

Marking Scheme
Mathematics (Term-I)
Class-XII (Code-041)

Q.N.	Correct Option	Hints / Solutions
1	d	$\sin\left(\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right) = \sin\left(\frac{\pi}{2}\right) = 1$
2	b	$\lim_{x \rightarrow 0} \left(\frac{1 - \cos kx}{x \sin x}\right) = \frac{1}{2}$ $\Rightarrow \lim_{x \rightarrow 0} \left(\frac{2 \sin^2 \frac{kx}{2}}{x \sin x}\right) = \frac{1}{2}$ $\Rightarrow \lim_{x \rightarrow 0} 2 \left(\frac{k}{2}\right)^2 \left(\frac{\sin \frac{kx}{2}}{\frac{kx}{2}}\right)^2 \left(\frac{x}{\sin x}\right) = \frac{1}{2}$ $\Rightarrow k^2 = 1 \Rightarrow k = \pm 1 \text{ but } k < 0 \Rightarrow k = -1$
3	d	$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
4	c	As A is singular matrix $\Rightarrow A = 0$ $\Rightarrow 2k^2 - 32 = 0 \Rightarrow k = \pm 4$
5	b	$f(x) = x^2 - 4x + 6$ $f'(x) = 2x - 4$ $\text{let } f'(x) = 0 \Rightarrow x = 2$ $\longleftarrow \quad \quad \longrightarrow$ $-\infty \quad (-) \quad 2 \quad (+) \quad \infty$ as $f'(x) > 0 \quad \forall \quad x \in (2, \infty)$ $\Rightarrow f(x)$ is Strictly increasing in $(2, \infty)$
6	d	as $ adj A = A ^{n-1}$, where n is order of the square matrix A $= (-4)^2 = 16$
7	b	(1, 2)
8	a	$2a + b = 4 \quad a - 2b = -3 \quad 5c - d = 11 \quad 4c + 3d = 24 \quad \} \Rightarrow a = 1 \quad b = 2 \quad c = 3 \quad d = 4$ $\therefore a + b - c + 2d = 8$
9	a	$f(x) = x + \frac{1}{x}, x > 0 \Rightarrow f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}, x > 0$ As normal to the curve $y = f(x)$ at some point (x, y) is \perp to given line $\Rightarrow \left(\frac{x^2}{1-x^2}\right) \times \frac{3}{4} = -1 \quad (m_1 \cdot m_2 = -1)$ $\Rightarrow x^2 = 4 \Rightarrow x = \pm 2$ But $x > 0, \therefore x = 2$ Therefore point = $\left(2, \frac{5}{2}\right)$
10	d	$\sin(\tan^{-1}x) = \sin\left\{\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right\} = \frac{x}{\sqrt{1+x^2}}$
11	a	{1, 5, 9}
12	c	$e^x + e^y = e^{x+y}$ $\Rightarrow e^{-y} + e^{-x} = 1$ Differentiating w.r.t. x:

		$\Rightarrow -e^{-y} \frac{dy}{dx} - e^{-x} = 0 \Rightarrow \frac{dy}{dx} = -e^{y-x}$				
13	b	3×5				
14	a	$y = 5\cos x - 3\sin x \Rightarrow \frac{dy}{dx} = -5\sin x - 3\cos x$ $\Rightarrow \frac{d^2y}{dx^2} = -5\cos x + 3\sin x = -y$				
15	c	$adj A = \begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix} \Rightarrow (adj A)' = \begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$				
16	c	$\frac{x^2}{9} + \frac{y^2}{16} = 1 \Rightarrow \frac{2x}{9} + \frac{2y}{16} \frac{dy}{dx} = 0$ \Rightarrow slope of normal at any point (x, y) to the curve $= \frac{-dx}{dy} = \frac{9y}{16x}$ As tangent to the curve at the point (x, y) is parallel to y -axis $\Rightarrow \frac{9y}{16x} = 0 \Rightarrow y = 0$ and $x = \pm 3$ \therefore points $= (\pm 3, 0)$				
17	b	$ A = -7$ $\therefore \sum_{i=1}^3 a_{i2}A_{i2} = a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32} = A = -7$				
18	d	$y = \log(\cos e^x)$ Differentiating w.r.t. x : $\frac{dy}{dx} = \frac{1}{\cos(e^x)} \cdot (-\sin e^x) \cdot e^x$ (chain rule) $\Rightarrow \frac{dy}{dx} = -e^x \tan e^x$				
19	d	Z is maximum 180 at points $C(15, 15)$ and $D(0, 20)$. $\Rightarrow Z$ is maximum at every point on the line segment CD				
20	c	$f(x) = 2\cos x + x, x \in [0, \frac{\pi}{2}]$ $f'(x) = -2\sin x + 1$ Let $f'(x) = 0 \Rightarrow x = \frac{\pi}{6} \in [0, \frac{\pi}{2}]$ $f(0) = 2$ $f(\frac{\pi}{6}) = \frac{\pi}{6} + \sqrt{3}$ $f(\frac{\pi}{2}) = \frac{\pi}{2} \Rightarrow$ least value of $f(x)$ is $\frac{\pi}{2}$ at $x = \frac{\pi}{2}$				
Section-B						
21	d	<table border="0" style="width: 100%;"> <tr> <td style="width: 50%; vertical-align: top;"> $let f(x_1)$ $= f(x_2)$ such that $x_1, x_2 \in R$ $\Rightarrow x_1^3 = x_2^3$ $\Rightarrow x_1 = x_2$ $\Rightarrow f$ is one - one </td> <td style="width: 50%; vertical-align: top;"> $Let y \in R(\text{codomain}).$ Then for any $x, f(x) = y$ if $x^3 = y$ i.e., $x = y^{\frac{1}{3}} \in R(\text{domain})$ i.e., every element $y \in R(\text{codomain})$ has a pre image $y^{\frac{1}{3}}$ in $R(\text{domain})$ $\Rightarrow f$ is onto </td> </tr> <tr> <td colspan="2" style="text-align: center;">$\therefore f$ is one-one and onto</td> </tr> </table>	$let f(x_1)$ $= f(x_2)$ such that $x_1, x_2 \in R$ $\Rightarrow x_1^3 = x_2^3$ $\Rightarrow x_1 = x_2$ $\Rightarrow f$ is one - one	$Let y \in R(\text{codomain}).$ Then for any $x, f(x) = y$ if $x^3 = y$ i.e., $x = y^{\frac{1}{3}} \in R(\text{domain})$ i.e., every element $y \in R(\text{codomain})$ has a pre image $y^{\frac{1}{3}}$ in $R(\text{domain})$ $\Rightarrow f$ is onto	$\therefore f$ is one-one and onto	
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22	a	$x = a \sec \theta \Rightarrow \frac{dx}{d\theta} = a \sec \theta \tan \theta$ $y = b \tan \theta \Rightarrow \frac{dy}{d\theta} = b \sec^2 \theta$ $\therefore \frac{dy}{dx} = \frac{b}{a} \operatorname{cosec} \theta$ $\Rightarrow \frac{d^2y}{dx^2} = \frac{-b}{a} \operatorname{cosec} \theta \cdot \cot \theta \cdot \frac{d\theta}{dx} = \frac{-b}{a^2} \cot^3 \theta$ $\therefore \left. \frac{d^2y}{dx^2} \right _{\theta = \frac{\pi}{6}} = \frac{-3\sqrt{3}b}{a^2}$				
23	c	Z is minimum -24 at $(0, 8)$				
24	a	$let u = \sin^{-1}(2x\sqrt{1-x^2})$				

		<p>and $v = \sin^{-1}x$, $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ $\Rightarrow \sin v = x \dots\dots(1)$ Using (1), we get : $= \sin^{-1}(2\sin v \cos v) = \sin^{-1}(\sin 2v)$ $\Rightarrow u = 2v, -\frac{\pi}{2} < 2v < \frac{\pi}{2}$ Differentiating u with respect to v, we get: $\frac{du}{dv} = 2$</p>								
25	d	$AB = 6I \Rightarrow B^{-1} = \frac{1}{6}A$								
26	b	$f'(x) = 6(x^2 - x - 6) = 6(x - 3)(x + 2)$ As $f'(x) < 0 \forall x \in (-2, 3)$ $\Rightarrow f(x)$ is strictly decreasing in $(-2, 3)$								
27	a	$\tan^{-1} \left(\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}} \right)$ $= \tan^{-1} \left(\frac{-\sqrt{2}\cos\frac{x}{2} + \sqrt{2}\sin\frac{x}{2}}{-\sqrt{2}\cos\frac{x}{2} - \sqrt{2}\sin\frac{x}{2}} \right)$, $\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$ $= \tan^{-1} \left(\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}} \right) = \tan^{-1} \left(\frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}} \right) = \tan^{-1} \left(\tan \left(\frac{\pi}{4} - \frac{x}{2} \right) \right)$ $= \frac{\pi}{4} - \frac{x}{2}$, $-\frac{\pi}{4} > \frac{\pi}{4} - \frac{x}{2} > -\frac{\pi}{2}$								
28	c	$A^2 = 2A$ $\Rightarrow A^2 = 2A $ $\Rightarrow A ^2 = 2^3 A $ as $ kA = k^n A $ for a square matrix of order n \Rightarrow either $ A = 0$ or $ A = 8$ But A is non-singular matrix $\therefore A = 8^2 = 64$								
29	b	$f'(x) = 1 - \sin x \Rightarrow f'(x) \geq 0 \forall x \in R$ \Rightarrow no value of b exists								
30	c	$a = b - 2$ and $b > 6$ $\Rightarrow (6, 8) \in R$								
31	a	$f(x) = \begin{cases} \frac{x}{-x} = -1, & x < 0 \\ -1, & x \geq 0 \end{cases}$ $\Rightarrow f(x) = -1 \forall x \in R$ $\Rightarrow f(x)$ is continuous $\forall x \in R$ as it is a constant function								
32	b	$kA = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$ $\Rightarrow k = -6, a = -4$ and $b = -9$								
33	c	Corner points of feasible region $Z = 30x + 50y$ <table style="margin-left: 40px;"> <tr><td>(5,0)</td><td>150</td></tr> <tr><td>(9,0)</td><td>270</td></tr> <tr><td>(0,3)</td><td>150</td></tr> <tr><td>(0,6)</td><td>300</td></tr> </table> <p>Minimum value of Z occurs at infinitely many points</p>	(5,0)	150	(9,0)	270	(0,3)	150	(0,6)	300
(5,0)	150									
(9,0)	270									
(0,3)	150									
(0,6)	300									
34	c	$f'(x) = \frac{-2x^2 - 10x + 100}{\sqrt{100 - x^2}}$ $f'(x) = 0 \Rightarrow x = -10$ or 5 , But $x > 0 \Rightarrow x = 5$ $f''(x) = \frac{2x^3 - 300x - 1000}{(100 - x^2)^{\frac{3}{2}}} \Rightarrow f''(5) = \frac{-30}{\sqrt{75}} < 0$ \Rightarrow Maximum area of trapezium is $75\sqrt{3} \text{ cm}^2$ when $x = 5$								
35	d	$(I - A)^3 - 7A = I + A + 3A + 3A - 7A = I$								
36	c	$\frac{-\pi}{2} < y < \frac{\pi}{2}$								

37	b	Since, distinct elements of A have distinct f-images in B. Hence, f is injective and every element of B does not have its pre-image in A, hence f is not surjective. $\therefore f$ is injective and is not surjective.
38	b	$ A = 7, \text{adj}A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ $\therefore 14A^{-1} = 14 \times \frac{1}{7} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$
39	b	$y = x^3 - 11x + 5 \Rightarrow \frac{dy}{dx} = 3x^2 - 11$ Slope of line $y = x - 11$ is 1 $\Rightarrow 3x^2 - 11 = 1 \Rightarrow x = \pm 2$ \therefore point is (2, -9) as (-2, 19) does not satisfy the equation of the given line
40	c	$A^2 = 3I$ $\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \Rightarrow 3 - \alpha^2 - \beta\gamma = 0$
Section C		
41	a	As Z is maximum at (30, 30) and (0, 40) $\Rightarrow 30a + 30b = 40b \Rightarrow b - 3a = 0$
42	b	$y = mx + 1 \dots (1)$ and $y^2 = 4x \dots (2)$ Substituting (1) in (2): $(mx + 1)^2 = 4x$ $\Rightarrow m^2x^2 + (2m - 4)x + 1 = 0 \dots (3)$ As line is tangent to the curve \Rightarrow line touches the curve at only one point $\Rightarrow (2m - 4)^2 - 4m^2 = 0 \Rightarrow m = 1$
43	c	Let $f(x) = [x(x - 1) + 1]^{\frac{1}{3}}, 0 \leq x \leq 1$ $f'(x) = \frac{2x-1}{3(x^2-x+1)^{\frac{2}{3}}}$ let $f'(x) = 0 \Rightarrow x = \frac{1}{2} \in [0, 1]$ $f(0) = 1, f\left(\frac{1}{2}\right) = \left(\frac{3}{4}\right)^{\frac{1}{3}}$ and $f(1) = 1$ \therefore Maximum value of $f(x)$ is 1
44	b	Feasible region is bounded in the first quadrant
45	d	$ A = 2 + 2\sin^2\alpha$ As $-1 \leq \sin\alpha \leq 1, \forall 0 \leq \alpha \leq 2\pi$ $\Rightarrow 2 \leq 2 + 2\sin^2\alpha \leq 4 \Rightarrow A \in [2, 4]$
46	d	Fuel cost per hour = $k(\text{speed})^2$ $\Rightarrow 48 = k \cdot 16^2 \Rightarrow k = \frac{3}{16}$
47	b	Total cost of running train (let C) = $\frac{3}{16}v^2t + 1200t$ Distance covered = 500km $\Rightarrow \text{time} = \frac{500}{v}$ hrs Total cost of running train 500 km = $\frac{3}{16}v^2\left(\frac{500}{v}\right) + 1200\left(\frac{500}{v}\right)$ $\Rightarrow C = \frac{375}{4}v + \frac{600000}{v}$
48	c	$\frac{dC}{dv} = \frac{375}{4} - \frac{600000}{v^2}$ Let $\frac{dC}{dv} = 0 \Rightarrow v = 80$ km/h
49	c	Fuel cost for running 500 km $\frac{375}{4}v = \frac{375}{4} \times 80 = \text{Rs. } 7500/-$
50	d	Total cost for running 500 km = $\frac{375}{4}v + \frac{600000}{v}$ $= \frac{375 \times 80}{4} + \frac{600000}{80} = \text{Rs. } 15000/-$