frac{\pi}{2}[\phi]_0^{1/2} = a$$

$$\Rightarrow \frac{\pi}{4} = a$$

11. Let $I = \displaystyle\int_0^{2n\pi} \left\{|\sin x| - \left|\frac{1}{2}\sin x\right|\right\} dx$

$$= \int_0^{2n\pi} \left\{|\sin x| - \frac{1}{2}|\sin x|\right\} dx$$

$$= \int_0^{2n\pi} \frac{1}{2}|\sin x|\, dx$$

$$= \frac{1}{2}\left[\int_0^{2\pi} |\sin x|\, dx + \int_{2\pi}^{4\pi} |\sin x|\, dx + \ldots\ldots + \int_{2(n-1)\pi}^{2n\pi} |\sin x|\, dx\right]$$

Now, $I_1 = \displaystyle\int_0^{2\pi} |\sin x|\, dx$

$$I_1 = \int_0^{\pi} \sin x\, dx - \int_{\pi}^{2\pi} \sin x\, dx$$

$= [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi} = -[-1-1] + [+1+1]$

$= 2 + 2$

$= 4$

$\therefore \qquad I = \frac{1}{2}[4 + 4 + 4 + \ldots n \text{ times}]$

$= \frac{1}{2}(4n) = 2n$

12. Given that $f(x) = \dfrac{x^2}{x^2+1}$

Since, it is an even function therefore its values is always greater than equal to 0 and we know

$$x^2 < x^2 + 1 \quad \text{or} \quad \frac{x^2}{x^2+1} < 1$$

$\therefore$ Required range is $[0, 1)$.

13. Given $\sin^{-1}(1-x) + 2\sin^{-1}x = \dfrac{\pi}{2}$

$$\Rightarrow \sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}(x)$$

$$\Rightarrow (1-x) = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right)$$

$$\Rightarrow (1-x) = \cos(2\sin^{-1}x)$$

$\Rightarrow \quad (1-x) = \cos[\cos^{-1}(1-2x^2)]$

$\Rightarrow \quad (1-x) = 1-2x^2$

$\Rightarrow \quad 2x^2 - x = 0$

$\Rightarrow \quad x = 0, \frac{1}{2}$

But $x = \frac{1}{2}$ does not satisfy the given equation,

So, $x = \{0\}$ is the answer.

14. Since $\sin A$, $\sin B$ and $\cos A$ are in GP

$\therefore \quad \sin^2 B = \sin A \cos A$ ...(i)

$x^2 + 2x \cot B + 1 = 0$ (given)

Now, $b^2 - 4ac = 4\cot^2 B - 4$

$= \frac{4\cos^2 B - 4\sin^2 B}{\sin^2 B} = \frac{4(1-\sin^2 B) - 4\sin^2 B}{\sin^2 B}$

$= \frac{4[1-2\sin^2 B]}{\sin^2 B}$

$= \frac{4[1-2\sin A\cos A]}{\sin^2 B}$ [from (i)]

$= 4\left(\frac{\sin A - \cos A}{\sin B}\right)^2 > 0$

$\therefore$ Roots are always real.

15. Let $I = \int_{\log 2}^{x} \frac{du}{(e^u - 1)^{1/2}}$

or $I = \int_{\log 2}^{x} \frac{e^u}{e^u (e^u - 1)^{1/2}} du$

Let $e^u - 1 = t^2 \Rightarrow e^u\, du = 2t\, dt$

$= \int_{1}^{\sqrt{e^x}-1} \frac{2t}{(t^2+1)t} dt = 2\int_{1}^{\sqrt{e^x}-1} \frac{dt}{(1+t^2)}$

$= [\tan^{-1} t]_1^{\sqrt{e^x - 1}} = 2\tan^{-1}\sqrt{e^x - 1} - \tan^{-1} 1]$

$\Rightarrow \quad 2\left[\tan^{-1}\sqrt{e^x - 1} - \frac{\pi}{4}\right] = \frac{\pi}{6}$ (given)

$\Rightarrow \quad \tan^{-1}\sqrt{e^x - 1} = \frac{\pi}{12} + \frac{\pi}{4} = \frac{\pi}{3}$

$\Rightarrow \quad \sqrt{e^x - 1} = \tan\left(\frac{\pi}{3}\right)$

$\sqrt{e^x - 1} = \sqrt{3}$

$\Rightarrow \quad e^x = 3 + 1 = 4$

16. Since

$(1+x)^{2n+1} = C_0 + C_1 x + \ldots + C_n x^n + C_{n+1} x^{n+1} + \ldots + x^{2n+1}$

$= 2(C_0 + C_1 + \ldots + C_n x^n)$

Put $x = 1$

$(1+1)^{2n+1} = 2(C_0 + C_1 + \ldots + C_n)$

$\Rightarrow \quad 2^{2n} = (C_0 + C_1 + \ldots + C_n)$

$\Rightarrow \quad 2^{2n} - 1 = C_1 + C_2 + \ldots + C_n$

$\Rightarrow \quad 2^{2n} - 1 = 63$

$\Rightarrow \quad 2^{2n} = 64 \Rightarrow 2^{2n} = 2^6$

$\Rightarrow \quad 2n = 6 \Rightarrow n = 3$

17. Given that

$x^2 = xy$

Let $x, y \in R$

$xRy = x^2 = xy$

and $yRz = y^2 = yz$

Now, $x^2y^2 = xy^2z$

$\Rightarrow \quad x^2 = xz$

$\Rightarrow \quad xRz$

$\therefore$ It is transitive.

18. Given that $\det(A) = 6$ ...(i)

Now, $B = 5A^2$

$\Rightarrow \quad \det(B) = \det(5A^2)$

$= 5\det(A^2) = 5\det(A)^2$

$= 5(6)^2$ (from (i))

$\Rightarrow \quad \det(B) = 180$

19. Given that

$f(x) = \begin{cases} 1 & \forall \quad x < 0 \\ 1 + \sin x & \forall \quad 0 \le x \le \pi/2 \end{cases}$

At $x = 0$

$\text{LHD} = \lim_{h\to 0} \frac{f(0-h) - f(0)}{-h}$

$= \lim_{h\to 0} \frac{1-1}{-h} = 0$

$\text{RHD} = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$

$= \lim_{h\to 0} \frac{1 + \sin(0+h) - 1}{h}$

$= \lim_{h\to 0} \frac{\sin h}{h} = 1$

$\Rightarrow \quad \text{LHD} \ne \text{RHD}$

$\therefore \quad f'(x)$ does not exist at $x = 0$.

20. Given curve is $y^2 = 4x$ ...(i)

Let the equation of line by $y = mx + c$

Since $\frac{dy}{dx} = m = 1$ and this line is passing through the point (0, 1).

$\therefore \quad 1 = 1(0) + c \Rightarrow c = 1$

$\therefore \quad y = x + 1$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$(x+1)^2 = 4x$$

$$\Rightarrow \quad (x-1)^2 = 0$$

$$\Rightarrow \quad x = 1 \text{ and } y = 2$$

This shows that line touch the curve at one point. So length of intercept is zero.

21. Given curves are

$$y = x^2 \quad \text{...(i)}$$

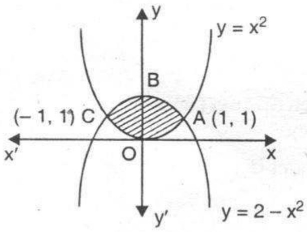
and $\quad y = 2 - x^2$

or $\quad x^2 = -(y-2) \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$x = -1, 1$$

and $\quad y = 1, 1$



$\therefore$ Required area = Area of curve $OABCO$

$$= 2 \text{ Area of curve } OABO$$

$$= 2\int_0^1 y\, dx$$

$$= 2\int_0^1 [(2-x^2)-(x^2)]\, dx$$

$$= 2\int_0^1 (2-2x^2)\, dx$$

$$= 4\left[x - \frac{x^3}{3}\right]_0^1$$

$$= 4\left[1 - \frac{1}{3}\right]$$

$$= \frac{8}{3} \text{ sq units.}$$

22. $\lim_{\theta \to 0} \dfrac{4\theta(\tan\theta - 2\theta\tan\theta)}{1-\cos 2\theta}$

$$= \lim_{\theta \to 0} \frac{4(\theta\tan\theta - 2\theta^2\tan\theta)}{1-\cos 2\theta}$$

Using L' Hospital's rule

$$= \lim_{\theta \to 0} \frac{4(\theta\sec^2\theta + \tan\theta - 4\theta\tan\theta - 2\theta^2\sec^2\theta)}{2\sin 2\theta}$$

Again using L' Hospital's rule

$$4(\sec^2\theta + 2\theta\sec^2\theta\tan\theta + \sec^2\theta - 4\tan\theta)$$

$$= \lim_{\theta \to 0} \frac{-4\theta\sec^2\theta - 4\theta\sec^2\theta - 4\theta^2\sec^2\theta\tan\theta}{4\cos 2\theta}$$

$$= \frac{4(1+0+1)}{4} = 2$$

23. Let $f(x) = 2x^3 - 3x^2 - 12x + 5$

$$\therefore \quad f'(x) = 6x^2 - 6x - 12$$

Put $f'(x) = 0$, for maxima or minima.

$$\therefore \quad 6x^2 - 6x - 12 = 0$$

$$\Rightarrow \quad x^2 - x - 2 = 0$$

$$\Rightarrow \quad x^2 - 2x + x - 2 = 0$$

$$\Rightarrow \quad (x-2)(x+1) = 0$$

$$\Rightarrow \quad x = -1, 2$$

Now, $f''(x) = 12x - 6$

$$f''(-1) = -12 - 6 = -18 < 0$$

$\therefore \quad f(x)$ is maximum at $x = -1$.

But $\quad x = 4$

$$f(x) = 37.$$

$\therefore$ The largest value of $f(x)$ is at $x = 4$

24. Let $z = x + iy$

$$\therefore \quad |z-1| = |z-2| = |z-i|$$

$$\Rightarrow \quad |(x-1)+iy| = |(x-2)+iy|$$

$$= |(x + i(y-1))|$$

$$\Rightarrow x^2 - 2x + 1 + y^2 = x^2 + 4 - 4x + y^2$$

$$= x^2 + y^2 + 1 - 2y$$

Taking Ist and IInd term

$$\Rightarrow \quad -2x + 1 = 4 - 4x$$

$$\Rightarrow \quad 2x = 3 \quad \text{...(i)}$$

Taking IInd and IIIrd term

$$\Rightarrow \quad 4 - 4x = 1 - 2y$$

$$\Rightarrow \quad 4x - 2y = 3 \quad \text{...(ii)}$$

Taking Ist and IIIrd term

$$\Rightarrow \quad -2x + 1 = 1 - 2y$$

$$\Rightarrow \quad 2x - 2y = 0$$

$$\Rightarrow \quad x = y \quad \text{...(iii)}$$

From (i) $x = \dfrac{3}{2}$

On putting value of x in Eq. (iii), we get

$$y = \frac{3}{2}$$

On putting the value of x and y in Eq. (ii), we get

$$4\left(\frac{3}{2}\right) - 2\left(\frac{3}{2}\right) = 3$$

$$\Rightarrow \quad 3 = 3$$

$\therefore$ One solution exist.

25. Given that, $P(A') = 0.3, P(B) = 0.4$

and $P(A \cap B') = 0.5$

$P(B') = 1 - P(B) = 1 - 0.4 = 0.6$

$P(A) = 1 - P(A') = 1 - 0.3 = 0.7$

$P(A \cup B') = P(A) + P(B') - P(A \cap B')$

$= 0.7 + 0.6 - 0.5 = 0.8$

26. $(10101101)_2$

$= 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 128 + 0 + 32 + 0 + 8 + 4 + 0 + 1$

$= 173$

27. Given that $f(x) = \dfrac{e^{2x} - 1}{e^{2x} + 1}$

On differentiating w.r.t. x, we get

$$f'(x) = \frac{2(e^{2x} + 1)(e^{2x}) - 2(e^{2x} - 1)(e^{2x})}{(e^{2x} + 1)^2}$$

$$= \frac{2(e^{2x} + e^{2x})}{(e^{2x} + 1)^2} = \frac{4e^{2x}}{(e^{2x} + 1)^2} > 0$$

$\therefore$ Function is an increasing

28. Given equation is

$$x^2 + y^2 - 2xy \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x^2 + y^2}{2xy} \quad \text{...(i)}$$

This is a homogeneous equation

$\therefore$ we put $y = vx$ and $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

The Eq. (i) is reduces to

$$v + x\frac{dv}{dx} = \frac{x^2(1 + v^2)}{2x^2 v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1 + v^2}{2v} - v = \frac{1 - v^2}{2v}$$

$$\Rightarrow \quad -\frac{2v}{1 - v^2}dv = -\frac{dx}{x}$$

On integrating both sides, we get

$\log(1 - v^2) = -\log x + \log c$

$\Rightarrow \log(x^2 - y^2) - 2\log x = -\log x + \log c$

$\Rightarrow \log(x^2 - y^2) = \log xc$

$\Rightarrow x^2 - y^2 = xc$

29. $\because f(x) = ax^2 + bx + c$

and $g(x) = px^2 + qx$

Since, $g(1) = f(1)$

$\Rightarrow \quad p + q = a + b + c \quad$ ...(i)

and $g(2) - f(2) = 1$

$\Rightarrow \quad 4p + 2q - 4a - 2b - c = 1 \quad$ ...(ii)

also $g(3) - f(3) = 4$

$\Rightarrow \quad 9p + 3q - 9a - 3b - c = 4 \quad$ ...(iii)

From Eqs. (i) and (ii)

$2p = 2a - c + 1$

Now, $g(4) - f(4)$

$= 16p + 4q - 16a - 4b - c$

$= 12p + 4(p + q) - 16a - 4b - c = 6 - 3c$

30. Since the given vectors $\alpha\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \hat{\mathbf{i}} + \beta\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \gamma\hat{\mathbf{k}}$ are coplanar, then

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \beta & 1 \\ 1 & 1 & \gamma \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_2$

$$\Rightarrow \begin{vmatrix} \alpha & 1-\alpha & 0 \\ 1 & \beta - 1 & 1 - \beta \\ 1 & 0 & \gamma - 1 \end{vmatrix} = 0$$

$$\Rightarrow (1-\alpha)(1-\beta)(1-\gamma)\begin{vmatrix} \dfrac{\alpha}{1-\alpha} & 1 & 0 \\ \dfrac{1}{1-\beta} & -1 & 1 \\ \dfrac{1}{1-\gamma} & 0 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (1-\alpha)(1-\beta)(1-\gamma)\left[\frac{\alpha}{1-\alpha}(1) - 1\left(-\frac{1}{1-\beta} - \frac{1}{1-\gamma}\right)\right] = 0$$

But $\alpha \neq 1, \beta \neq 1$ and $\gamma \neq 1$

$$\therefore \quad \frac{1}{(1-\alpha)} - 1 + \frac{1}{1-\beta} + \frac{1}{1-\gamma} = 0$$

$$\Rightarrow \quad \frac{1}{1-\alpha} + \frac{1}{1-\beta} + \frac{1}{1-\gamma} = 1$$

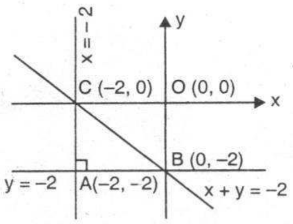
31. Given equation of lines are

$xy + 2x + 2y + 4 = 0$

or $(x + 2)(y + 2) = 0$

or $x + 2 = 0, y + 2 = 0 \quad$ ...(i)

and $x + y + 2 = 0 \quad$ ...(iii)



These three lines makes an right triangle *CAB* right angled at *A*.

The circumcentre of a triangle is the mid point of *BC* i.e. $(-1, -1)$.

32. The centre and radius of the first circle $x^2 + y^2 + 2x + 8y - 23 = 0$ are $C_1\,(-1, -4)$ and $r_1 = \sqrt{40}$

Similarly, the centre and radius of second circle $x^2 + y^2 - 4x - 10y + 9 = 0$ are $C_2\,(2, 5)$ and $r_2 = \sqrt{20}$

Now, $C_1 C_2 = \sqrt{(2+1)^2 + (5+4)^2}$

$= \sqrt{9 + 81} = \sqrt{90}$

and $\quad r_1 + r_2 = \sqrt{40} + \sqrt{20}$

also $\quad r_1 - r_2 = \sqrt{40} - \sqrt{20}$

Here, $r_1 - r_2 < C_1 C_2 < r_1 + r_2$

$\therefore$ Two common tangents can be drawn.

33. Since the line $\frac{x}{\alpha} + \frac{y}{\beta} = 1$ touches the circle $x^2 + y^2 = a^2$.

$\therefore$ The perpendicular distance from centre (0, 0) to the tangent = radius of the circle

$$\Rightarrow \quad \frac{|-1|}{\sqrt{\frac{1}{\alpha^2} + \frac{1}{\beta^2}}} = a$$

$$\Rightarrow \quad \frac{1}{a^2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

The locus of $\left(\frac{1}{\alpha}, \frac{1}{\beta}\right)$ is

$$\frac{1}{a^2} = \frac{1}{x^2} + \frac{1}{y^2}$$

$\therefore$ It represents a circle.

34. Given equation of line is

$$\frac{x-1}{3} = \frac{y+2}{4} = \frac{z-3}{-2} = k \quad \text{(say)}$$

Any point on the line is

$(3k + 1, 4k - 2, -2k + 3)$.

If the given line intersect the plane $2x - y + 3z - 1 = 0$, then any point on the line lies in the plane.

$\therefore 2(3k + 1) - (4k - 2) + 3(-2k + 3) - 1 = 0$

$\Rightarrow \quad -4k + 12 = 0 \Rightarrow k = 3$

$\therefore \quad$ Point is $(9 + 1, 12 - 2, -6 + 3)$

i.e., $(10, 10, -3)$.

35. Equation of director circle of given hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ is $x^2 + y^2 = 25 - 16$

$\Rightarrow \quad x^2 + y^2 = 9 \quad$ ...(i)

This circle passes through $(2\sqrt{2}, 1)$ and we know that director circle is the locus of point of intersection of perpendicular tangents drawn to a hyperbola.

Thus the angle between the tangents is $\pi/2$.

36. Given that, it is given

$\alpha\beta\gamma\delta = 1 \quad$ ...(i)

As, we know A.M. ≥ G.M.

$\Rightarrow \quad \frac{1+\alpha}{2} \geq \sqrt{\alpha}$

$\Rightarrow \quad 1 + \alpha \geq 2\sqrt{\alpha} \quad$ ...(ii)

Similarly, $1 + \beta \geq 2\sqrt{\beta} \quad$ ...(iii)

$1 + \gamma \geq 2\sqrt{\gamma} \quad$ ...(iv)

and $\quad 1 + \delta \geq 2\sqrt{\delta} \quad$ ...(v)

Multiplying Eqs. (ii), (iii), (iv) and (v), we get

$(1+\alpha)(1+\beta)(1+\gamma)(1+\delta) \geq 16\sqrt{\alpha\beta\gamma\delta}$

$\Rightarrow \quad (1+\alpha)(1+\beta)(1+\gamma)(1+\delta) = 16$

37. Given that

$$\sum_{k=1}^{6}\left(\sin\left(\frac{2k\pi}{7}\right) - i\cos\left(\frac{2k\pi}{7}\right)\right)$$

$$= -i\sum_{k=1}^{6}\cos\left(\frac{2k\pi}{7}\right) + i\sin\left(\frac{2k\pi}{7}\right)$$

$$= -i\sum_{k=1}^{6}\left(e^{\frac{2\pi i}{7}}\right)^k$$

$$= -i\sum_{k=1}^{6} r^k \qquad \left(\text{let } r = e^{\frac{2\pi i}{7}}\right)$$

$$= -i\,(r^1 + r^2 + \dots + r^6)$$

$$= -i\,r\,\frac{(1 - r^6)}{1 - r} = \frac{-i(r - r^7)}{1 - r}$$

$$= \frac{-i\,(r-1)}{1-r} = i \qquad [\because r^7 = e^{2\pi i} = 1]$$

38. Given that

$$y\,(x) = 1 + \frac{dy}{dx} = \frac{1}{1\cdot 2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{1\cdot 2\cdot 3}\left(\frac{dy}{dx}\right)^3 + \dots$$

or $$y\,(x) = 1 + \frac{1}{1!}\left(\frac{dy}{dx}\right) + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + \dots$$

$y\,(x) = e^{dy/dx}$

Taking log on both sides, we get

$$\log y\,(x) = \frac{dy}{dx}$$

$\therefore$ The degree of this equation is 1.

39. Given x_1, x_2 are the roots of the equation

$x^2 + 2x - 3 = 0$

$\Rightarrow \quad x^2 + 3x - x - 3 = 0$

$\Rightarrow \quad x(x+3) - 1(x+3) = 0$

$\Rightarrow \quad (x-1)(x+3) = 0$

$\Rightarrow \quad x_1 = -3, \; x_2 = 1$

and y_1, y_2 are the roots of the equation

$y^2 + 4y - 12 = 0$

$\Rightarrow \quad y^2 + 6y - 2y - 12 = 0$

$\Rightarrow \quad y(y+6) - 2(y+6) = 0$

$\Rightarrow \quad (y-2)(y+6) = 0$

$\Rightarrow \quad y_1 = -6, y_2 = 2$

$\therefore$ Points are $P(-3, -6)$ and $Q(1, 2)$.

Since, P and Q are the end points of a diameter.

$\therefore$ Centre = mid point of PQ

$$= \left(\frac{-3+1}{2}, \frac{-6+2}{2}\right)$$

$$= (-1, -2)$$

40. The equation of any plane through (2, –1, 3) is

$a(x-2) + b(y+1) + c(z-3) = 0$ ...(i)

where a, b and c are direction ratios, Since Eq. (i) is parallel to $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$

$\therefore \quad 3a + 0b - c = 0$ ...(ii)

and $\quad -3a + 2b - 2c = 0$ ...(iii)

Solving Eqs. (ii) and (iii), we get

$$\frac{a}{2} = -\frac{b}{6-3} = \frac{c}{6} = k \quad \text{(say)}$$

$\Rightarrow \quad a = 2k, b = -3k, c = 6k$

Putting the values of a, b and c in Eq. (i), we get

$2k(x-2) - 3k(y+1) + 6k(z-3) = 0$

$\Rightarrow \quad 2x - 3y + 6z - 25 = 0$

which is a required equation of a plane.

41. Equation of parabola is

$$y^2 = -4x$$

$\therefore$ focus is (–1, 0).

The equation of line passing through (–1, 0) is

$y - 0 = m(x+1)$ ...(i)

Since, the line makes an angle $\theta = 120°$

$\therefore \quad m = \tan\theta = \tan 120°$

$\Rightarrow \quad m = -\sqrt{3}$

On putting the value of m in Eq. (i), we get

$$y = -\sqrt{3}(x+1)$$

42. Given that

$x = \alpha = \beta,\ y = \alpha\omega + \beta\omega^2,\ z = \alpha\omega^2 + \beta\omega$

Now, $xyz = (\alpha+\beta)(\alpha\omega + \beta\omega^2)(\alpha\omega^2 + \beta\omega)$

$= (\alpha+\beta)(\alpha^2\omega^3 + \alpha\beta\omega^2 + \alpha\beta\omega^4 + \beta^2\omega^3)$

$= (\alpha+\beta)(\alpha^2 + \alpha\beta(\omega^2+\omega) + \beta^2)$

$\left[\because 1 + \omega + \omega^2 = 0 \text{ and } \omega^3 = 1\right]$

$= (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= \alpha^3 + \beta^3$

43. Given that

$$r = \left[2\phi + \cos^2\left(2\phi + \frac{\pi}{4}\right)\right]^{1/2}$$

On differentiating w.r.t ϕ, we get

$$\frac{dr}{d\phi} = \frac{\left[2 - 2\cos\left(2\phi + \frac{\pi}{4}\right)\sin\left(2\phi + \frac{\pi}{4}\right)\cdot 2\right]}{2\sqrt{2\phi + \cos^2\left(2\phi + \frac{\pi}{4}\right)}}$$

$$= \frac{\left[1 - \sin\left(4\phi + \frac{\pi}{2}\right)\right]}{\sqrt{2\phi + \cos^2\left(2\phi + \frac{\pi}{4}\right)}}$$

$$\Rightarrow \left(\frac{dr}{d\phi}\right)_{\phi = \pi/4} = \frac{\left[1 - \sin\left(\pi + \frac{\pi}{2}\right)\right]}{\sqrt{2\cdot\frac{\pi}{4} + \cos^2\left(\frac{\pi}{2} + \frac{\pi}{4}\right)}}$$

$$= \frac{1+1}{\sqrt{\frac{\pi}{2} + \frac{1}{2}}} = 2\sqrt{\frac{2}{1+\pi}}$$

44. Since, $\vec{\alpha}$ lie in the plane of $\vec{\beta}$ and $\vec{\gamma}$.

It means that all three vectors are coplanar.

$$\therefore \quad [\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = 0$$

45. Given that

$$\vec{\alpha} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}},\ \vec{\beta} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$$

and $\quad \vec{\gamma} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Now,

$$\vec{\alpha} \times \vec{\beta} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & -1 \\ -1 & 2 & -4 \end{vmatrix}$$

$= \hat{\mathbf{i}}(-12+2) - \hat{\mathbf{j}}(-8-1) + \hat{\mathbf{k}}(4+3)$

$= -10\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$

and $(\vec{\alpha} \times \vec{\gamma}) = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & -1 \\ 1 & 1 & 1 \end{vmatrix}$

$= \hat{\mathbf{i}}(3+1) - \hat{\mathbf{j}}(2+1) + \hat{\mathbf{k}}(2-3)$

$= -4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$

Now, $(\vec{\alpha} \times \vec{\beta}) \cdot (\vec{\alpha} \times \vec{\gamma})$

$= (-10\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 7\hat{\mathbf{k}}) \cdot (4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= -40 - 27 - 7$

$= -74$

46. Gravitational acceleration is given by

$$g = \frac{GM}{R^2}$$

where G =gravitational constant

$$\therefore \quad \frac{g}{G} = \frac{M}{R^2}$$

47. In this case the internal force is applied on the system, so he will not succeed. According to Newton's law the state of a body can only be changed if some external force is applied on it.

48. $y = \frac{\text{stress}}{\text{strain}} = \text{N/m}^2$ or pascal (in SI system)

and $y = \frac{\text{dyne}}{\text{cm}^2}$ (in CGS System)

Thus, Nm^{-1} is not the unit of Young's modulus.

49. According to Stefan's law the energy emitted by a body per second is directly proportional to the fourth power of the temperature of the body. Here, the temperature of blue glass is more than that of red glass, so it will look brighter.

50. Chemical energy reduced

$$= VIt$$
$$= 6 \times 5 \times 6 \times 60$$
$$= 10800$$
$$= 1.08 \times 10^4 \text{ V}$$

51. Let the original resistance is $R\,\Omega$.

$$\therefore \quad V = IR$$
$$V = 5 \times R = 5R \quad \text{...(i)}$$

When 2Ω resistance is inserted, then total resistance $= (R + 2)\,\Omega$

$$\therefore \quad V = I'(R + 2) = 4(R + 2) \quad \text{...(ii)}$$

From Eqs. (i) and(II), we get

$$5R = 4(R + 2)$$
$$\therefore \quad R = 8\Omega$$
$$= 100 + \frac{75}{4} = \frac{475}{4}\Omega$$
$$= 118.75\Omega$$

52. Let S be the large and R be the smaller resistance.

From formula for metre bridge

$$S = \left(\frac{100 - l}{l}\right) R$$
$$= \frac{100 - 20}{20} R = 4R$$

Again,

$$S = \left(\frac{100 - l}{100}\right)(R + 15)$$
$$= \frac{100 - 40}{40}(R + 15)$$
$$= \frac{3}{2}(R + 15)$$
$$\therefore \quad 4R = \frac{3}{2}(R + 15)$$
$$\frac{8R}{3} - R = 15 \Rightarrow \frac{5R}{3} = 15$$
$$R = 9\Omega$$

53. If we take $R_1 = 4\Omega\; R_2 = 12\Omega$,

then in series resistance

$$R = R_1 + R_2$$
$$= 4 + 12$$
$$= 16\Omega$$

In parallel, resistance $R = \frac{4 \times 12}{4 + 12} = 3\Omega$

So, $R_1 = 4\Omega$ and $R = 12\Omega$

54. Let the resistance of voltmeter is $G\,\Omega$.

$\therefore$ Total resistance of the circuit

$$R = \left(\frac{G \times 100}{G + 100} + 50\right)\Omega$$

Total current $i = \frac{V}{R}$

$$= \frac{10}{\left(\frac{G \times 100}{G + 100} + 50\right)}$$

Voltage across $100\,\Omega$ resistance

$$= i\left(\frac{G \times 100}{G + 100}\right) = \frac{10}{\left(\frac{G \times 100}{G + 100} + 50\right)} \times \left(\frac{G \times 100}{G + 100}\right)$$

Reading of voltmeter = 5 V

$\therefore$ Voltage across $100\,\Omega$ = 5 V

$$\therefore \quad 5 = \frac{10}{\left(\frac{G \times 100}{G + 100} + 50\right)} \times \left(\frac{G \times 100}{G + 100}\right)$$

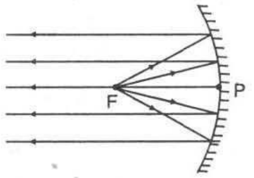
On solving $G = 100\,\Omega$.

55. According to Wien's law

$$\lambda \propto \frac{1}{T}$$

i.e., it depends on the temperature of the surface.

56. It lamp is placed at the focus of concave mirror, then we get parallel beam of light.



58. Here, $E = 1500 \text{V/m}, B = 0.4 \text{ Wb/m}^2$

Minimum speed of electron along the straight line $v = \frac{E}{B}$

$$= \frac{1500}{0.4}$$
$$= 3750$$
$$= 3.75 \times 10^3 \text{m/s}$$

59. Shunt resistance

$$S = \frac{I_g G}{I - I_g}$$
$$= \frac{0.1 G}{1 - 0.1}$$
$$= \frac{G}{9}$$

60. Diamagnetic materials have negative susceptibility. Thus, (c) is wrongly stated.

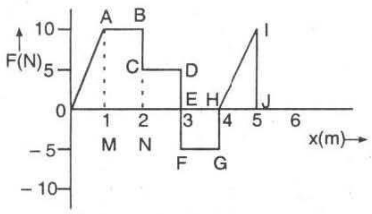
61. The induction coil works on the principle of mutual induction.

62. We know $f = \frac{1}{2\pi\sqrt{LC}}$

or $\sqrt{LC} = \frac{1}{2\pi f} = \text{time.}$

Thus, $\sqrt{LC}$ has the dimension of time.

63. Work done = area enclosed by F-x graph.

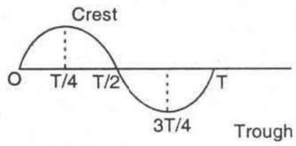


= area of $ABNM$ + area of $CDEN$ – area of $EFGH$ + area of HIJ

$$= 1 \times 10 + 1 \times 5 - 1 \times 5 + \frac{1}{2} \times 1 \times 10$$
$$= 10 + 5 - 5 + 5 = 15 \text{J}$$

64. Both the stones will have the same speed when they hit the ground.

66. The time taken by the particle to come to mean position from the trough = $\frac{T}{4}$



67. Speed = 360 rev/min

$$= \frac{360}{60} \text{ rev/s}$$
$$= 6$$

$\therefore$ Frequency $= 6 \times 60$

$$= 360$$

68. Velocity of sourd $v = \sqrt{\frac{\gamma RT}{M}}$

$$\frac{v_H}{v_O} = \sqrt{\frac{M_O}{M_H}}$$
$$= \sqrt{\frac{16}{1}}$$
$$= 4 : 1$$

69. For sonometer $n \propto \frac{1}{l}$

$$\therefore \quad \frac{n_1}{n_2} = \frac{l_2}{l_1} \Rightarrow \frac{256}{n_2} = \frac{16}{25}$$
$$n_2 = \frac{256 \times 25}{16}$$
$$= 400 \text{ Hz}$$

70. Wave theory of light was first proposed by Christian Huygens.

71. For the liquids, which do not wet the glass, the liquid meniscus is convex upward, so angle of contact is obtuse.

72. Radius of path of electron

$$r = \frac{mv}{Bq}$$

m and q remain unchanged.

So, $\frac{r_1}{r_2} = \frac{v_1}{v_2} \cdot \frac{B_2}{B_1}$

$$= \frac{v}{2v} \cdot \frac{B/2}{B} = \frac{1}{4} \Rightarrow r_2 = 4r$$

73. As $I \propto a^2$ or $a \propto \sqrt{I}$

$$\therefore \quad \frac{a_1}{a_2} = \sqrt{\frac{I}{4I}}$$

$\Rightarrow \quad \dfrac{a_1}{a_2} = \dfrac{1}{2}$

$$\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$$

$$= \left(\frac{1+2}{1-2}\right)^2 = \frac{9}{1}$$

$\therefore \quad I_{max} = 9I, I_{min} = I$

74. In a hydrogen atom the time period is given by

$$T \propto n^3$$

$$\frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^2 \Rightarrow \frac{8}{1} = \left(\frac{n_1}{n_2}\right)^3$$

$\therefore \quad \dfrac{n_1}{n_2} = \dfrac{2}{1}$

Thus, $\quad n_1 = 4$ and $n_1 = 2$

75. On increasing the forward bias voltage, the barrier energy decreases. This results in the flow of majority charge carriers. Hence, width of depletion region decreases.

76. Nuclear forces are charge independent so,

$$F_1 = F_2 = F_3.$$

79. Potential $V = \dfrac{Q}{C} \Rightarrow V = \dfrac{Q}{\dfrac{A\,\varepsilon_0}{d}}$

Hence, potential depends on the amount of charge, area or geometry and size of the conductor.

78. The potential at each point on the circular path will be equal.

So, work done $= q \times$ potential difference

$= q \times 0$

$= 0$

79. Capacitance with air

$$C = \frac{A\varepsilon_0}{d}$$

When interspace between the plates is filled with wax, then

$$C' = \frac{KA\varepsilon_0}{2d}$$

or $\quad C' = \left(\dfrac{A\varepsilon_0}{d}\right)\dfrac{K}{2}$

or $\quad C' = C\dfrac{K}{2}$

$\therefore \quad 6 = 2 \cdot \dfrac{K}{2} \Rightarrow K = 6$

80. de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$\therefore \quad \dfrac{\lambda_1}{\lambda_2} = \sqrt{\dfrac{E_2}{E_1}} \Rightarrow \dfrac{1 \times 10^{-9}}{0.5 \times 10^{-9}} = \sqrt{\dfrac{E_2}{E_1}}$

$\Rightarrow \quad 2 = \sqrt{\dfrac{E_2}{E_1}} \Rightarrow \dfrac{E_2}{E_1} = 4$

$\therefore \quad E_2 = 4E_1$

$\therefore$ Energy to be added $= E_2 - E_1$

$= 4E_1 - E_1 = 3E_1$

81. Half-life $T/2 = \dfrac{T}{1.44} = \dfrac{100}{1.44}$ s

$= 69.44$ s

$= \dfrac{69.44}{60} \approx 1.155$ min

82. Radioactive decay does not depend upon the time of creation.

84. Coulomb's law is appliable for charged particles, it is not responsible to bind the protons and neutrons in the nucleus of an atom.

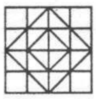
85. If unpolarised light is incident at polarising angle, then reflected light is completely, *i.e,* 100% polarised.

Reasoning

141. (c) Second is the result of the first.

142. (b) In all other groups, the first second and third letters are respectively moved one, five and one step forward to obtain second, third and fourth letters respectively.

143. Clearly, fig. (d) when placed in the blank space of fig (x) will complete the pattern, as shown below.



Hence, the answer is (d).

145. In fig. X, the right half of the rectangular paper sheet is folded over the left half. In fig. Y, two semicircles are punched into the folded paper. When the paper is unfolded, the semicircles in the two halves will join to form circles. Thus, two circles will appear in the unfolded position of fig. Y.

Hence, fig. (d) is the correct answer.

$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{1}{2}$$

$$\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$$

$$= \left(\frac{1+2}{1-2}\right)^2 = \frac{9}{1}$$

$$\therefore \quad I_{max} = 9I, I_{min} = I$$

74. In a hydrogen atom the time period is given by

$$T \propto n^3$$

$$\frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^2 \Rightarrow \frac{8}{1} = \left(\frac{n_1}{n_2}\right)^3$$

$$\therefore \quad \frac{n_1}{n_2} = \frac{2}{1}$$

Thus, $n_1 = 4$ and $n_1 = 2$

75. On increasing the forward bias voltage, the barrier energy decreases. This results in the flow of majority charge carriers. Hence, width of depletion region decreases.

76. Nuclear forces are charge independent so,

$$F_1 = F_2 = F_3.$$

79. Potential $V = \frac{Q}{C} \Rightarrow V = \frac{Q}{\frac{A\,\varepsilon_0}{d}}$

Hence, potential depends on the amount of charge, area or geometry and size of the conductor.

78. The potential at each point on the circular path will be equal.

So, work done $= q \times$ potential difference

$$= q \times 0$$

$$= 0$$

79. Capacitance with air

$$C = \frac{A\varepsilon_0}{d}$$

When interspace between the plates is filled with wax, then

$$C' = \frac{KA\varepsilon_0}{2d}$$

or $$C' = \left(\frac{A\varepsilon_0}{d}\right)\frac{K}{2}$$

or $$C' = C\frac{K}{2}$$

$$\therefore \quad 6 = 2\cdot\frac{K}{2} \Rightarrow K = 6$$

80. de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}} \Rightarrow \frac{1 \times 10^{-9}}{0.5 \times 10^{-9}} = \sqrt{\frac{E_2}{E_1}}$$

$$\Rightarrow \quad 2 = \sqrt{\frac{E_2}{E_1}} \Rightarrow \frac{E_2}{E_1} = 4$$

$$\therefore \quad E_2 = 4E_1$$

$\therefore$ Energy to be added $= E_2 - E_1$

$$= 4E_1 - E_1 = 3E_1$$

81. Half-life $T/2 = \frac{T}{1.44} = \frac{100}{1.44}$ s

$$= 69.44 \text{ s}$$

$$= \frac{69.44}{60} \approx 1.155 \text{ min}$$

82. Radioactive decay does not depend upon the time of creation.

84. Coulomb's law is appliable for charged particles, it is not responsible to bind the protons and neutrons in the nucleus of an atom.

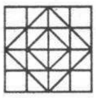
85. If unpolarised light is incident at polarising angle, then reflected light is completely, *i.e,* 100% polarised.

Reasoning

141. (c) Second is the result of the first.

142. (b) In all other groups, the first second and third letters are respectively moved one, five and one step forward to obtain second, third and fourth letters respectively.

143. Clearly, fig. (d) when placed in the blank space of fig (x) will complete the pattern, as shown below.



Hence, the answer is (d).

145. In fig. X, the right half of the rectangular paper sheet is folded over the left half. In fig. Y, two semicircles are punched into the folded paper. When the paper is unfolded, the semicircles in the two halves will join to form circles. Thus, two circles will appear in the unfolded position of fig. Y.

Hence, fig. (d) is the correct answer.