

Mathematics
Sample Question Paper
Class XII

Class:12
Time 3hrs

Max Mks:100
No of pages:4

General Instructions:

- All questions are compulsory.
- The question paper consists of 29 questions divided into three sections - A, B and C.
- Section - A comprises of 10 questions of one mark each.
- Section - B is of 12 questions of four marks each.
- Section - C comprises of 7 questions of six marks each.
- Internal choice has been provided in four marks question and six marks question.
- You have to attempt any one of the alternatives in all such questions .
- Use of calculator not permitted.

SECTION A

1. If A is an invertible matrix of order 5 and $|A| = 5$, then find $[\text{adj}, A]$
2. Show that $f(x) = x^2 + x$ is an odd function

3. Find the value of x, if

$$\begin{bmatrix} 4x + y & -y \\ 2y - x & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -5 & 6 \end{bmatrix}$$

4. If $|\vec{a}| = 2$, $|\vec{b}| = \sqrt{3}$ and angle between \vec{a} and \vec{b} is 45° , find $\vec{a} \cdot \vec{b}$.

5. What is the value of $\tan^{-1} \frac{3\pi}{4}$.

6. Evaluate $\sin(\tan^{-1}(x))$, $|x| < 1$.

7. Evaluate $\int \left(\frac{2x^2}{1+3x} \right) dx$

8. Write the domain of $\tan^{-1}(x)$.

9. Write the order of the matrix $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 6 & 4 \end{bmatrix}$

10. Find the value of λ , so that $\begin{bmatrix} 7 & 1 \\ 2 & \lambda \end{bmatrix}$

SECTION B

11. Prove that $\tan^{-1} \left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \right) = \frac{\pi}{2}$

12. Using properties of determinants, prove that:

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a^2 + b^2 + c^2)(a+b+c)(b-c)(c-a)(a-b)$$

13. Find $\frac{dy}{dx}$ if $y = \sqrt{\frac{1+\tan x}{1-\tan x}}$

14. State whether the function is one-one, onto or objective. Justify your answer

a) $f: R \rightarrow R$ defined by $f(x) = 4 + 5x$

b) $f: R \rightarrow R$ $f(x) = 4x + 5x^2$

15. A line makes angles $\alpha, \beta, \gamma, \delta$ with the diagonals of a cube, prove that

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$$

16. A bag contains 5 red, 6 white and 7 black balls. Two balls are drawn at random. What is the probability that (i) both are black (ii) both are red?

17. Determine if the function f defined by:
$$f(x) = \begin{cases} x^2 \sin x & x \neq 0 \\ 0 & x = 0 \end{cases}$$
 is continuous.

18. Solve the differential equation:

$$x \, dy - y \, dx = \sqrt{x^2 + y^2} \text{ or}$$

$$\frac{dy}{dx} = \log(x+2)^2$$

19. Determine k s.t. the function f defined by:

$$f(x) = \begin{cases} kx + 1, & x \leq 5 \\ 3x - 5, & x > 5 \end{cases} \text{ is continuous at } x=5.$$

20. Show that $y = \frac{4\sin\theta}{2 + \cos\theta} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$.

21. Define dot product of vectors and find the value of λ such that

$$\vec{a} \perp \vec{b} \text{ where } \vec{a} = 3\vec{i} + 3\vec{j} + \lambda\vec{k} \text{ and } \vec{b} = \lambda\vec{i} - \vec{j} + 4\vec{k}$$

22. Integrate $\int \sin 3x \cos 5x \, dx$.

SECTION C

23. Evaluate $\int \frac{1-x^2}{x(1-2x)} \, dx$.

24. A manufacturer produces two types of steel trunks. He has machines A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type of trunk requires 3 hours on machine A and 2 hours on machine B. Machine A and B can work for 18 and 15 hours at most in a day. He earns a profit of Rs. 30 on the first trunk and a profit of Rs.25 per trunk on the second trunk. How many trunks of each type must be made each day to maximize his profit.

25. Find the area of the region bounded by $y^2 = 4x, x = 1, x = 4$ and x axis in the first quadrant.

or

Evaluate $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n (x^2 + x + 1)$ as limit of a sum.

26. Find the area bounded by $x^2 = 4y$ and the line $x = 4y - 2$.

27. Find the equation of the plane passing through the $(-1, -1, 2)$ and perpendicular to each of the following planes $2x + 3y - 3z = 2$ and $5x - 4y + z = 6$

or

Find the point on the curve $y^2 = 4x$ which is the nearest point $(-1, -8)$

28. A can hit a target 4 times in 5 shots, B 3 times in 4 shots and C, 2 times in 3 shots. Calculate the probability that: a) A, B, C all may hit b) none of them ii hit the target.

29. Show that the height of a closed cylinder of given volume and minimum surface area is equal to its diameter.